basic radio

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FUNDAMENTALS OF RADIO TELEMETRY

VOL. 2

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PREFACE

The purpose of this book is to fill a need for a text stating in plain, everyday language, what many people consider a complex technical subject. A technical subject need not be complex. Careful filling in of the background with essential information, and then leading step by step to the final explanation, provides a logical method of explaining the most difficult subject.

It would be impossible to cover in a single book or series of books, the immense scope implied in the word *electronics*. However, an understanding of radio circuits serves as a foundation for advanced study in all fields of electronics, such as television, radar, computers, etc. For teaching radio, the all-important basic tool of electronics, most available textbooks are woefully inadequate. One type contains information so brief as to acquaint rather than instruct. Another type is based on the premise that teaching a student to design a circuit is the best method of having him understand that circuit's operation.

Basic Radio represents the neglected middle ground. It is a course in radio communications, as distinct from a general course in electronics. The text deals with the circuitry and techniques used for the transmission and reception of intelligence via radio energy. Assuming no prior knowledge of electricity or electronics, the six volumes of this course "begin at the beginning" and carry the reader in logical steps through a study of electricity and electronics as required for a clear understanding of radio receivers and transmitters. Illustrations are used on every page to reinforce the highlights of that page. All examples given are based on actual or typical circuitry to make the course as practical and realistic as possible. Most important, the text provides a solid foundation upon which the reader can build his further, more advanced knowledge of electronics.

The sequence of *Basic Radio* first establishes a knowledge of d-c electricity. Upon this is built an understanding of the slightly more involved a-c electricity. Equipped with this information the reader is ready to study the operation of electron tubes and electron tube circuits, including power supplies, amplifiers, oscillators, etc. Having covered the components of electronic circuitry in Volumes 1 through 3, we assemble these components

in Volume 4, and develop the complete radio receiver, AM and FM. In Volume 5 we recognize the development of the transistor, and devote the entire volume to the theory and circuitry of transistor receivers and semiconductors. The last volume of the course, Volume 6, covers the long-neglected subject of transmitters, antennas, and transmission lines.

No prior knowledge of algebra, electricity, or any associated subject is required for the understanding of this series; it is self-contained. Embracing a vast amount of information, it cannot be read like a novel, skimming through for the high points. Each page contains a carefully selected thought or group of thoughts. Readers should take advantage of this, and study each individual page as a separate subject.

Whenever someone is presented with an award he gives thanks and acknowledgement to those "without whose help . . ." etc. It is no different here. The most patient, and long-suffering was my wife Celia, who typed, and typed, and typed. To her, the editorial staff of John F. Rider, and others in the "background", my heartfelt thanks and gratitude for their assistance and understanding patience.

MARVIN TEPPER

Malden, Mass.
September 1961

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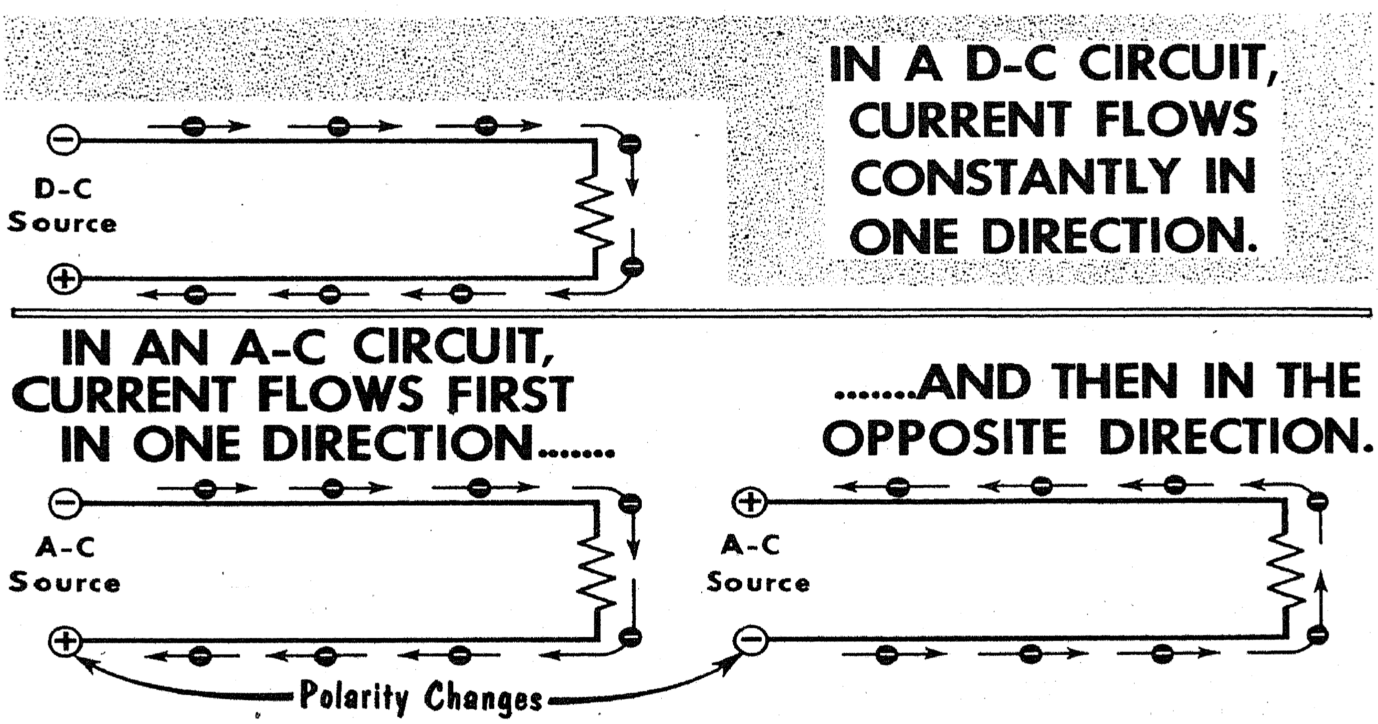
VOL. II — BASIC RADIO

Introduction to AC

In Volume 1, we covered the subject of d-c electricity. In d-c circuits, the polarity of the voltage source remained constant, as did the difference of potential, or voltage. Under these conditions, electron flow was always in one direction, from minus to plus, and of constant quantity. In alternating current (a-c) electricity, we have a condition where the polarity of the voltage source is constantly changing. What was the positive terminal at one instant becomes the negative terminal some time later; what was the negative terminal at one instant becomes the positive terminal some time later. As a result of the constantly changing polarity of the voltage source, the direction of electron flow in the circuit also keeps reversing. In addition to reversing direction, current in an a-c circuit will also keep varying in quantity--from zero to maximum in one direction and back to zero, and from zero to maximum in the opposite direction and then back to zero. Thus, the alternating voltage will cause an alternating current.

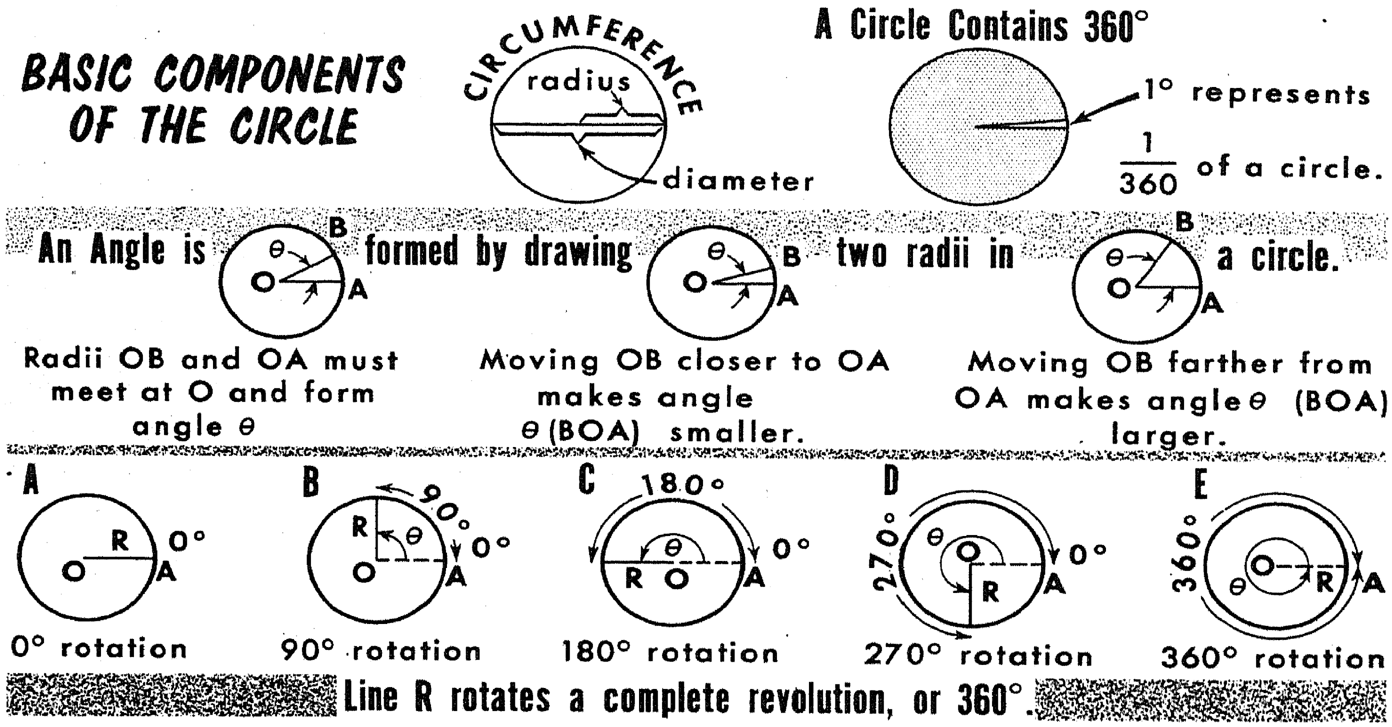


A-c electricity is not "better" than d-c; it is another type of electricity that has certain advantages. With a-c we can use transformers which enable us to transform a-c voltage to as high or as low a voltage as we wish. This permits efficient distribution of electrical power. In addition, there are many kinds of electrical components and devices that can do certain "jobs" in a-c circuits that cannot be done in a d-c circuit. An important point to remember is that a-c does not replace d-c.

A-c makes possible radio communication. However, most of the circuits in a-c communications equipment are controlled by d-c voltages. Because a-c involves constantly changing voltages and currents, we must give a little more thought to them. Thus, we begin our study of a-c with some fundamental mathematics which will help us in this study.

The Circle--Angular Rotation

The circle is a simple figure, and yet it represents an important consideration in our study of a-c electricity. Let us review it briefly. The constantly curving line that forms the circle is called the circumference. If we draw a straight line from the center of a circle to any point on the circumference, that line is called a radius. Any line drawn through the center of the circle and dividing the circle in half is called the diameter. Looking further into the circle, we find that the circle is divided into degrees; let us see how they are formed.

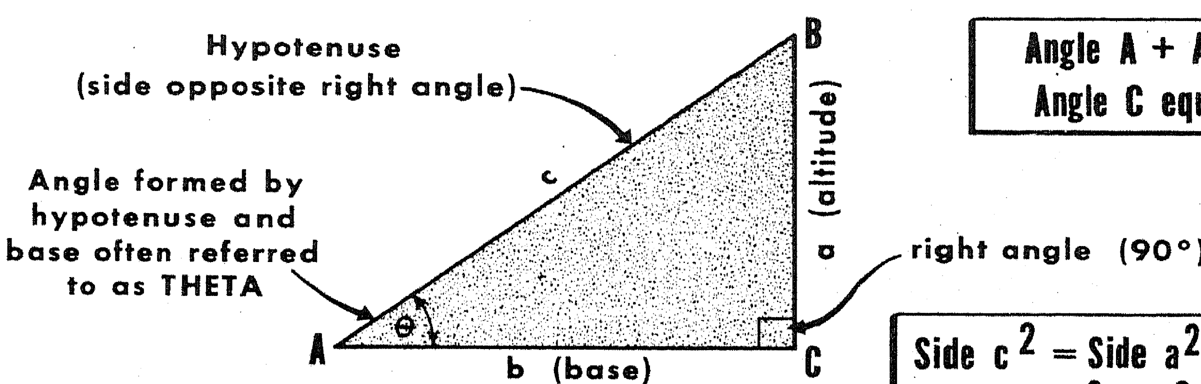


From the center of the circle, we draw a radius to the circumference and call that line OA. We will keep this line in this position and use it as a reference line. We now draw a second radius, OB. The position of OB to OA forms an angle. We refer to this angle as angle BOA, with point O being the vertex or origin of lines OA and OB. If we move line OB closer to OA, the angle thus formed becomes smaller; if OB is moved farther away from OA, the angle becomes larger. If line OB is rotated farther and farther away from our reference line OA, it will spin past the entire circumference and end up overlapping line OA. The entire rotation of a radius from one point on the circumference, all around the circumference, and back to the starting point, covers 360°. Thus, if line R started at position OA and rotated until it pointed straight up on the page, it would have rotated 90°, and we would say that it formed a 90° angle with line OA. If we keep rotating the radius 90° more, line R would now form a diameter together with OA, and we call this a straight angle, or 180°. As line R now moves downward to the bottom of the page, it has gone through 90° more, or a total of 270°. Finally, we rotate it 90° more until it reaches its starting point. In one complete rotation, line OB has moved through 360°. The angle formed by the rotating line R with respect to line OA is given the name "theta," after the Greek letter (θ).

The Right Triangle

The right triangle is a special triangle in that one of its three angles is a right (90°) angle. The number of degrees included in the three angles of any triangle is 180. Thus, since one of the angles of a right triangle is equal to 90°, the sum of the remaining two angles must be 90°. In studying the right triangle, we assign a particular group of names to the various sides and angles. The side that lies horizontal to the page is called the base (b), the vertical side is called the altitude (a), and the side opposite the right angle is called the hypotenuse. The length of these sides has a particular relationship--the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides ($c^2 = a^2 + b^2$). Also, the length of the hypotenuse is greater than the length of either of the other sides but is less than their sum.

THE RIGHT TRIANGLE



Angle A + Angle B + Angle C equals 180°

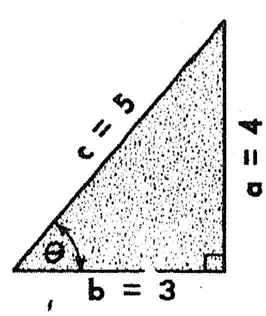
Side c^2 = Side a^2 + Side b^2 or $c^2 = a^2 + b^2$

RIGHT TRIANGLE RELATIONSHIPS

$$\text{Sine of } \theta = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\text{Cosine of } \theta = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{Tangent of } \theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$$



$$\text{Sine } \theta = \frac{4}{5} \qquad \text{Cosine } \theta = \frac{3}{5} \qquad \text{Tangent } \theta = \frac{4}{3}$$

$$\boxed{c^2 = a^2 + b^2}$$

$$5^2 = 4^2 + 3^2$$

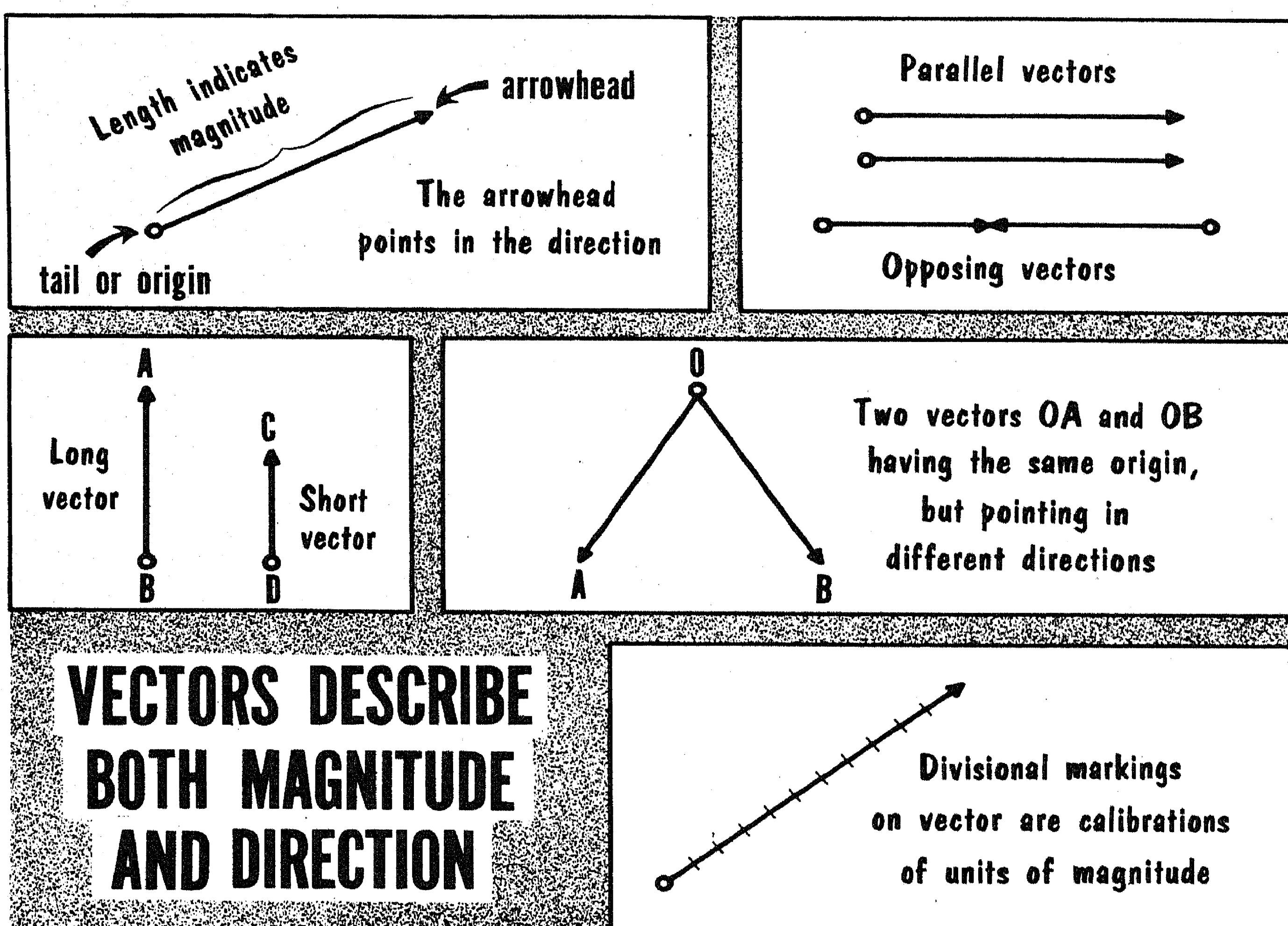
$$25 = 16 + 9$$

$$25 = 25$$

The angle formed by the base and hypotenuse is often referred to as the angle theta (θ). With regard to this angle, we will often refer to the side opposite angle θ, the side adjacent to angle θ, and the hypotenuse. These relationships are referred to as the sine, cosine, and tangent of angle θ. The sine of this angle is equal to the side opposite divided by the hypotenuse; the tangent of θ is equal to the side opposite divided by the side adjacent. In rotating from 0° to 90°, the sine of θ will vary from a value of 0 to 1; the cosine of θ will vary from 1 down to 0; and the tangent of θ will vary from 0 to infinity.

Vectors

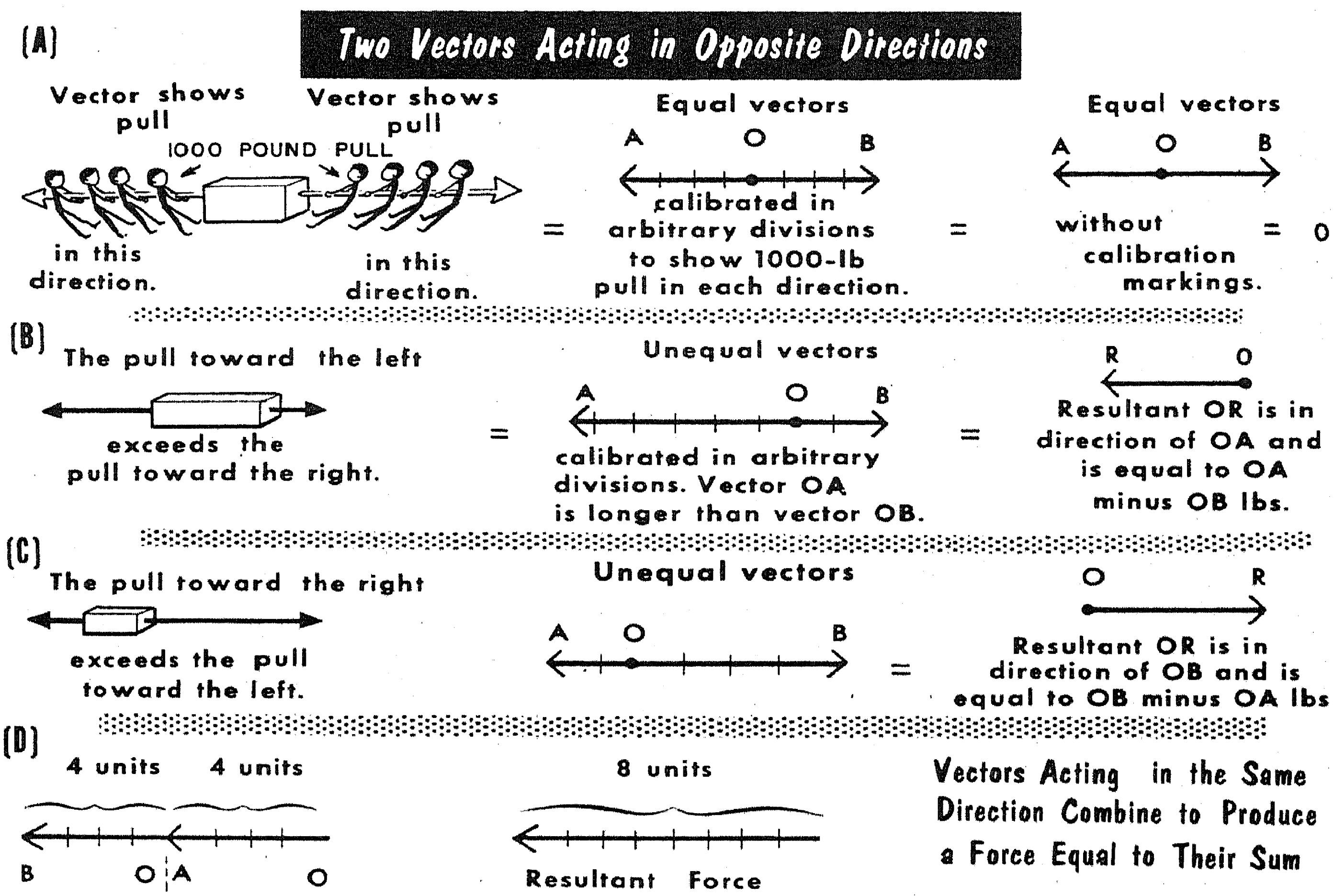
Some things or situations can be expressed by a single number, (e.g., the population of a town, the number of feet in a mile, or the number of chairs in a room). Anything which can be described fully by a single number is called a scalar quantity. There are, however, many situations or actions that cannot be described in this manner. For example, the movement of a jet plane. To say that it is flying at 600 miles an hour is not sufficient--the direction in which it is going also must be stated. If we desire to identify the force being applied to an object, it is not sufficient to say it has a force of 100 pounds. Is the force being applied upward or downward, to the right or to the left? Both magnitude (amount) and direction must be stated. Any situation or action that requires mention of both magnitude and direction to describe it is called a vector quantity. Alternating voltage and current and related phenomena are vector quantities.



A vector is a straight, or directed line with an arrowhead at one end. This is the head end; the other end is the origin or the tail of the vector. The length of the vector identified in any suitable units indicates the magnitude of the quantity, whereas the direction in which the arrowhead points is the direction of the quantity. Letters assigned to the head and to the tail of the vector readily identify the vectors, such as OA, OB, AB, CD, etc.

Multiple Vectors

Assume two like teams of men pulling on an object as in (A). One team exerts a pull of 1000 pounds in one direction while the other team is exerting a similar pull of 1000 pounds in the opposite direction. The situation can be illustrated by two vectors OA and OB of equal length (equal magnitude of force) directed in opposite directions. One pull cancels the other, hence, the net force acting on the object is zero.



If vector OB is shorter than vector OA as in (B), it means that the amount of force applied in one direction (vector OA) exceeds the amount of force applied in the other direction (vector OB). The net force is established by subtracting vector OB from OA. The resultant force (OR) acting on the object is in the direction of the greater force--vector OA. The same method is used to establish the resultant when the greater force is in the direction of OB as in (C). The resultant is vector OR in the direction of vector OB. The vector subtraction obviously is simple arithmetical subtraction.

Vectors representing forces acting in the same direction can be added to each other, as shown in (D). They are joined head to tail, as shown by the tail of OB being joined to the head of OA. The resultant is indicated by the sum of the lengths of OA and OB interpreted in whatever units express the magnitudes of OA and OB. Again, the vector addition is simple arithmetical addition.

The Parallelogram

Forces do not always counteract or aid each other completely. Sometimes, they act on the same object in directions which are at right angles to each other. This condition can be shown graphically by two vectors, OA and OB, having a common origin and forming a right angle. Four positions of the vectors are shown in (A) on page 2-7. Each of the vectors in the presentation is referred to as a component vector. Also, one of them is selected as the reference vector, usually the one which is positioned horizontally. Vector OB typifies this.

Two forces acting at right angles to each other produce a resultant force which also has magnitude and direction. It is established in a particular way known as the parallelogram method. The parallelogram is formed by using the vectors OA and OB as adjacent sides and adding two new sides, BC and AC, shown by the dotted lines in the figures shown in (B). Side BC is parallel and equal to side OA, and side AC is parallel and equal to side OB. The diagonal drawn between the origin (point O) of the component vectors and the opposite corner of the parallelogram is the resultant. If the resultant is calibrated in the same units that are used for the two component vectors OA and OB, the magnitude of the resultant can be interpreted directly from the length of the resultant OC.

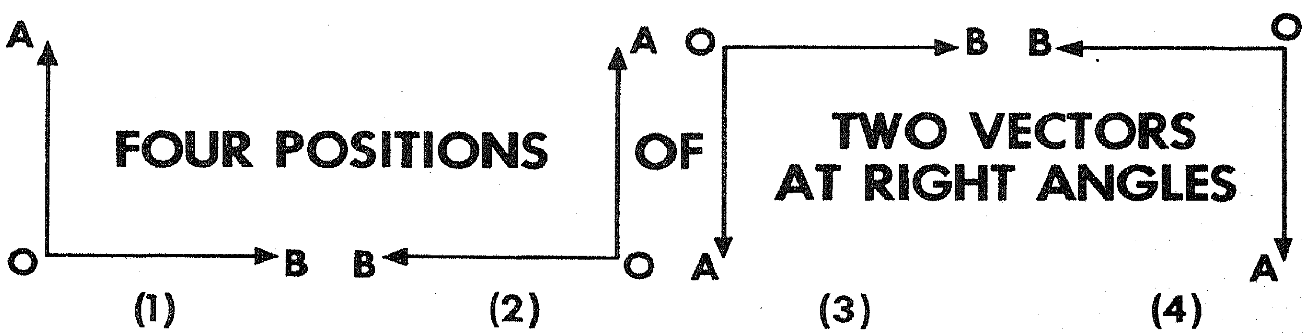
As can be seen, the resultant has a direction of action which differs from that of the two component vectors OA and OB. The original right angle (90°) relationship between OA and the reference vector OB is modified, and the resultant now has an angular relationship COB relative to the direction of vector OB.

The magnitude of the resultant is a function of the relative magnitudes of the two components. When the two components have equal magnitudes (equal length), the resultant is the smallest possible, but even then it is greater than either of the components but not equal to their sum. When one component vector exceeds the other, the resultant is always greater than the larger of the two components. Examples are given in (C) on page 2-7.

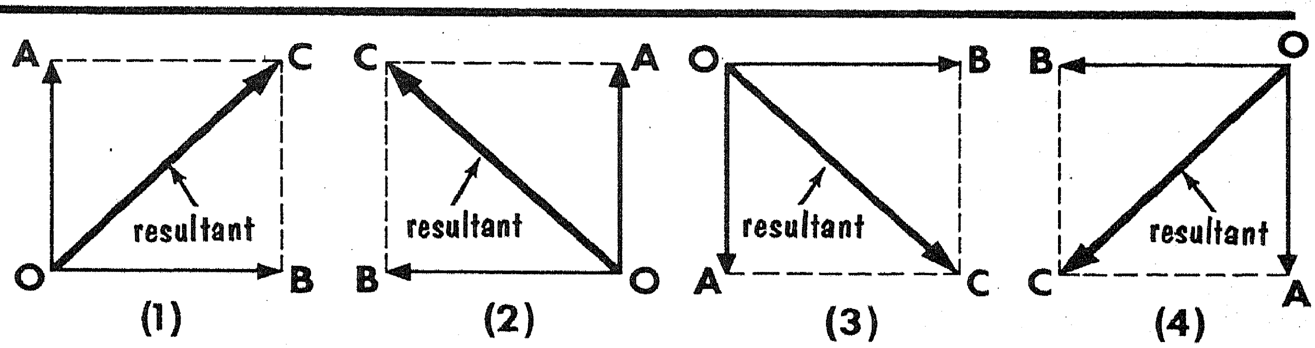
As to the direction of the resultant OC, it too is a function of the relative magnitudes of the two component vectors. When the component vectors are of equal length, the direction of the resultant is mid-way between the directions of the individual components. These are 90° apart; the resultant is always at 45° see the first example in (C) . When the vertically directed component (in these examples it is OA) is greater in magnitude than the horizontally-directed component OB as shown in (C), examples 2 and 4 , the resultant has a direction which is closer to the vertically directed component, and the angle COB exceeds 45°. When the situation is reversed and the horizontally directed component vector OB is the larger of the two components (C), example 3 the resultant is directed more in the direction of the larger component. As can be seen, the angle COB is then less than 45°. As long as the two component vectors are present, the angle COB will never be 0°, nor will it ever be 90° -- its angle will always have some value in between these limits.

VECTORS IN DIRECTIONS FORMING A 90° ANGLE

(A)

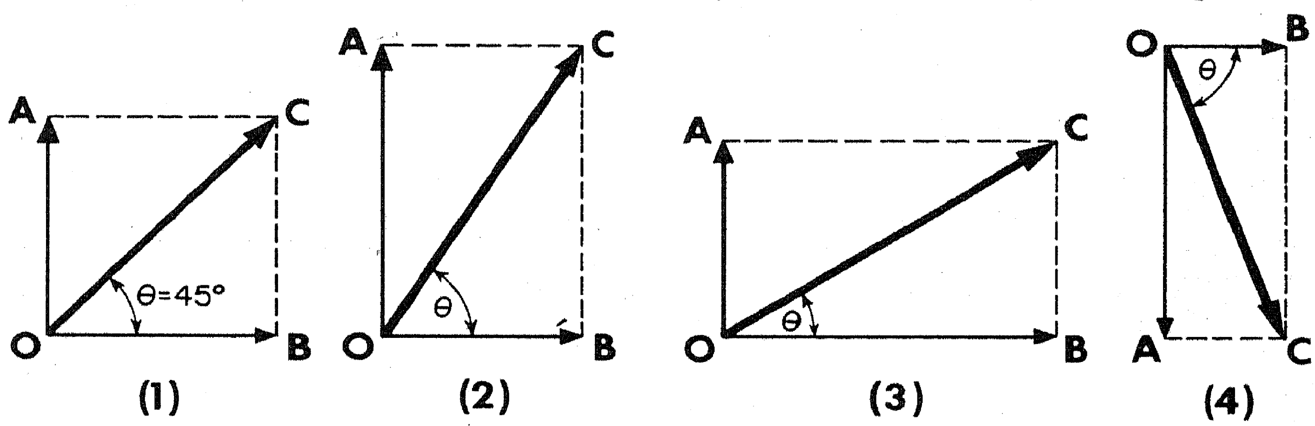


(B)



THE RESULTANT ALWAYS IS LARGER THAN THE LARGER OF TWO UNEQUAL COMPONENTS

(C)

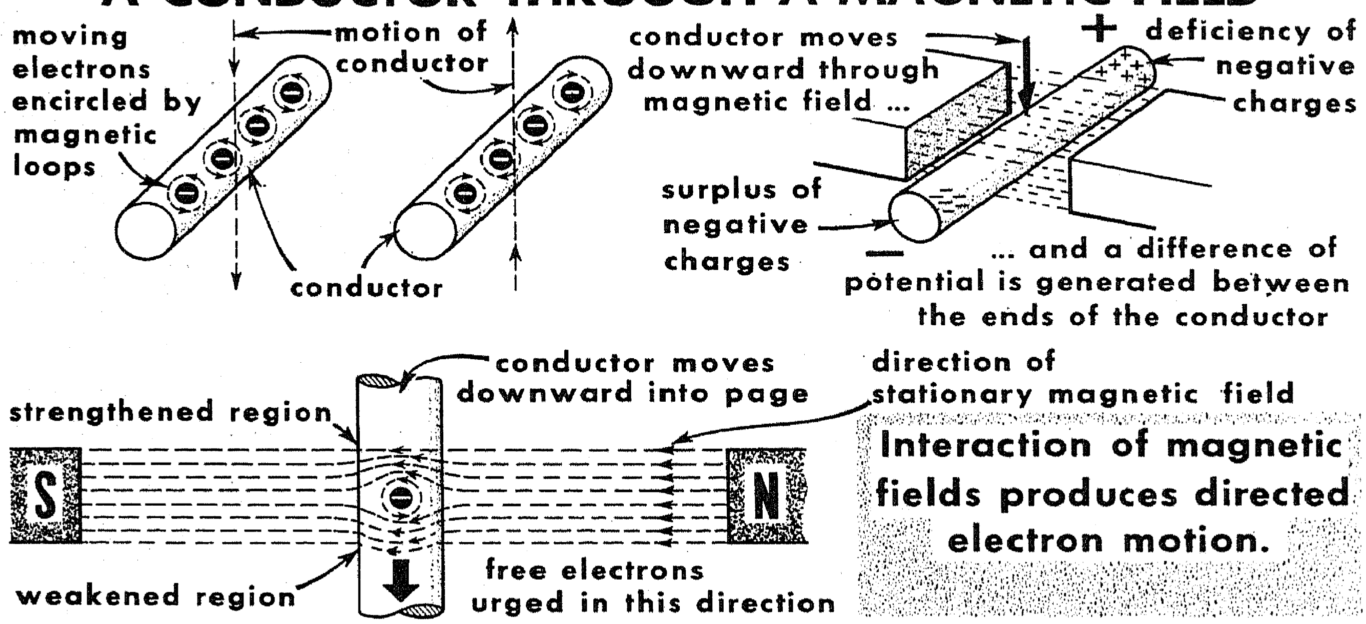


OA = OB, $\theta = 45°$	OA > OB, θ=more than 45°	OA < OB, θ=less than 45°	OA > OB, θ=more than 45°

Generating a Voltage in a Moving Conductor

There are several ways of generating an a-c voltage. The basic method is to induce an emf in a conductor by moving it across the lines of force of a stationary magnetic field, as we discussed in Volume 1. We will now consider this a little more closely. When a conductor is moved, the free electrons it contains move with it regardless of which direction the conductor moves. Every moving electron is encircled by magnetic loops of force, and these loops always position themselves at right angles, or perpendicular to the direction of the moving negative charge. When a conductor moves downward, the electrons it contains move downward with it. Thus, the magnetic loops encircling the electrons are perpendicular to the downward motion, or in a horizontal plane. Applying the left-hand rule to the motion of the electrons, the magnetic loops will rotate counterclockwise around the electrons (viewing the electrons from the top down). When a conductor moves upward, the reverse occurs.

GENERATING A VOLTAGE BY PASSING A CONDUCTOR THROUGH A MAGNETIC FIELD

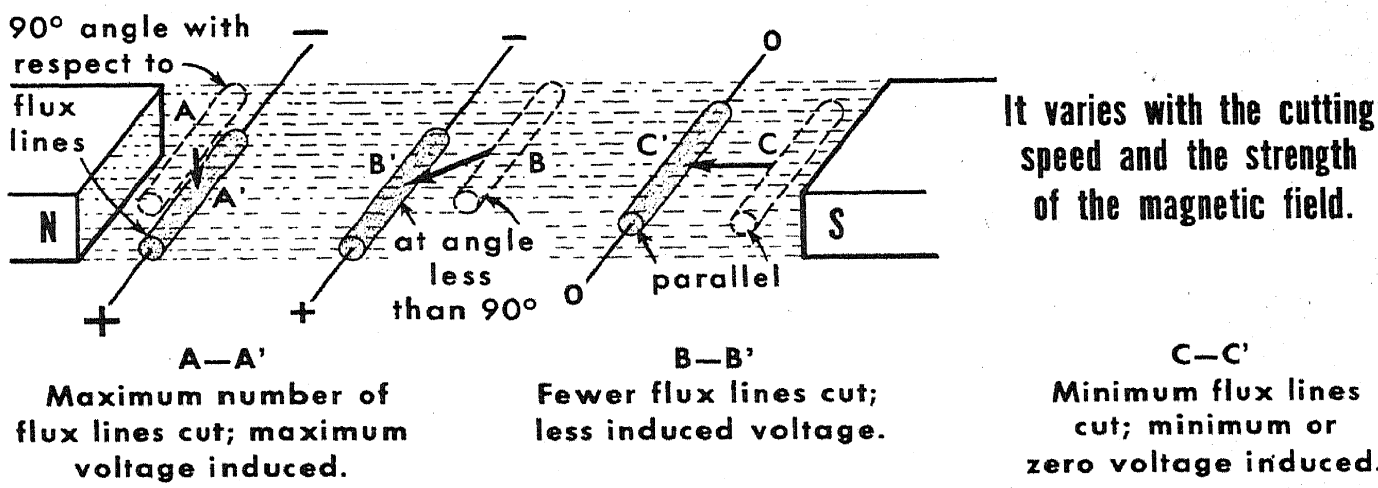


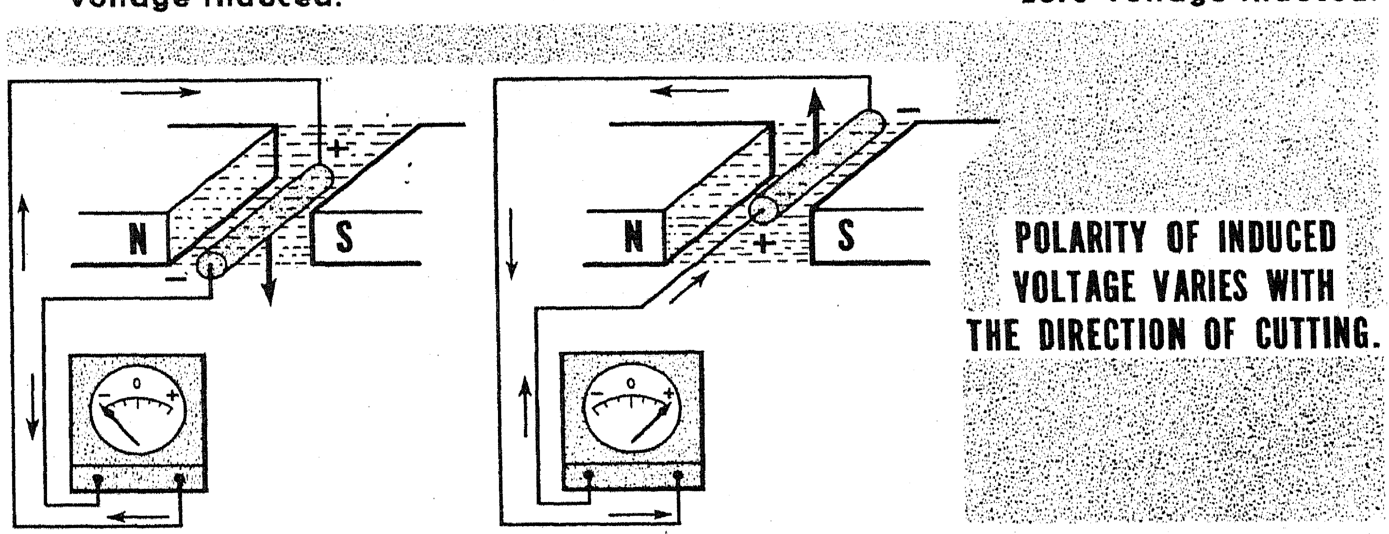
When a conductor cuts through a stationary magnetic field, there is interaction between this field and the magnetic loops encircling the moving electrons. This interaction produces a resultant magnetic field around the free electrons, which in turn produces a strengthened and a weakened region on opposite sides of the electrons. The strengthened region results from the two magnetic fields aiding each other; the weakened region from the two magnetic fields opposing each other. The free electrons present are thus "urged" from the strengthened region in the direction of the weakened region, creating an accumulation of free electrons at one end and a corresponding shortage at the other. The area of electron accumulation is called the negative end of the conductor; the area of electron shortage is called the positive end. Thus, a potential difference is produced between the ends of the moving conductor, and the conductor becomes a voltage source.

Generating a Voltage in a Moving Conductor (Contd.)

The polarity of the electromotive force induced in a conductor cutting through magnetic lines of force is a function of the relative directions of the lines of force of the stationary magnetic field and the direction of motion of the conductor. The direction of the stationary field is fixed – from the north pole to the south pole. Therefore, the direction of motion of the moving conductor is the controlling factor in determining polarity.

Maximum Voltage is Induced in a Conductor When It Cuts a Maximum Number of Lines of Force Per Unit Time.





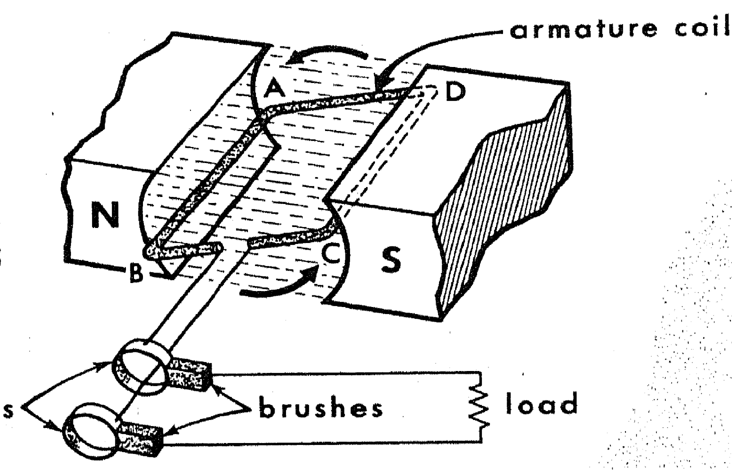
The amount of voltage that is induced or generated depends on the velocity at which the lines of force of the stationary field are cut by the conductor, and the strength or flux density of the magnetic field. Assuming the conductor velocity remains constant, the rate at which the stationary flux lines are cut then will depend on the angle at which the conductor cuts through the flux lines. Minimum or zero voltage is induced when the conductor moves parallel to the lines of force. Maximum voltage is induced when the conductor cuts the lines of force at right angles, or 90°. This is the greatest possible cutting angle, and the angle at which maximum flux lines are cut per unit time. Between these two points, an intermediate amount of voltage is induced.

The Basic A-C Generator

The basic generator of alternating voltage is a pivoted-loop armature having two coil sides, A-B and C-D, which rotate between the two pole pieces of a horseshoe magnet with uniform velocity and through a uniform stationary magnetic field. The rotating motion causes the coil sides to cut the flux lines of the stationary field. Because the voltage generated in the two sides of the rotating loop is equal (though opposite in polarity), we can examine the process of voltage generation by considering one coil side only. For this purpose, we select the side C-D and use its slip-ring as the voltage reference.

BASIC A-C GENERATOR

AS ARMATURE COIL ROTATES BETWEEN MAGNETIC POLES, A-C VOLTAGE IS GENERATED BETWEEN SLIP RINGS. BRUSHES MAINTAIN CONTACT ON RINGS AND CARRY CURRENT TO AND FROM LOAD.

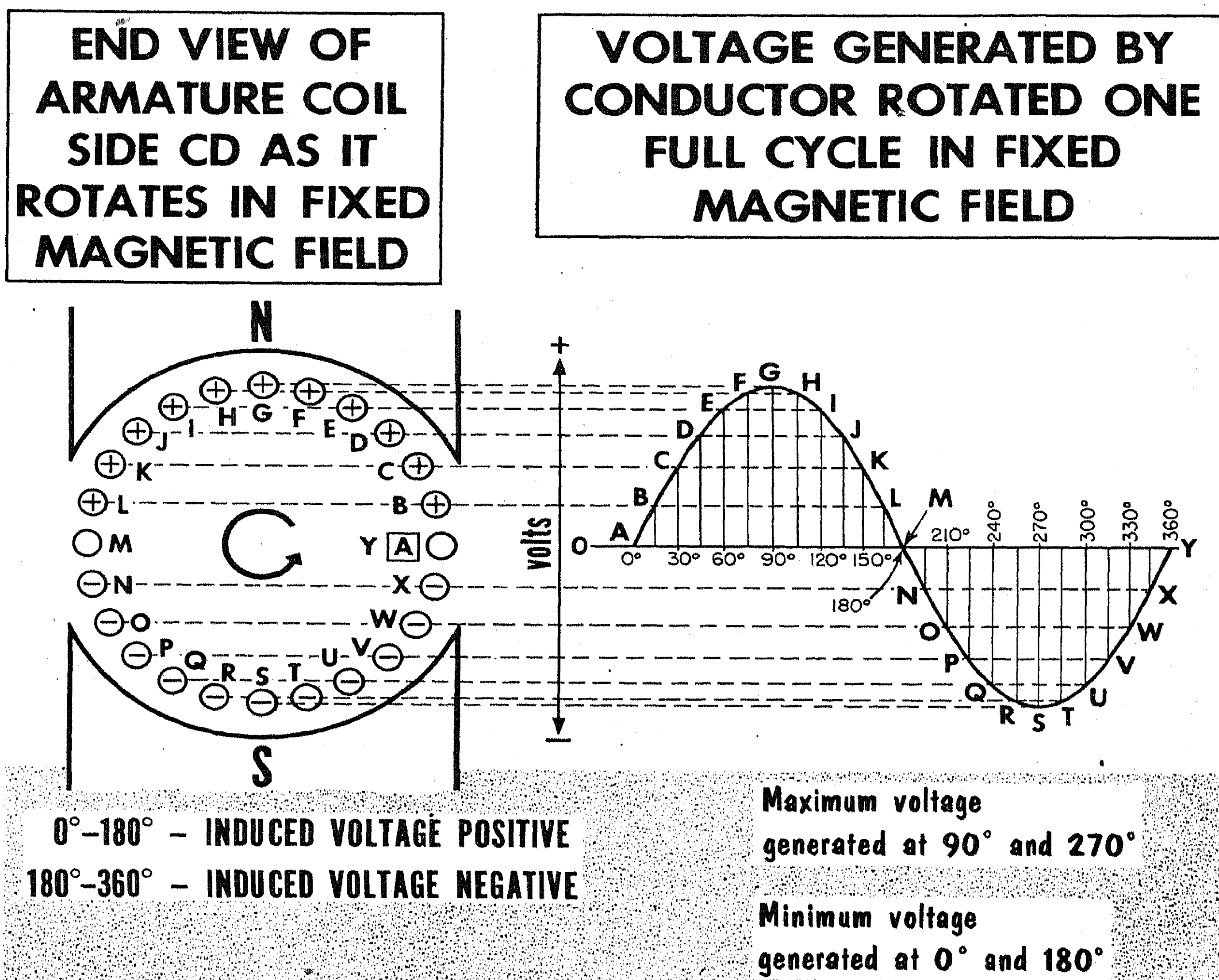


Assume that the action begins with side C-D momentarily positioned at A. At this moment, the coil side is moving parallel to the flux lines of the stationary field. Thus, the angle of cutting of the flux lines is zero (or the rate of cutting of the flux lines is zero); hence, the voltage induced in the coil side is zero. As rotation continues, the coil side moves <u>upward</u> and passes through progressively increasing angles of rotation, as shown by points B, C, D, E, F, and G, which correspond to 15°, 30°, 45°, 60°, 75°, and 90°. In doing so, the angle of cutting of the flux lines by the rotating coil side increases from 0° at A to a maximum of 90° at G; therefore, the rate of cutting of flux lines increases and the output voltage increases. Maximum output voltage is developed at G, or when the angle of cutting is 90°, thus completing one-quarter turn. A plot of the output voltage in steps of 15° of angular time between 0° (A) and 90° (G) is shown.

As the coil side continues rotating, it inscribes increasing angles of rotation; 105° (H), 120° (I), 135° (J), 150° (K), 165° (L), and 180° (M), but the angle and the rate of cutting of the flux lines decreases progressively from G to M. And so does the voltage output, reaching zero at M. Here, the coil side again is moving parallel to the flux lines. Note that the amount of decrease in voltage for each 15° change in angular rotation between 90° and 180° (G to M) is exactly the same as the amount of increase in voltage between 0° and 90° (A to G). Note also that the output voltage remains positive while the coil side is completing the half turn from 0° to 180° of angular time, the reason being that the motion of the coil side through the flux lines (past the N pole) continues throughout, except at the angles of 0° (A) and 180° (M).

The Basic A-C Generator (Cont'd)

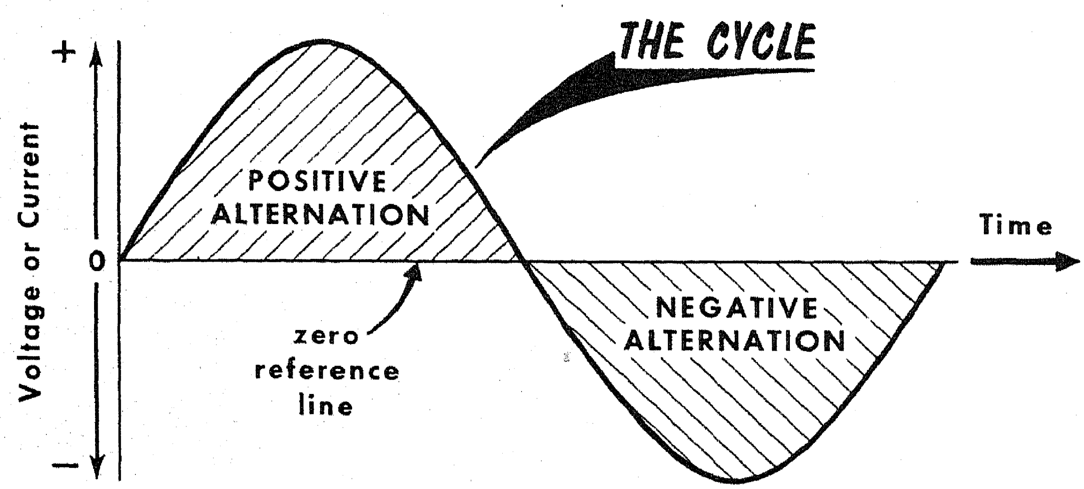
As the coil side moves past the 180° point, the angle of rotation continues to increase, as does the angle at which the conductor cuts the field. The conductor now is moving downward past the S pole. The direction of electron flow in the conductor is the opposite of before and the induced voltage is negative.

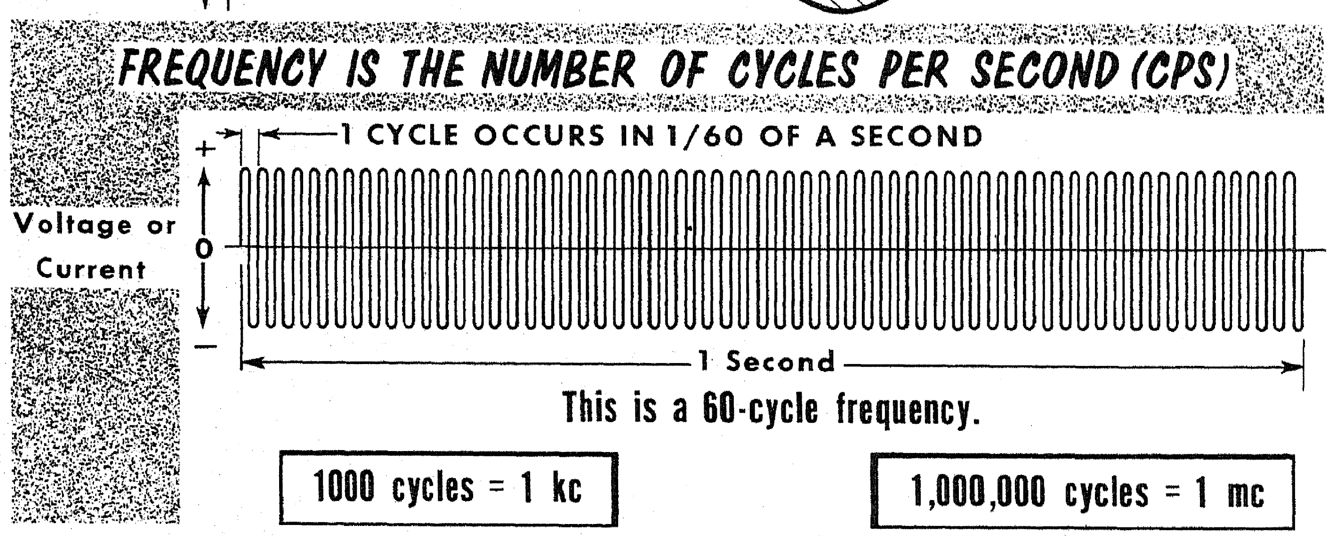


Now we show the voltage below the zero voltage reference line. The output voltage has been reversed because of the change in the relative direction of the moving conductor and the stationary flux lines. The change in output voltage of negative polarity for angular steps of 15° between 180° (M) and 270° (S) is exactly the same as the voltage between 0° (A) and 90° (G) of positive polarity. That is, the 90° (G) and 270° (S) points are maximum voltage points of opposite polarity. Further movement of the coil side from the 270° (S) position to the 360° (Y) position results in a fall in voltage from maximum negative to zero. The angular rotation increases but the rate of cutting of the flux lines decreases from the maximum at the 270° (S) point to 0 at the moment of 360° (Y) of rotation. The coil has completed one full turn. It corresponds to 360° of rotation and is the equivalent of 1 cycle of the output voltage.

The Cycle--Frequency

When the armature in the basic a-c generator completes one full turn (360°) of rotation, it has generated one cycle of voltage. The voltage has gone from a starting value of zero, risen to a maximum positive value, fallen back to zero, risen to a maximum negative value, and fallen back to zero. A cycle refers to a complete chain or sequence of events. A cycle of voltage applied to a resistance load will cause a similar cycle of current to flow in a circuit. The cycle of current will go through the same fluctuation as the cycle of voltage. When all the voltage or current values are joined together, they form a "picture" or pattern called a waveform.



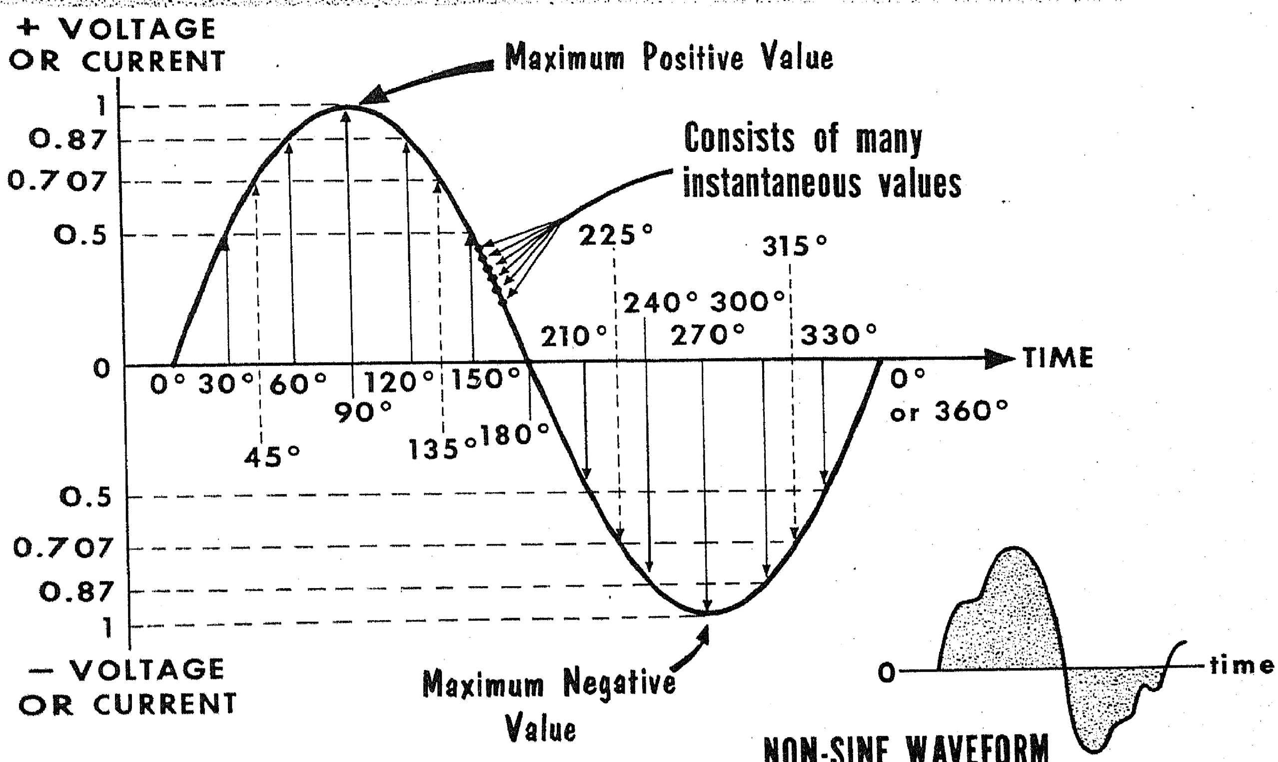


The number of complete cycles that occur in one second is called the frequency of the waveform. When a voltage or current waveform passes through 60 cps, it is called a 60-cycle frequency. Each half-cycle is called an alternation; each complete cycle thus contains two alternations--a positive and a negative. Since a 60-cycle frequency represents 60 complete cycles per second, the time duration of each cycle is 1/60 of a second. In high frequencies such as 1 megacycle (1, 000, 000 cycles per second), the time duration of each cycle is 1/1, 000, 000 of a second. The faster our basic generator rotates, the more cycles per second will be generated, and the higher will be the output frequency. The strength of the magnetic field will determine the strength or amplitude of the output waveform, but not its frequency.

The Sine Waveform--Voltage and Current

The sine or sinusoidal waveform is a pattern of instantaneous changes in the value of an alternating voltage or current. The word "sine" is taken from the sine table (see Appendix) used in mathematics because the amplitude of the sine wave varies from zero to maximum in the same manner as the values in the sine table. When we refer to a waveform as a sine wave, it indicates that we are considering only a single frequency. When various sine waves of different frequencies are combined, they form a complex waveform which is not a sine wave.

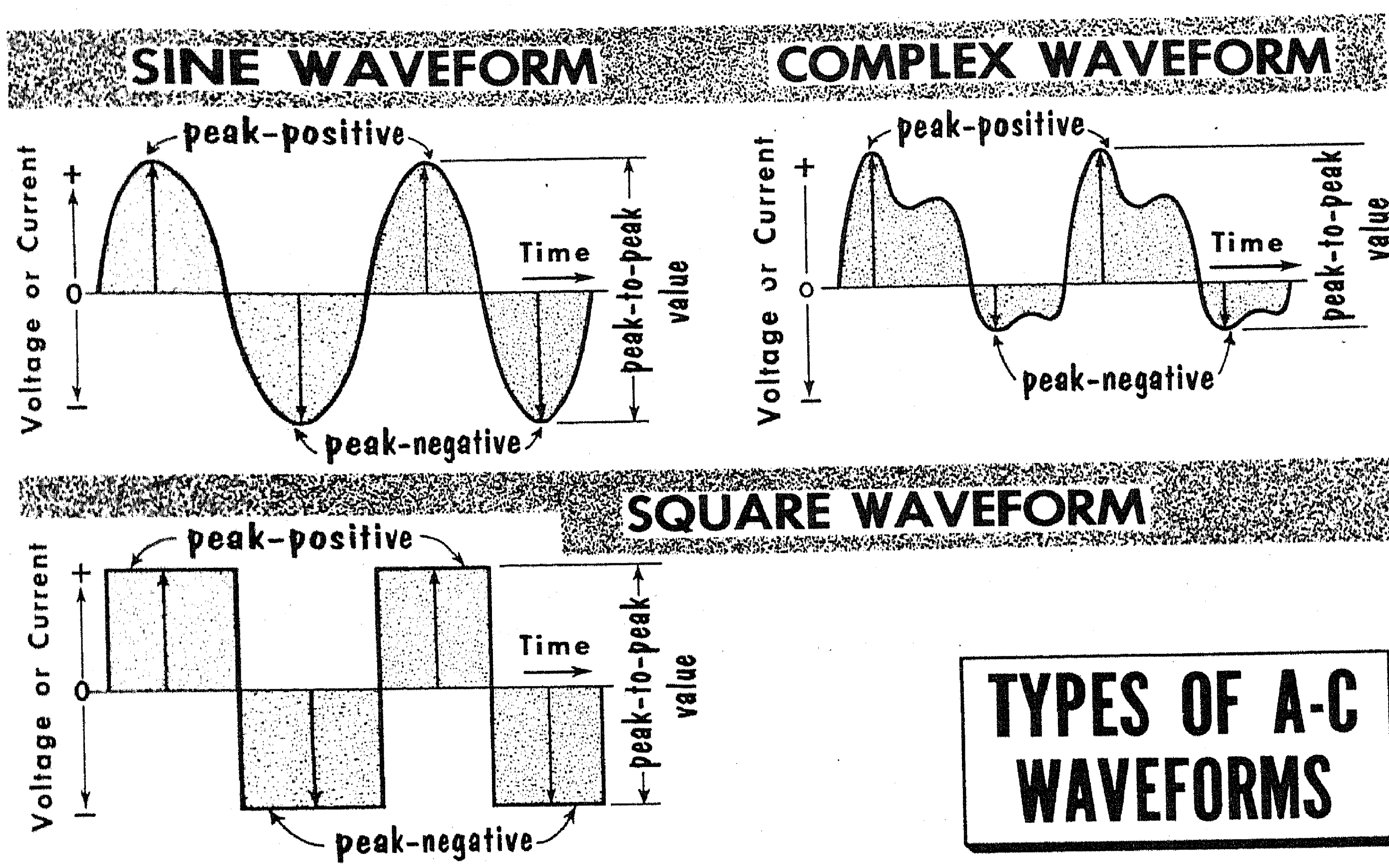
The Sine Waveform



An important characteristic of the sine waveform is that the positive and negative half-cycle are mirror images of each other. The rate of rise and fall of both alternations is identical: At 0°, we see that the value of the sine wave (voltage or current) is zero. At 30° along the zero time axis, the sine waveform value has climbed to 0.5, or half its maximum value. At 45°, the sine wave is at 0.707 of its maximum value, and at 60° a value of 0.87 of maximum is reached. Finally, at 90° or one quarter of the entire cycle, the maximum value of the sine wave is reached. In going from 90° to the half-way point at 180°, the sine wave decreases in value in a manner opposite from the way it increased going from 0° to 90°. The second half-cycle, from 180° to 360°, has identical rise and fall values to those of the first half-cycle except that they are in the opposite direction. Actually, a sine wave consists of many more values than are shown. There are an infinite number of values, and the sine wave is a picture of all their instantaneous values joined together in time.

Instantaneous and Peak Values of A-C Voltage and Current

The continually changing values of alternating voltage and current necessitate the use of special terminology to express the amount of the voltage and the current. The instantaneous value of an a-c voltage or current is that value which exists at any specific instant of time. It can have any amplitude between 0 and maximum, and can be of a positive or a negative polarity. When an instantaneous value is indicated, small letter "e" is used to express voltage and small letter "i" to express current. For example, the instantaneous value of a sine waveform voltage at 45° is stated as e = 28.3 volts. Similarly, the instantaneous value of a sine waveform current at 345° might be stated as i = 0.259 ampere. Reference to an instantaneous value must be accompanied by identification of its time in the cycle, because the value of a sine wave changes constantly. An a-c voltage at 33° is different from an a-c voltage at 34°, and is still different at 34.5°.

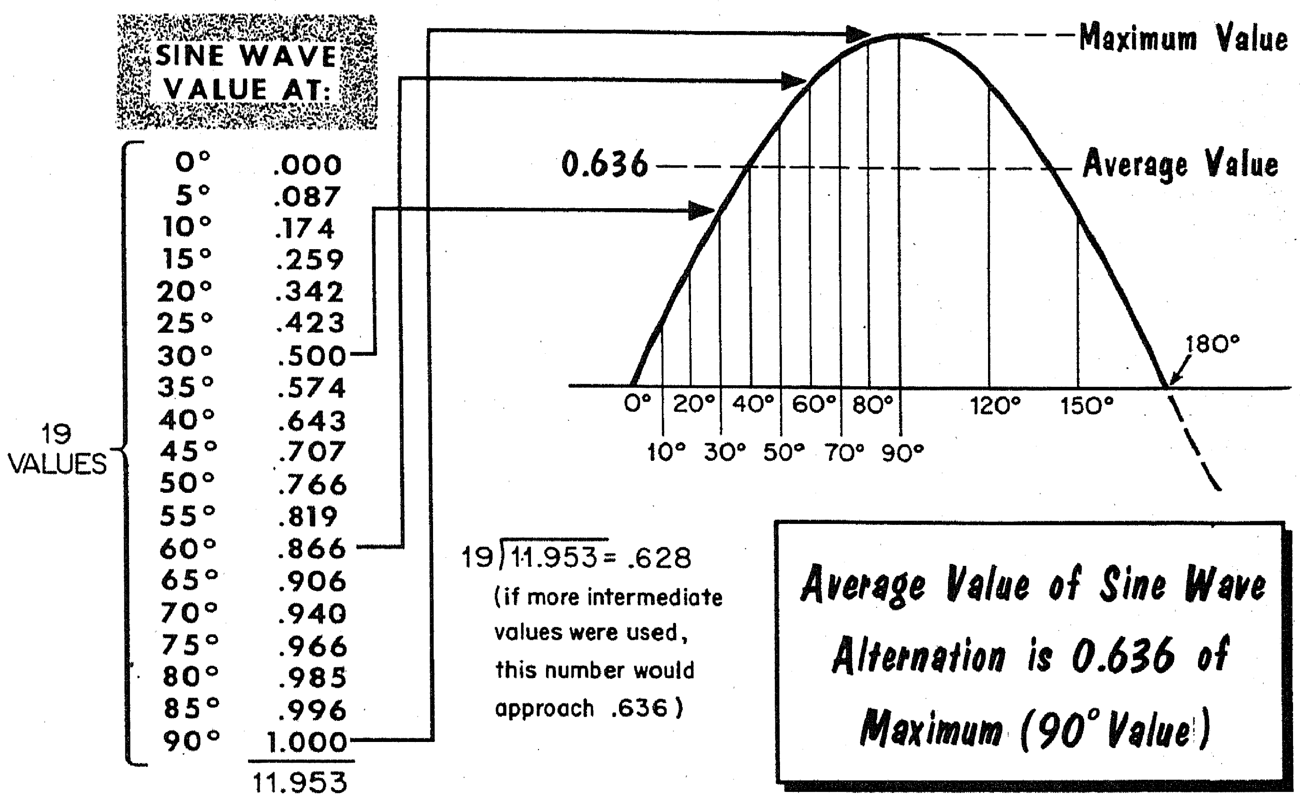


TYPES OF A-C WAVEFORMS

The peak value of an alternating voltage or current is the highest value reached by the quantity during a cycle. Maximum and peak value have the same meaning. A peak amplitude of 10 volts means the same as a maximum amplitude of 10 volts. This holds true for any type of waveform. Of course, the polarity may also be referred to, such as peak-positive or peak-negative value. In the sine waveform, the positive-and negative-peak values are always alike; this is not so for voltage or current waveforms which are not of sine waveform variation. The peak-to-peak value (sometime abbreviated p-p), is simply the sum of the positive-peak and negative-peak values, regardless of the waveform. A voltage that has a 10-volt peak-positive value and a 10-volt peak-negative value has a 20 volt (10 + 10) peak-to-peak value.

Average Value of a Sine Waveform Voltage or Current

If we add all the instantaneous values of the positive half-cycle and the following negative half-cycle of a sine waveform, and then find the average of these values, we find it to be zero. The reason for this is that the two adjacent half-cycle are of opposite polarity (one being plus and the other minus) and, when we add a plus quantity to a minus quantity of equal value, the result is zero. So a general statement can be made--the average value of a cycle of a sine waveform is zero.



A different situation prevails if we think in terms of a half-cycle, either positive or negative. Imagine that the peak value of a sine waveform voltage or current is 1 volt or 1 ampere. If we add up the instantaneous values of voltage or current prevailing at each 5° of the cycle, the total is 22.90. Since the half-cycle waveform is made up of 36 instantaneous values, then 22.9/36 gives us an answer of 0.636 volt or ampere. Therefore, 0.636 is the average value of a half-cycle of a sine waveform voltage or current when the peak value is 1 volt or 1 ampere. We say then, that the average value of a sine wave is 0.636 of its maximum value.

$$E_{av} = 0.636 \times E_{max}, \text{ or } I_{av} = 0.636 \times I_{max}$$

Knowing the average value of a sine wave, we can calculate the peak or maximum value. It is the average value multiplied by 1.57 or

$$E_{max} = 1.57 \times E_{av}, \text{ or } I_{max} = 1.57 \times I_{av}$$

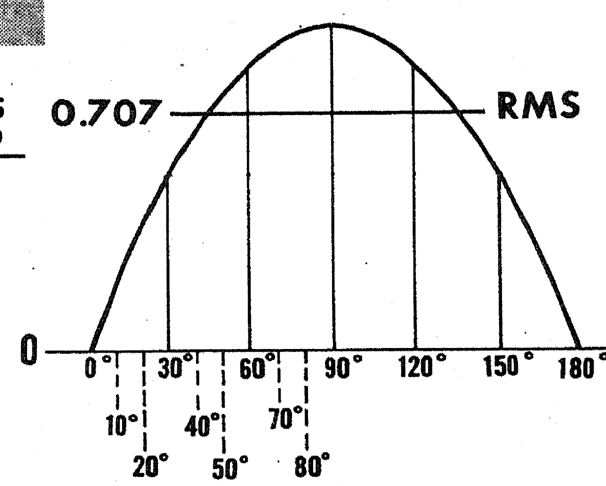
For instance, if the average value of a sine waveform voltage is 140 volts, the maximum (or peak) value is $E_{max} = 1.57 \times 140 = 219.8$ volts.

The Effective or RMS Value of a Sine Wave

You have learned that when direct current flows through a resistance, the amount of heat developed is proportional to I^2, or the square of the current. When alternating current flows through a resistor, the heat developed is proportional to the square of the instantaneous values of current. We can see why this is so. Alternating current changes constantly in value--it changes every instant. First, it rises from zero to a maximum value and then, it falls back to zero. Following this, it rises to a maximum in the opposite direction in a circuit and then, it again falls back to zero. Because of this constant variation, we must find a value that will be equivalent to some value in direct current. This equivalent value is called the effective value, because the effective value of an a-c sine wave tells us that that value of alternating current will do as much work as the same value of direct current. Unless we have an effective value, it would be difficult to discuss a-c voltages and currents in comparison to d-c voltages and currents.

ROOT MEAN SQUARE (EFFECTIVE VALUE) OF SINE WAVE EQUALS 0.707 OF MAXIMUM VALUE

SINE WAVE VALUE AT		INSTANTANEOUS VALUE		INSTANTANEOUS VALUE SQUARED
0°	=	0.00	→	0.0000
10°	=	0.17	→	0.0289
20°	=	0.34	→	0.1156
30°	=	0.50	→	0.2500
40°	=	0.64	→	0.4096
50°	=	0.77	→	0.5929
60°	=	0.87	→	0.7569
70°	=	0.94	→	0.8836
80°	=	0.98	→	0.9604
90°	=	1.00	→	1.0000
				4.9979

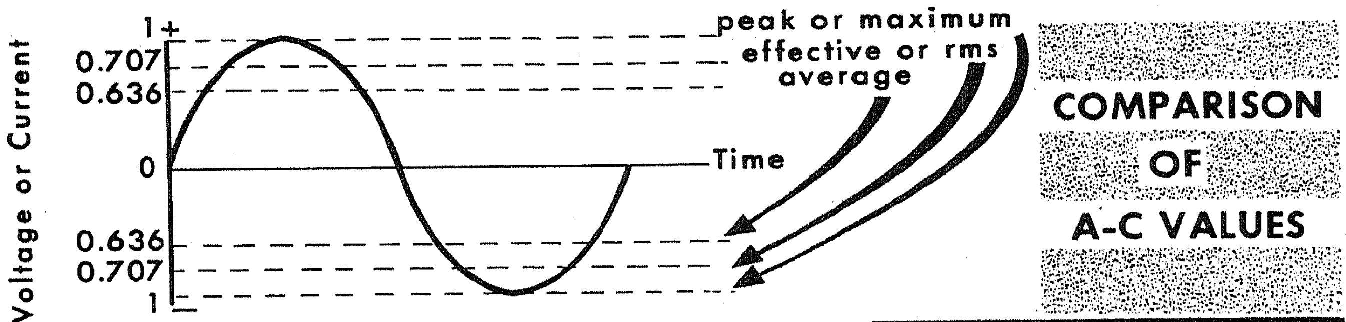


$4.9979 \div 10 = 0.49979$ (mean square)

$\sqrt{0.49979} = 0.7069$ — (root mean square)
or 0.707

The effective value of a sine wave is obtained as follows: First, we take a large number of instantaneous values of a sine wave and square each one. Then, we add up all the squared instantaneous values and divide this total by the number of values used. This gives us the average or mean square. Finally, we take the square root of the mean square, which gives us the root mean square, or rms value. This rms value is very important, because the rms value of an a-c sine wave indicates that a specific voltage or current will do as much work as the same value of d-c.

The Meaning of Effective Value



AVERAGE = Max. × 0.636
Effective × 0.9

EFFECTIVE = Max. X 0.707
Average X 1.11

MAXIMUM = Effective × 1.414
Average × 1.57

When we plug a soldering iron into a 120-volt a-c source, it will reach a certain temperature, depending upon its wattage rating. If we plug this same soldering iron into a 120-volt d-c source, it will arrive at the same temperature. This is because the 120 volts a-c is the effective value of the a-c waveform. Its peak value is much higher than 120 volts, and for much of each alternation its instantaneous values are less than 120 volts. Actually, the effective value of a sine wave is 0.707 of its maximum value. An alternating voltage with a peak or maximum value of 1 volt will have an effective value of 1 × .707 volt, and it will produce the same heat in a given resistor as .707 volt d-c.

There are two simple formulas that can be used: One, to find the effective value of a sine wave knowing its maximum value; the other, to find the peak value knowing the effective value.

$$\text{Effective Value} = \text{Maximum Value} \times 0.707$$
$$\text{Maximum Value} = \text{Effective Value} \times 1.414$$

At the ordinary 120-volt 60-cycle house outlet, the peak value is:

$$E_{max} = 120 \times 1.414 = 169.68 \text{ volts.}$$

If an a-c current has a peak value of 5 amperes, the rms value would be:

$$I_{eff} = 5 \times .707 = 3.535 \text{ amperes}$$

As another example, if the effective value of a current is 180 milliamperes, and the effective value of a voltage is 690 millivolts, their peak values are:

$$I_{max} = 180 \times 1.414 = 254.5 \text{ milliamperes}$$

or

$$.18 \times 1.414 = .2545 \text{ amperes}$$

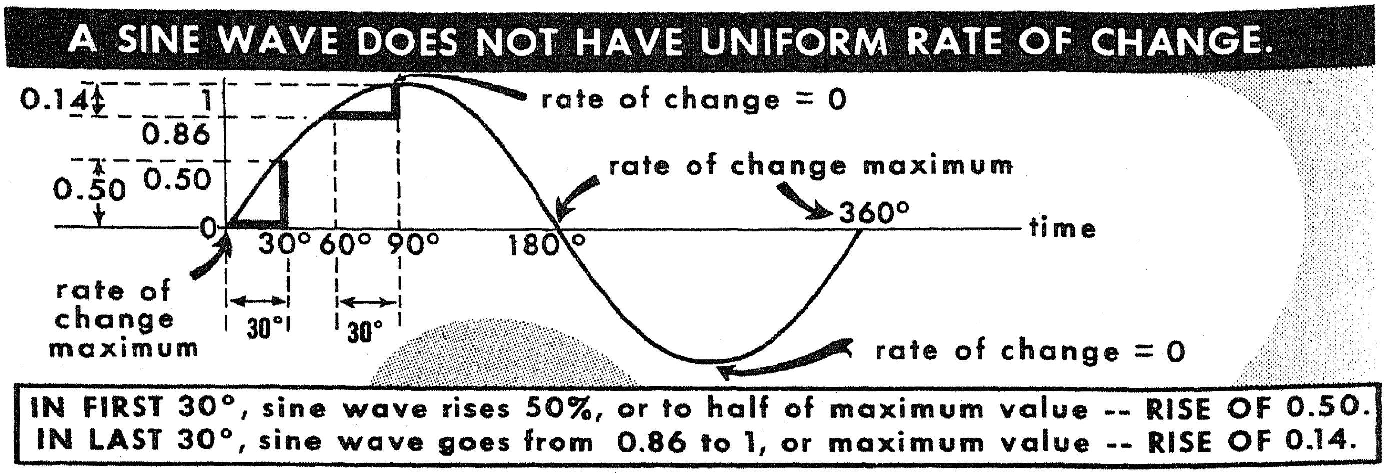
$$E_{max} = 690 \times 1.414 = 975.7 \text{ millivolts}$$

or

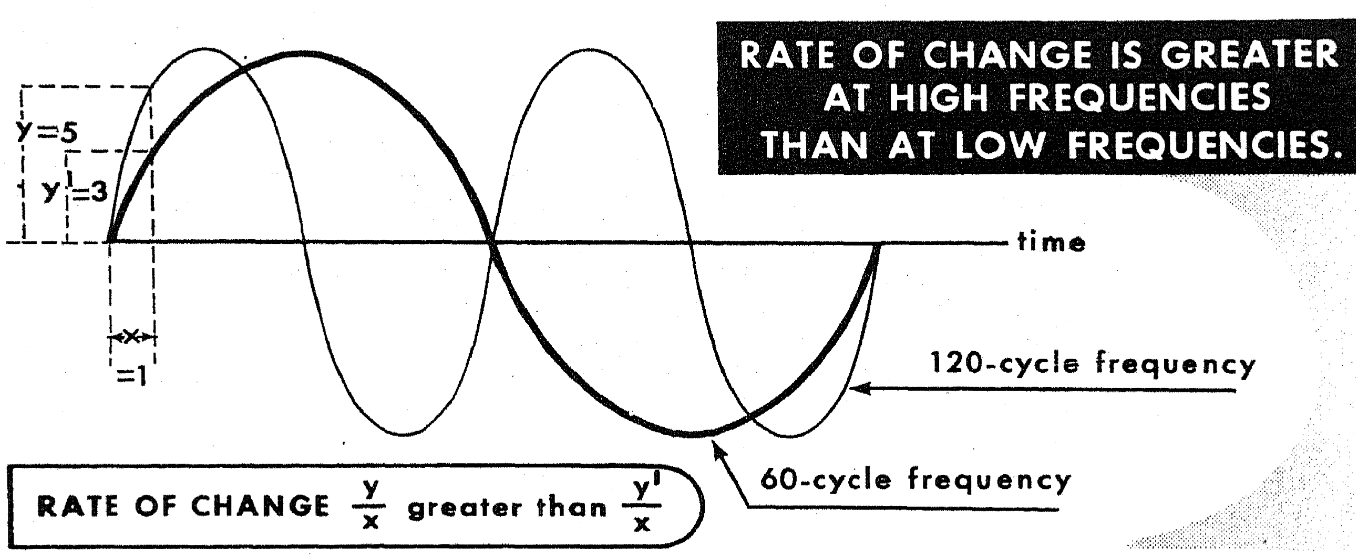
$$.69 \times 1.414 = .9757 \text{ volts}$$

Whenever an a-c voltage or current is stated, it always is taken to mean the effective or rms value, unless otherwise indicated. The usual a-c voltmeters and ammeters are calibrated to read rms values.

Rate of Change



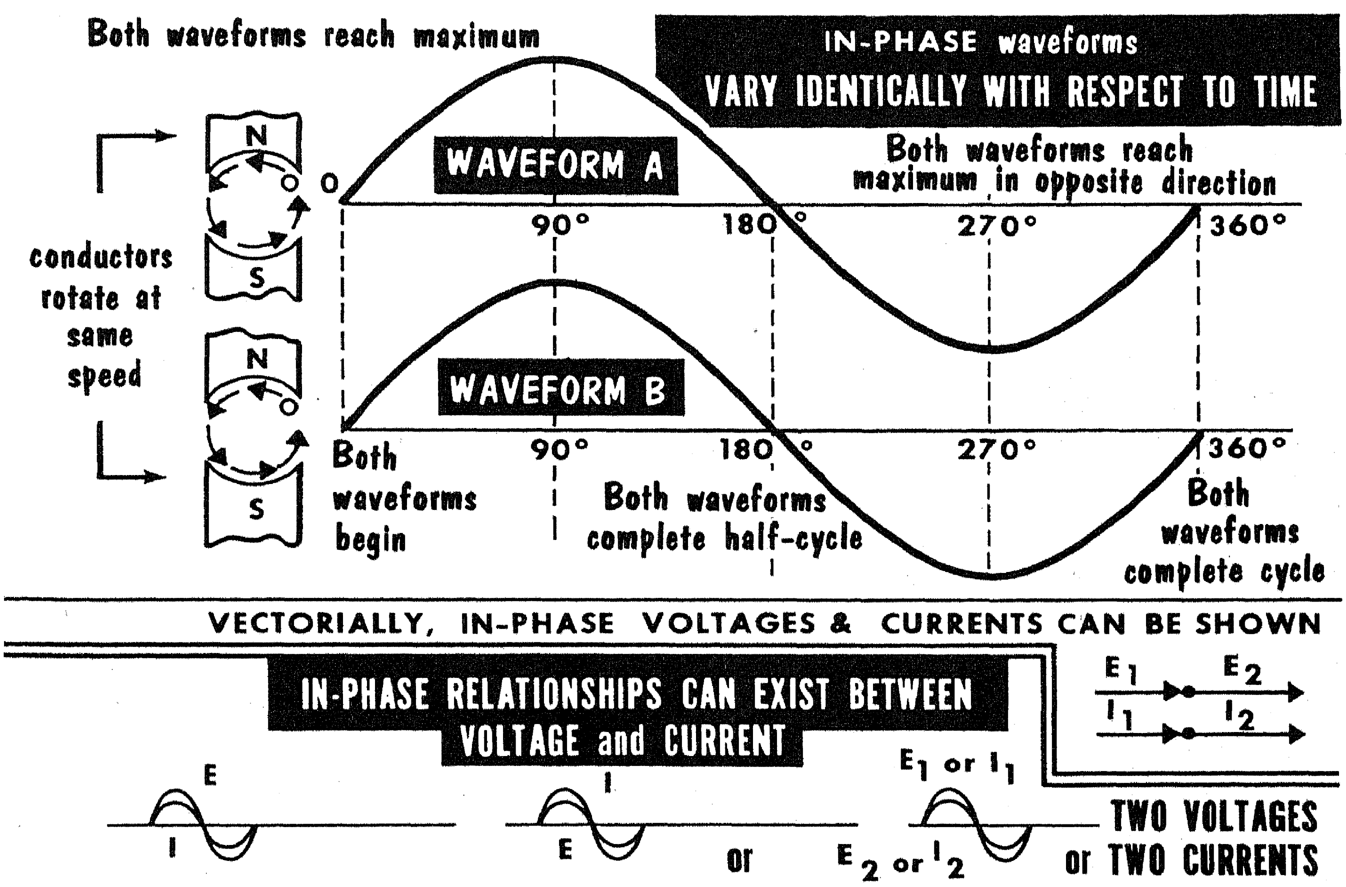
Related to the behavior of alternating current and voltage is a mathematical term known as rate of change. It refers to the relative change in value of an a-c voltage or current in a unit period of time. For example, if a current (or voltage) changes a great deal in value in a given small interval of time, its rate of change is high; if its value changes only a little in the same period of time, its rate of change is low. If an alternating current (or voltage) waveform is plotted on a graph, the slope or steepness of the waveform as it increases or decreases at any point on the graph, relative to the horizontal axis or time scale, is an indication of the relative rate of change. From this, it is evident that the higher the frequency of a voltage or current, the faster the rate of change.



The rate of change of a sine waveform quantity is maximum at the instant that the current (or voltage) is passing through zero in both the positive and negative-polarity directions. It is minimum (zero) at the moment when the waveform is passing through its maximum amplitude. At this instant, the quantity is neither decreasing nor increasing. Thus, the maximum rate of change occurs at 0°, 180°, and 360°; the minimum rate of change occurs at 90° and 270°.

'he Concept of Phase (In Phase)

Phase," sometimes referred to as "phase displacement," "phase differ- nce", or "phase relation," is a concept of a time relationship between two lternating quantities--voltage, currents, or a current and a voltage. By ime relationship in a-c, we mean the extent to which the two quantities re- ıain in step or go out of step as their amplitudes change in value. In a d-c ircuit, a change in current keeps in step (phase) with a change in voltage; his is not necessarily the case in a-c circuits. Circuit components other han resistance cause changes in phase. When the voltage and current hanges keep in step with each other, they are said to be in phase. This al- ays takes place in a resistive circuit, since voltage and current in a resist- nce are in phase.



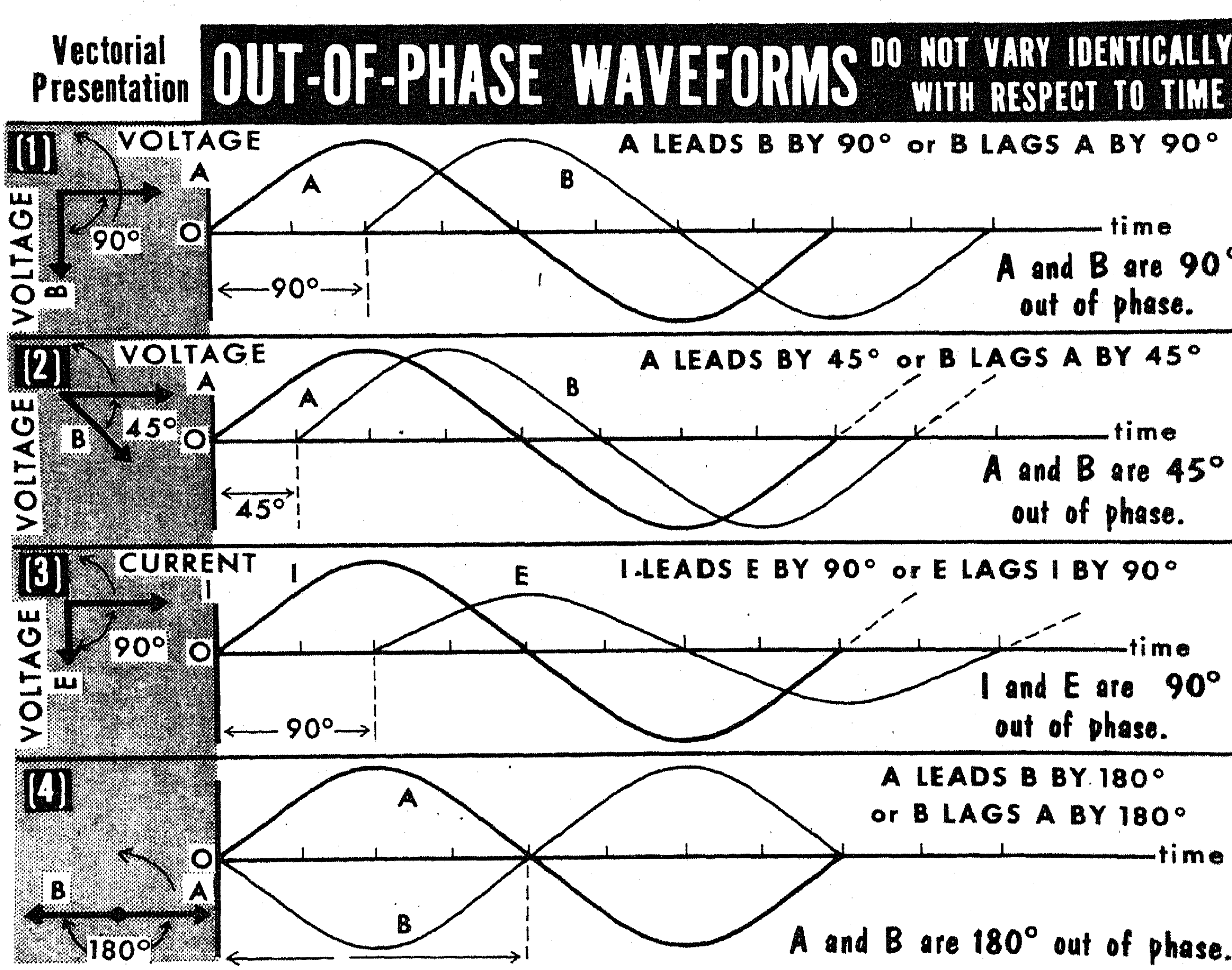
Imagine two identical generators that start functioning at the same instant, with their armatures revolving at the same speed. Each will generate a sine waveform voltage in which zero and maximum amplitudes occur together, and where their relative intermediate values will occur at the same time. We describe such behavior of two generators as being in step, or in phase, and producing two voltages which are in phase. Another way of stating this is to say that the two voltages have 0° phase difference, or that the phase angle of voltage A relative to voltage B (or vice versa) is 0°. When considering the phase relation between two quantities, a suitable point of reference is the instant when the two quantities pass through zero amplitude in the same direction. When shown as vectors, the in-phase quantities have a common origin and lie along the same plane, each vector head having its own identity.

Concept of Phase (Lead and Lag)

Out of phase is a broad expression which indicates that the identical amplitude variations of the waveforms do not occur at the same time. Like two runners (A and B) who are racing, A can arrive at a selected point ahead of B, or lead B, which automatically places B behind A, or lagging A. In electrical considerations, the point of reference is the instant when the two waveforms being compared pass through zero amplitude in the same direction. Whichever quantity passes through this point first is leading the other.

Out-of-phase conditions are expressed in electrical degrees, because this manner of expression is much more convenient than referring to a fractional part of a cycle. It is preferable to say a 90° phase difference than 1/4 cycle; using the term 45° phase difference is preferable to 1/8 cycle.

If you examine (1) below, you will note that voltage A passes through zero going in the positive direction one-quarter of a cycle, or 90° before voltage B. In (2), the two voltages (A and B) have a phase difference of only 45° with A leading B, which is the same as B lagging A by this amount. In (3), current I is leading voltage E by one-quarter of a cycle, or 90°, or E is lagging I by 90°. In (4), voltages A and B are 180° out of phase. They pass through maximum and zero points at the same time but in opposite directions.



Any line drawn through the center of a circle which divides the circle in half is called the diameter.
The constantly curving line that forms a circle is called the circumference.
The three sides of a right triangle are: the base (horizontal side); the altitude (vertical side); and the hypotenuse (side opposite the right angle).
In a right triangle, the angle formed by the base and the hypotenuse is referred to as the angle theta (θ).
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides ($C^2 = A^2 + B^2$).
A vector relationship is used to describe any situation or action that involves both magnitude and direction.
The basic method of inducing an emf in a conductor is to move it across the lines of force of a stationary magnetic field.
Maximum voltage is induced in a conductor when the conductor cuts the magnetic lines of force at right angles, or 90°.
The basic a-c generator is a pivoted-loop armature having two coil sides which rotate between magnetic poles with uniform velocity and through a uniform stationary magnetic field.
A cycle is one complete series of changes in an a-c current or voltage. Frequency is the number of cycles which occur in one second.
The sine waveform is a pattern of instantaneous changes in the value of an alternating voltage or current.
The instantaneous value of an a-c voltage or current is that value which exists at any specific instant of time. The peak (maximum) value is the highest value reached by a quantity during a cycle.
The average value of a sine wave equals .636 of its maximum value.
The root mean square (RMS) or effective value of a sine wave equals .707 of its maximum value.
The higher the frequency of a voltage or current, the faster the rate of change.
Phase refers to a time relationship between two alternating quantities--voltages, currents, or voltages and currents.
Out of phase broadly indicates that the identical amplitude variations of two waveforms do not occur at the same time.

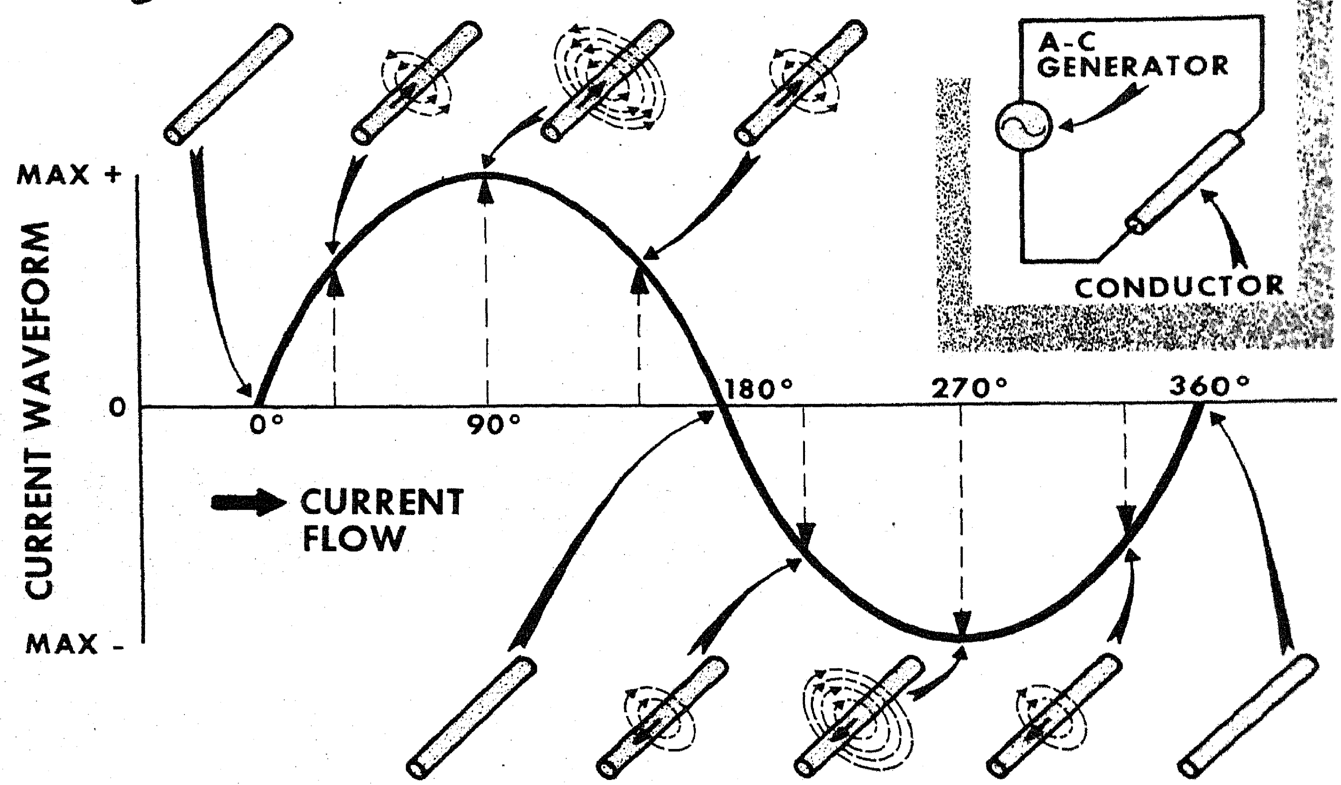
REVIEW QUESTIONS

1. What are the three sides of a right triangle?
2. Explain the parallelogram method and its application to vector analysis.
3. In a right triangle, what is the angle formed by the base and the hypotenuse referred to as?
4. What is the relationship of the three sides of a right triangle to each other?
5. What is a vector relationship used to describe?
6. Give three factors which determine the magnitude of an induced emf.
7. What is the basic method used to induce an emf in a conductor?
8. Explain the principle of operation of the basic a-c generator.
9. What is a sine waveform?
10. Define a cycle. Define frequency.
11. What is meant by the rms or effective value of a sine wave and what is it equal to? What is the average value of a sine wave equal to?
12. Explain what is meant by rate of change.

Magnetic Field around Alternating Current

Alternating current is encircled by loops of magnetic force that change in number instant by instant and periodically change direction.

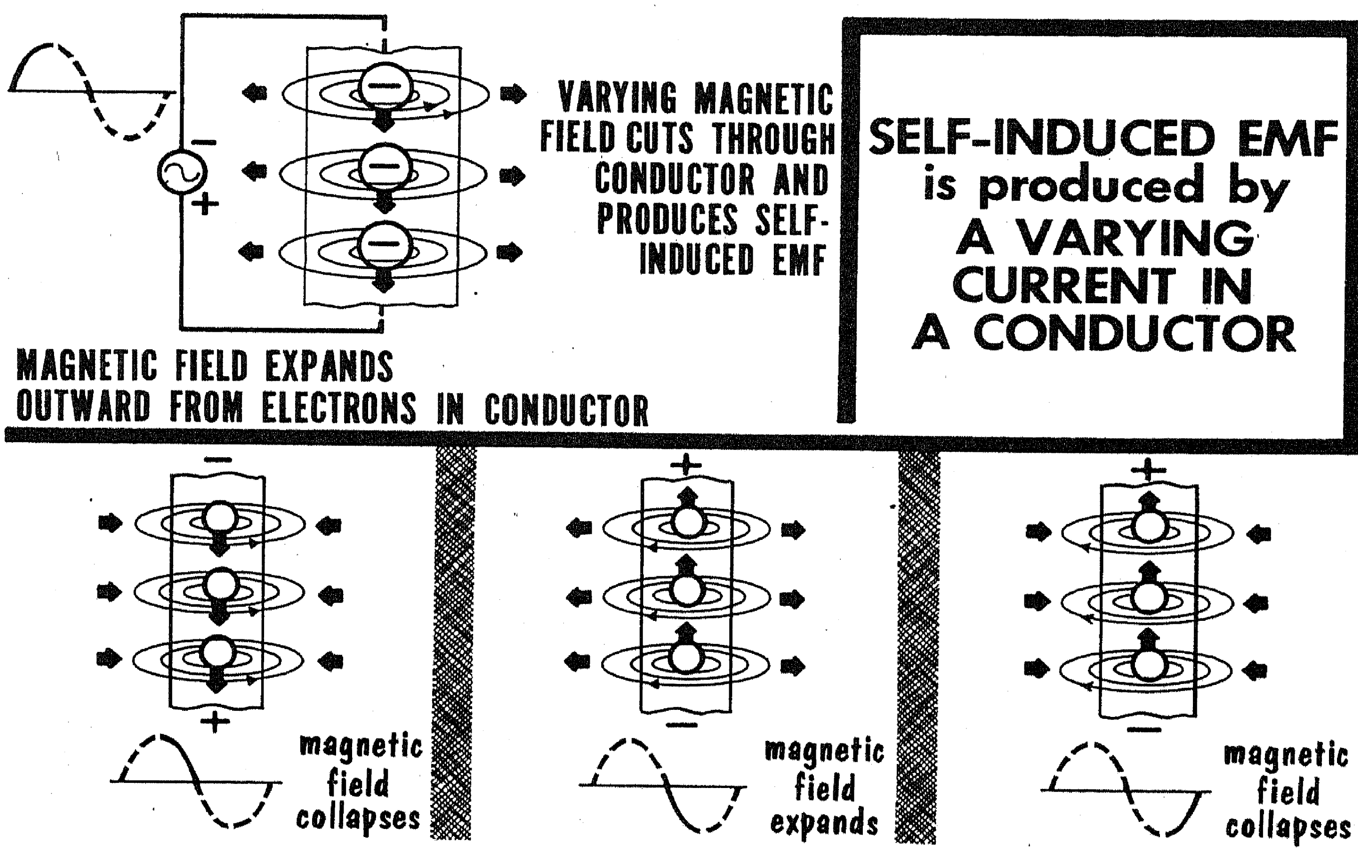
Magnetic Field around a Current-Carrying Conductor during 1 Complete Cycle



Assume a sine waveform current. (Whatever happens during one cycle occurs during the others.) When the current is zero, there is no magnetic field around the conductor. As current begins to increase, the magnetic field builds up in density, reaching maximum value coincident with the maximum current point in the positive half-cycle. The direction of the field is determined by the left-hand rule. We show the field counterclockwise; during the positive alternation, as the current starts to decrease in value but is still in the same direction, the intensity of the magnetic field also decreases. To all intents and purposes, the lines of force of the previously higher value of current in the vicinity of the conductor fall back into the conductor, i.e., the field collapses, reaching zero intensity when the current reaches zero. At this instant, the direction of flow reverses. As the current begins to flow in the opposite direction, increasing in value moment by moment, the magnetic field starts increasing in intensity--but now it has a direction that is the opposite of what existed before. We show it as having a clockwise direction. Maximum field strength is again reached at the peak point of the negative half-cycle; then, the magnetic field begins to decrease, again collapsing into the conductor, reaching zero at the instant the cycle has been completed. An alternating current produces a constantly changing magnetic field around the conductor in which it is flowing.

Self-Induction of EMF

You have learned that relative motion between a magnetic field and a conductor will induce an emf in a conductor. In the examples shown so far, the moving or changing magnetic field was produced by one component (a magnet), and the conductor in which the emf was induced represented another component.

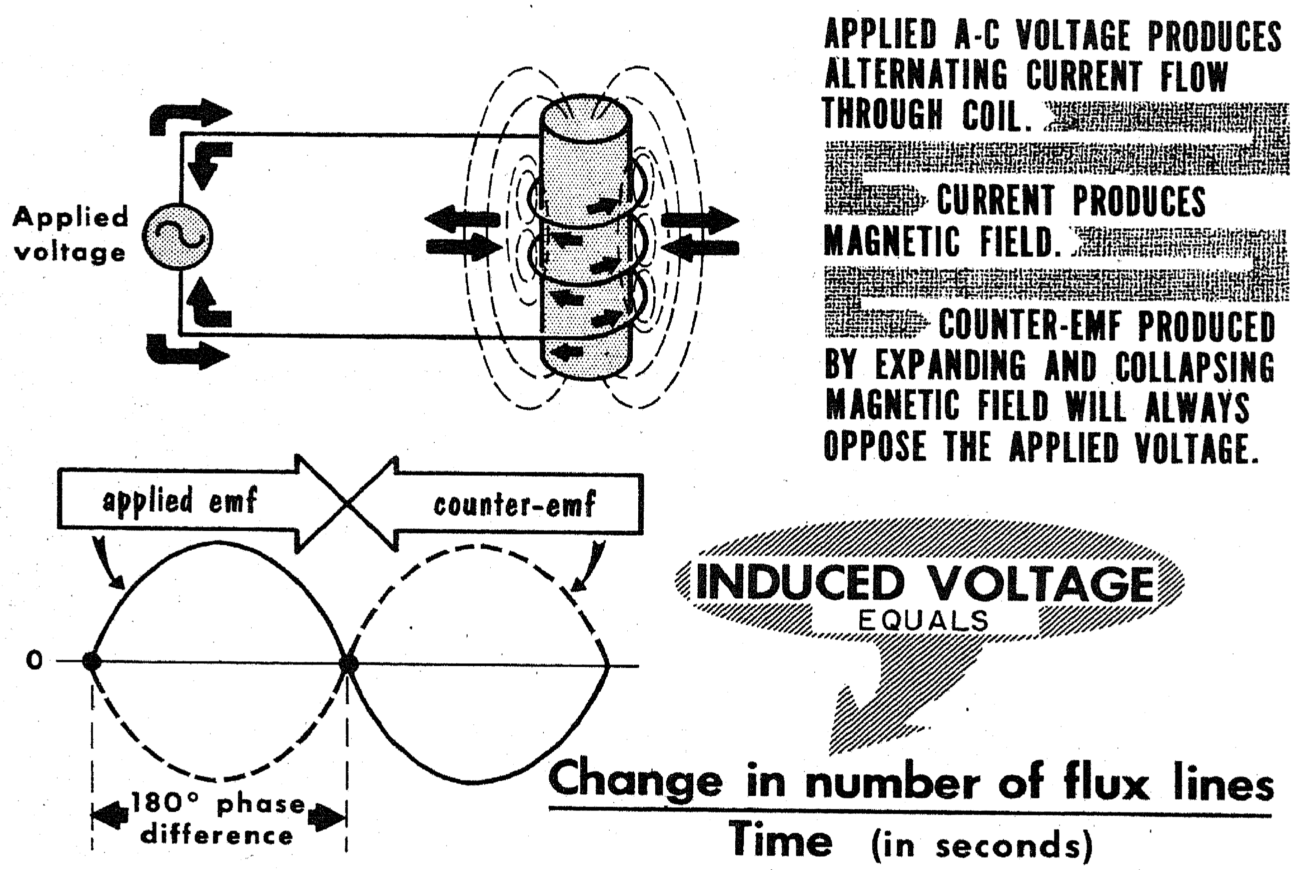


Self-induction of emf involves a changing current, a changing magnetic field, and a conductor; but now, the path of the current, the place of origin of the magnetic field, and the conductor in which it induces a voltage are one and the same. This is why we call the emf that is produced a self-induced emf. To visualize the action, think of it this way. An increasing current creates an increasingly intense magnetic field. The magnetic field originates at the free electrons inside the wire. As the field increases and expands from inside the wire to outside the wire, it must first move through the wire. It is during the time that the expanding magnetic field is cutting the wire that the emf of self-induction is generated.

Now imagine the current to be decreasing. The surrounding field collapses into the wire, i.e., returns to its place of origin, the free electrons. While moving back through the conductor to the electrons, the shrinking loops of flux cut the conductor and induce an emf--a self-induced emf. In one case, the emf of self-induction is generated by lines of force that move outward; in the other case, the emf of self-induction is generated by lines of force that move inward. If one direction of cutting due to a rising current generates a voltage of one polarity, the opposite direction of cutting due to a falling current generates an emf of opposite polarity.

The Action of Self-Induced EMF (Lenz's Law)

Assume an a-c circuit in which the current is increasing. The self-induced emf produced would have a polarity opposite to the applied voltage and, therefore, acts in opposition to this voltage and tends to retard the build-up of the circuit current. When the circuit current decreases, the self-induced emf has a polarity which aids the applied voltage and so tends to maintain the current; that is, prevent it from falling together with the decrease in voltage. Because the action of the self-induced emf is opposite to that of the applied voltage, it is often referred to as counter-emf or back-emf.



The behavior of self-induced emf was first explained by H. F. Emil Lenz and has since become known as Lenz's law. Although stated in different ways, Lenz's law states: "A changing current induces an emf whose polarity is such as to oppose the change in current." Counter-emf is not readily measurable, but its effects can be observed. When a circuit in which a substantial amount of current is flowing through coils is suddenly opened, the sudden collapse of the magnetic field induces a counter-emf which can be greater than the originally applied voltage. In fact, the counter-emf may even cause a momentary arc to bridge the gap where the circuit was opened.

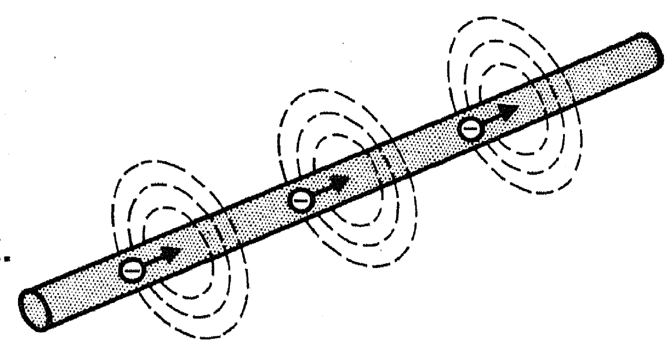
The amount of counter-emf produced will depend upon the rate of change at which the expanding and contracting magnetic lines of force cut the conductor. The greater the current, the more lines of force cutting per unit time; the higher the frequency, the more rapidly the magnetic field moves and, again, the more lines of force cutting per unit time.

Inductance

The physical shape of the conductor in which current flows, sometimes referred to as the "geometry of the inductor," also has a bearing on the control of the current. The loops of flux lines associated with current in a straight wire cut only that conductor during expansion and contraction of the surrounding magnetic field. The number of flux linkages between the lines of force and the conductor is the same as the number of loops of force produced by the current in the conductor. If, however, the conductor is coiled to form a solenoid, each turn links not only with the flux lines from that turn, but also with flux lines from adjacent and nearby turns.

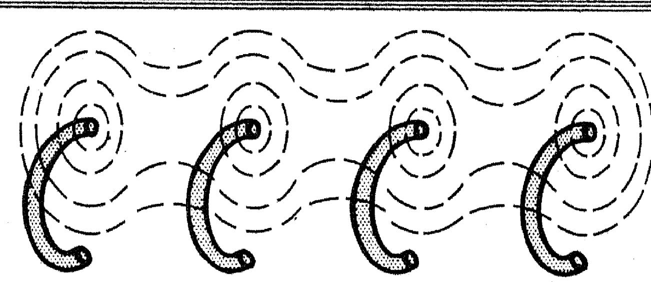
Inductance is that Property of a Circuit or Component which Opposes a Change in Current.

In expanding and contracting, flux linkages induce a counter-emf in a conductor. Induced voltage opposes both a rise and fall of circuit current.



A STRAIGHT CONDUCTOR CONTAINS INDUCTANCE.

Cross sectional view of a coil shows flux linkages between turns of the coil.



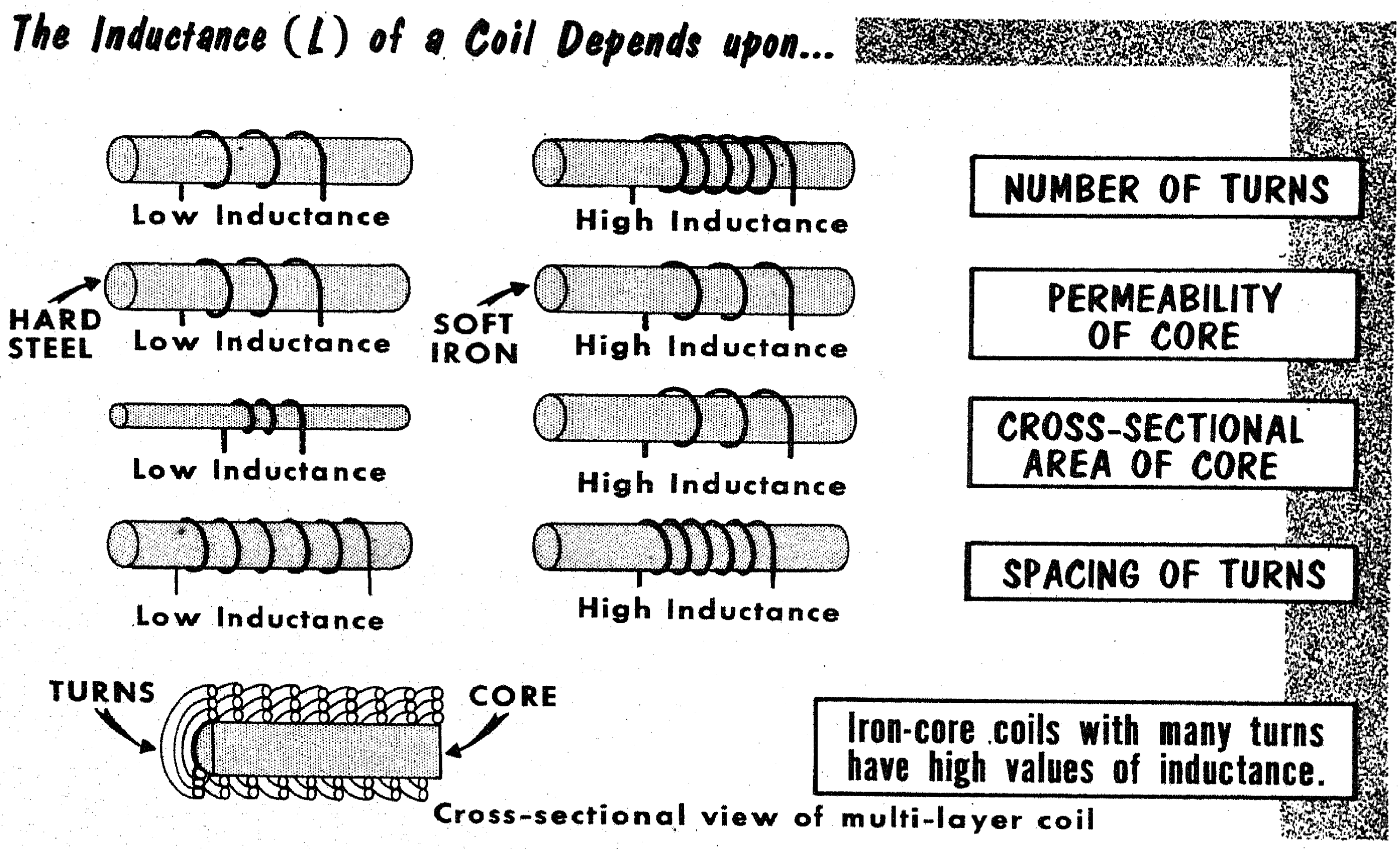
A COIL CONTAINS MORE INDUCTANCE BECAUSE INCREASED NUMBER OF FLUX LINKAGES PROVIDES GREATER COUNTER-EMF.

The total number of flux linkages with each turn is, therefore, several times the number of flux lines from a given current in the turn. If 10 flux lines originate from each of three adjacent turns and the lines link with each turn, the total number of flux linkages is 90, whereas the total number of lines that originate from the three turns is only 30. The greater the number of flux linkages per unit time, the greater the emf induced in each turn; hence, in the solenoid as a whole. For any given current in a coil, the counter-emf induced in the coil is a function of the number of flux lines multiplied by a constant that arises from the shape of the coil. The constant is symbolized by the capital letter L and is called self-inductance, or simply inductance.

Inductance (Cont'd)

The greater the inductance of a coil, the higher is the induced emf and the greater is the opposition to the increase and decrease of current in the solenoid.

Every conductor--short or long--has inductance. When the frequency is low (say up to several hundred cycles), the effect of the inductance of any reasonably long length of straight conductor is negligible. When the straight conductor is coiled, its inductance increases substantially. Even if the conductor is not coiled but its length is great, the amount of inductance possessed by the straight wire can be sufficient to influence current flow. This situation can be a problem for even a reasonably short length of wire when the operating frequency is high.



The greater the number of turns in a coil, the higher is its inductance. The closer the coil turns are to each other, the higher the coil inductance, because the flux linkages increase in number. If the core of a solenoid is made of a high-permeability material, such as soft iron, the inductance increases still more. On the other hand, if a coil is wound by doubling the wire back on itself, the inductance is held to a minimum. The self-induced emf generated in one half of the total length of the conductor offsets the self-induced emf generated in the other half of the conductor; hence, the coil as a whole displays minimum or even negligible inductance. Such a winding is known as a "non-inductive" winding. This method of winding is used to form wire-wound resistors wherein d-c resistance is desired but where inductance is an undesirable effect.

Unit of Inductance--The Henry

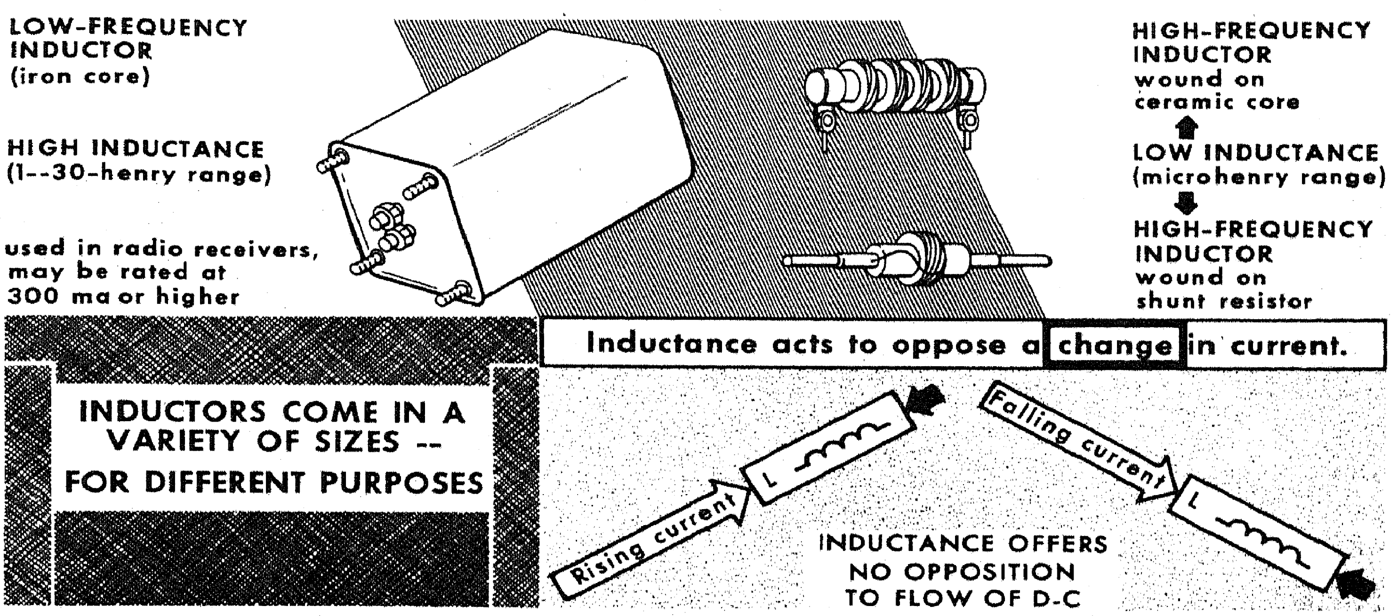
The unit of inductance is the henry, named after the American physicist, Joseph Henry. By definition, a conductor, or coil, has an inductance of 1 henry when a current which changes at the rate of 1 ampere per second induces an emf of 1 volt. The number of flux lines corresponding to this rate of change in the current is 100,000,000, or 10^8. In defining inductance as flux linkages per ampere of current producing the flux, we can say:

$$\text{Inductance (in henries)} = \frac{\text{flux linkages}}{\text{current producing flux}} \times 10^{-8}$$

Having established the above, we can now study the formula for determining the magnitude of a counter-emf:

$$\text{Counter-emf (induced voltage caused by changing current)} = -L \times \frac{\text{change in current}}{\text{change in time}}$$

The minus sign means that the voltage developed is a counter voltage and opposes the force producing it. From this, we can see that the greater the inductance or the faster the rate of current change, the greater the counter-emf induced in the circuit.

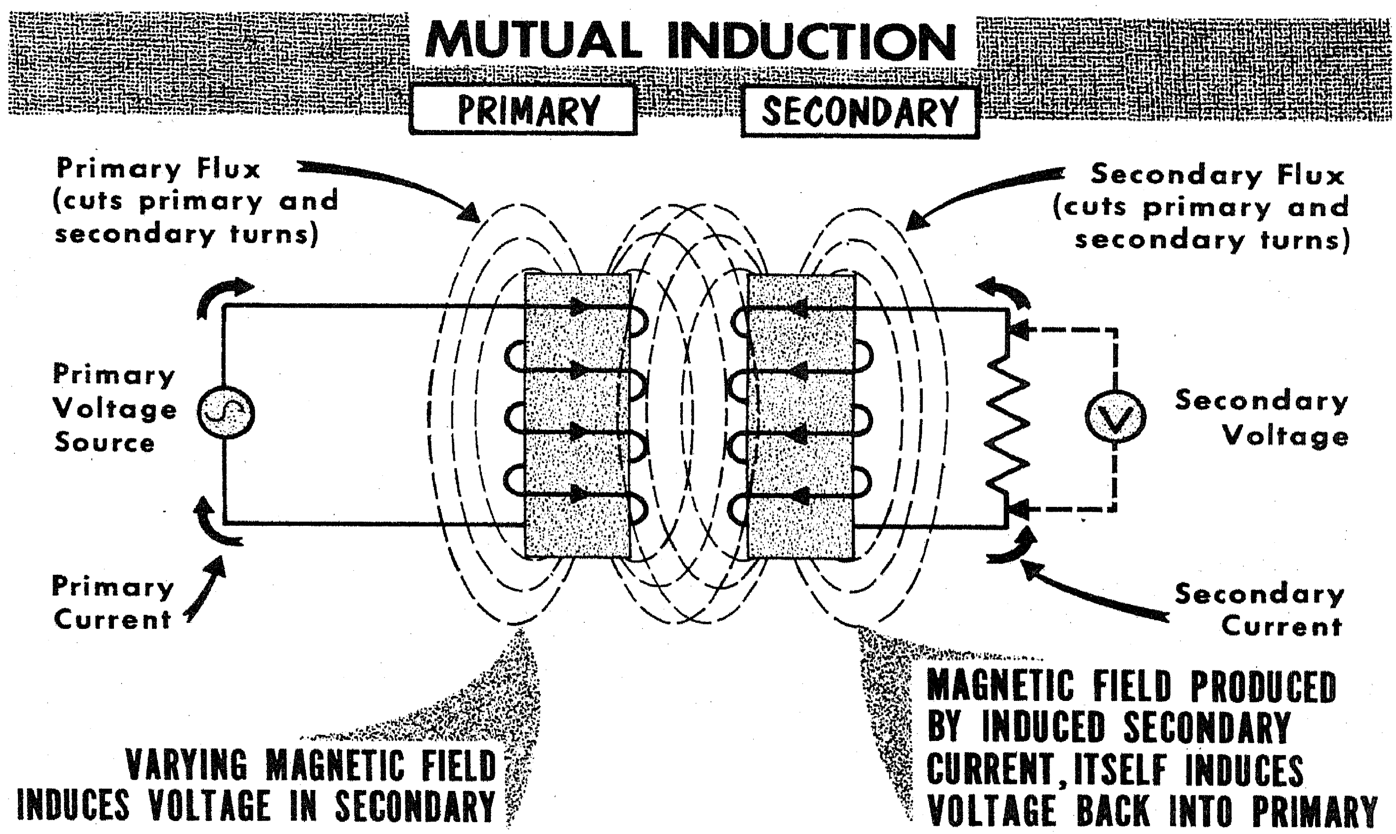


The inductance of coils used in radio communications equipment ranges from extremely small air-core units of 1 microhenry and less to large iron-core multilayer units of 30 henries and more. Inductors are also usually rated at some specific current. When excessive current flows through an iron-core inductor, the core may become "saturated" and the inductance decreases.

From what we have learned, we see that in addition to resistance, another circuit property, inductance, also is involved in the control of current. Of course, we must remember that while resistance opposes the flow of both a-c and d-c, the effect of inductance comes into play only under conditions of alternating or changing current.

Mutual Induction

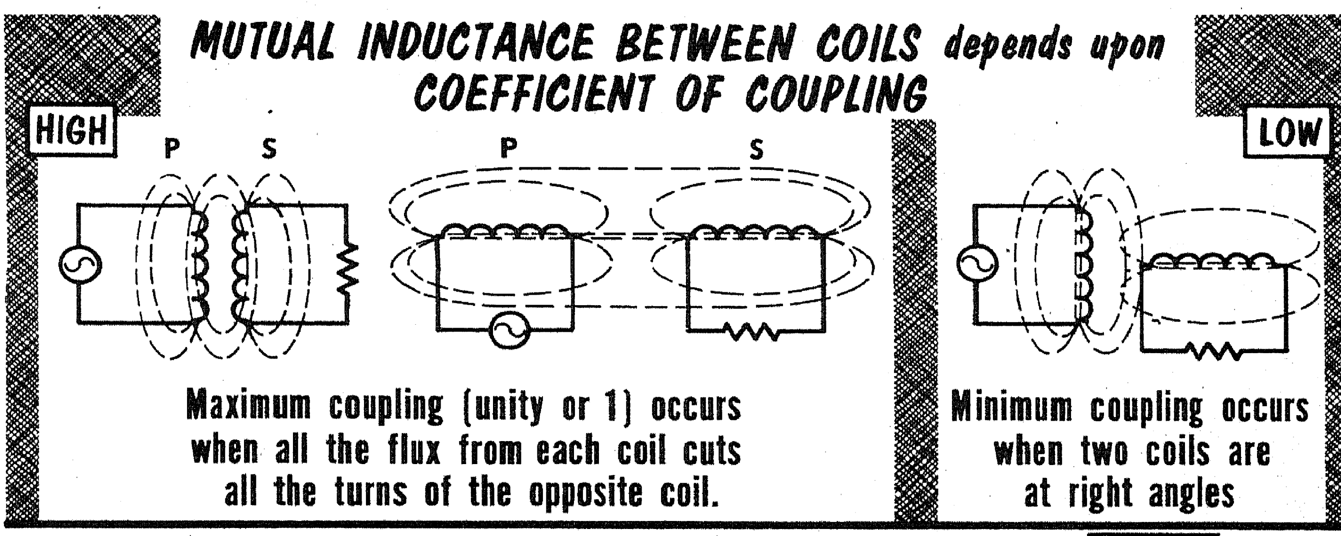
When a changing magnetic field produced by one coil cuts the turns of a second coil and induces an emf in the second winding, the action is known as mutual induction. The winding from which the flux originates is called the primary, usually indicated by the letter P. The voltage that is applied to the primary winding and causes current to flow is called the primary voltage. The changing current that flows in the primary winding and produces the changing flux is the primary current, sometimes referred to as the inducing current. The winding in which the emf is induced by the changing magnetic field is known as the secondary, usually indicated by the letter S. The emf induced in the secondary winding is known as the secondary voltage. If the secondary is part of a closed circuit wherein current flows, this current is called the secondary current. Mutual induction is a basis for transferring electrical energy from one circuit to another by means of a changing magnetic field. This is the basis of transformer operation.



The amount of emf induced in the secondary is, among other conditions, a function of the physical positioning between the primary and secondary windings. The positioning determines the flux linkages between the windings and, therefore, the rate of cutting by the lines of force. This physical relationship is known as "coupling." Coupling, flux linkage, and induced emf are maximum when the primary and secondary turns are interwound, or when the primary and secondary windings are one above the other and very close together. The more the number of turns of the secondary winding that are cut by the changing flux from the primary, the higher the emf induced in the secondary. The emf induced in each turn of the secondary winding is additive to the others.

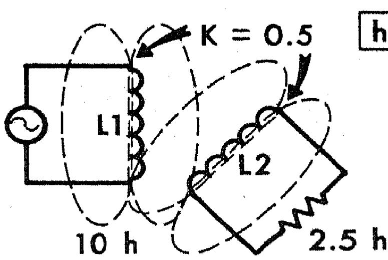
Mutual Inductance

If a sine waveform voltage is applied to the primary winding of a primary-secondary assembly, a similarly varying current will flow in the primary winding. During the time that the primary current increases, its magnetic field expands, cutting the turns of the secondary and inducing a voltage in it. The secondary voltage, in turn, causes the flow of secondary current which has such direction (opposite to the primary current) as to create a magnetic field that opposes the field produced by the primary current. This action conforms with Lenz's law.



MUTUAL INDUCTANCE (M) = K (coefficient of coupling) $\sqrt{L1 \times L2}$

henries — decimal from 0.0 to 1.0 — henries



$$M = 0.5\sqrt{10 \times 2.5} = 0.5\sqrt{25} = 0.5 \times 5 = \underline{2.5 \text{ HENRIES}}$$

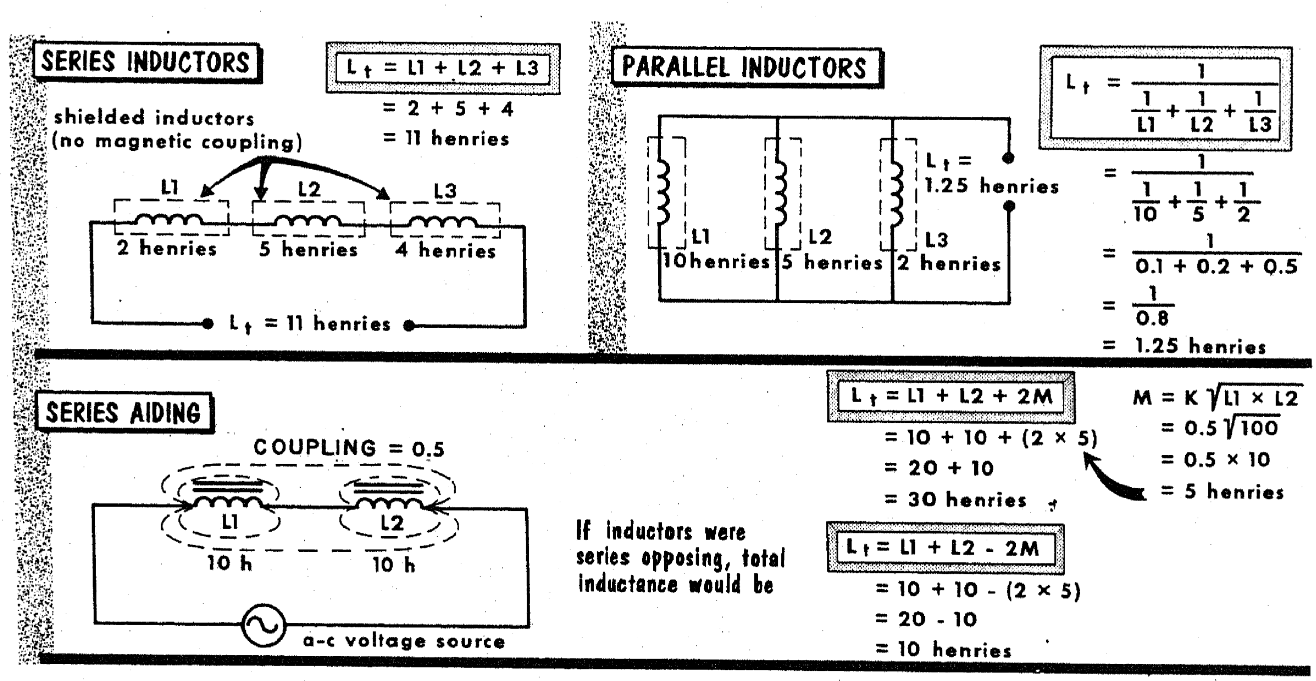
The magnetic field produced by the secondary current expands and, in so doing, cuts the turns of the primary winding. Here, it induces an emf which acts in opposition to the emf that is self-induced by the primary current. The resultant voltage of these two oppositely acting voltages is lower in value than the original self-induced voltage. Therefore, the primary current rises higher than it would were the field from the secondary current absent. During the period of decreasing primary current, the collapsing magnetic field cuts the secondary winding and induces an emf. The secondary current now has a direction that produces a magnetic field which tends to offset the collapsing field around the primary; i.e., it aids the self-induced emf in the primary and thus tends to prevent the primary current from falling.

The control of the primary current is presumed to be the result of magnetic lines of force common to both the primary and secondary windings. These common flux linkages are given the name mutual inductance, which is designated by the letter M and uses the henry as its unit. Any two coils positioned so that flux from one links with the other have mutual inductance.

Inductance in Series and Parallel

In order to achieve certain desired amounts of inductance, it is sometimes necessary to conduct inductors in series or parallel. When connecting inductors in series, the total inductance will be the sum of all the individual inductances: $L_t = L1 + L2 + L3 + \text{etc.}$ This formula holds true, however, only when the inductors are shielded from each other, or so positioned physically that there is no mutual inductance between them. If however, two inductors are located so that the flux lines from each cut the turns of the other, then the total inductance must take into consideration the mutual inductance between them.

We use the formula: $L_t = L1 + L2 \pm 2M$. The plus-or-minus 2M is used to take into consideration that the two inductors can be connected either series aiding or series opposing. In series aiding, the two inductors are arranged so that their flux lines move in the same direction and thus aid each other. This additional mutual inductance adds to the basic inductances of L1 and L2. When two inductances are arranged so that their flux lines oppose each other, we say that they are connected series opposing. Thus, the coefficient of coupling and the direction of magnetic fields help determine the total inductance. We can note an interesting point here. If we double the number of turns of a coil and assume perfect coupling (K - 1), we can achieve four times the inductance.



The total inductance of a circuit containing inductances in parallel is calculated in the same manner as resistances in parallel:

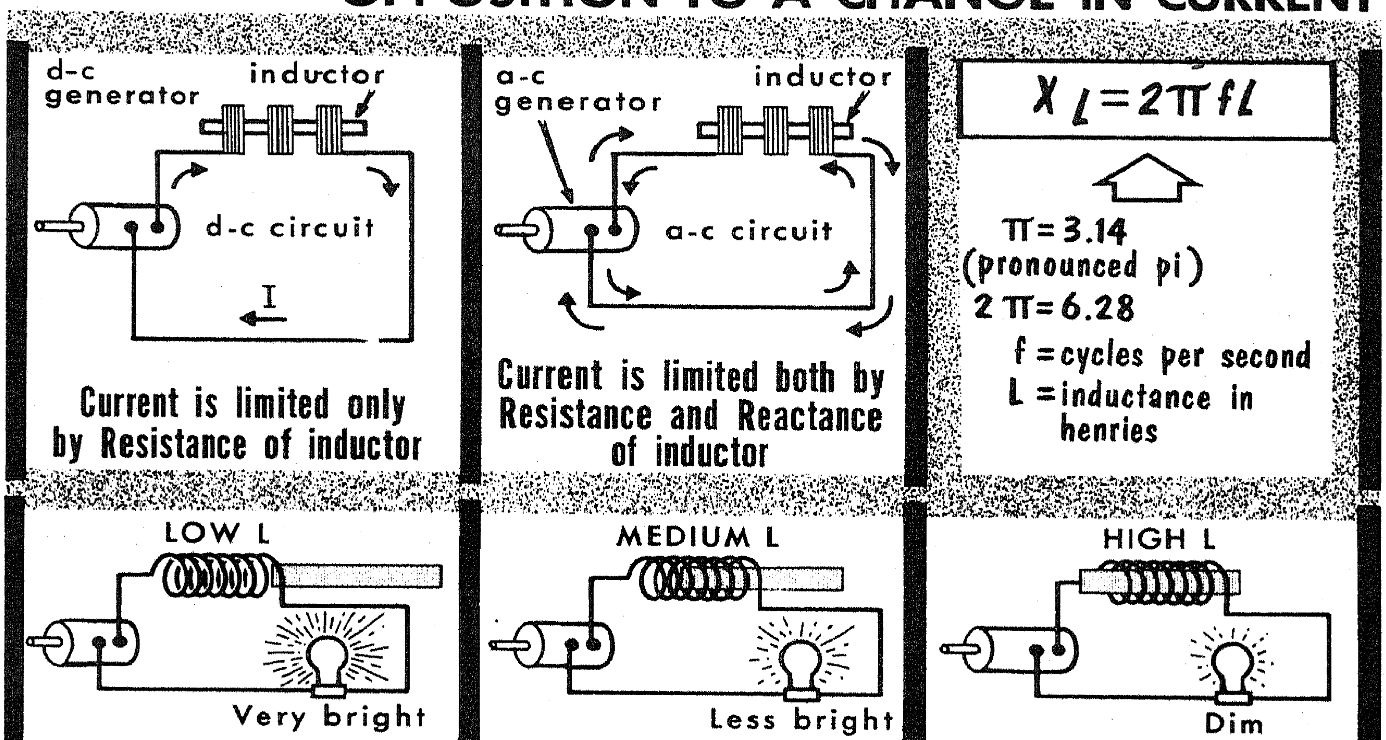
$$L_t = \frac{1}{\frac{1}{L1} + \frac{1}{L2} + \frac{1}{L3} + \text{etc.}}$$

The above formula is valid, as in series opposing, only when each inductor is shielded from the other. Any mutual inductance existing between inductors in parallel tends to reduce the total inductance.

Inductive Reactance

The opposition offered by an inductance to a change in current is measured at any given instant in terms of counter-emf (the voltage opposing the applied voltage). We saw that the characteristic of an inductance was to oppose any change in current, be it an increase or a decrease. This opposition presented by an inductance to an a-c or changing current is called inductive reactance (indicated as X_L). It can be compared somewhat to resistance (R). In d-c circuits and in a-c circuits containing only resistance, the total opposition to the flow of current is the resistance, in ohms ($R = E/I$).

INDUCTIVE REACTANCE -- OPPOSITION TO A CHANGE IN CURRENT

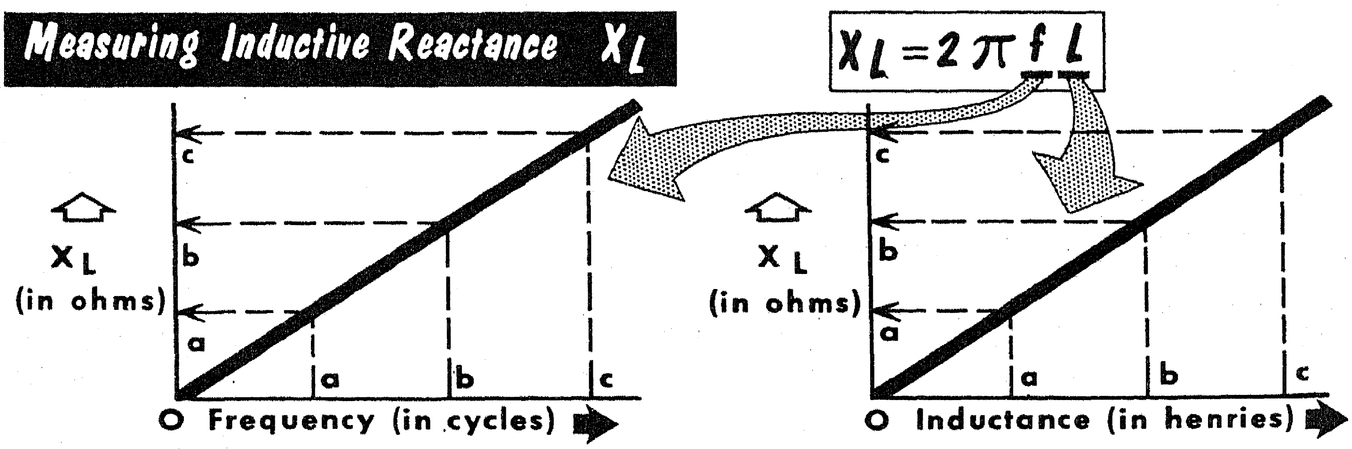


As inductance of circuit increases, reactance increases.

Reactance comes into play only under varying conditions. It represents an opposition to the flow of a varying current. Thus, the opposition offered by an inductor is called inductive reactance and, like resistance, is also measured in ohms. (Later in this course, we will study the reactance presented by a capacitor, called capacitive reactance.) Since the magnitude of induced emf depends on the amount of inductance and the frequency (rate of change) of the current, the formula for inductive reactance takes both into account. Inductive reactance is calculated: $X_L = 2\pi fL$. The $2\pi f$ represents the rate of change of the current. There are 2π(6. 28) radians in a cycle, so $2\pi f$ represents the rate of change in current per second (angular velocity). Frequency (f) is in cycles per second, and L is equal to the inductance in henries. From this formula, we can see that the higher the frequency or the greater the inductance, the greater will be the inductive reactance. This is logical, since an increase in either will cause flux lines to be cut at a greater rate, and produce a greater counter-emf.

Inductive Reactance--Solving Problems

Having established the formula for inductive reactance: $X_L = 2\pi fL$, let us solve some problems concerning inductive reactance so as to gain a greater familiarity with this "new" type of opposition to current flow. In each of the following problems, we will assume that the resistance of the inductor is zero ohms. Actually, this is never the case. Since an inductor is wound with turns of wire, there must be some d-c resistance. Later, we will discuss the practical inductor which contains both resistance and reactance.



As FREQUENCY increases or decreases, inductive reactance increases or decreases. **As inductance increases or decreases, inductive reactance increases or decreases.**

In a simple circuit containing a 60-cycle voltage source and a 10-henry coil we will find the inductive reactance of the coil using the basic formula: $X_L = 2\pi fl = 6.28 \times 60 \times 10 = 3768$ ohms.

Leaving L fixed at 10 henries, and doubling the frequency to 120 cycles, $X_L = 2\pi fL = 6.28 \times 120 \times 10 = 7536$ ohms.

We see that doubling the circuit frequency doubled the inductive reactance. Leaving L at 10 henries but changing the frequency to 30 cycles,

$$X_L = 2\pi fL = 6.28 \times 30 \times 10 = 1884 \text{ ohms.}$$

We see that halving the circuit frequency halved the inductive reactance. Just as doubling and halving the circuit frequency doubled and halved the inductive reactance, the same would be true with doubling and halving the inductance of the coil. We say, then, that the inductive reactance of a coil varies directly with the frequency and with the inductance.

Working with a small 50-microhenry coil at a frequency of 4 megacycles, we get an inductive reactance through the coil of:

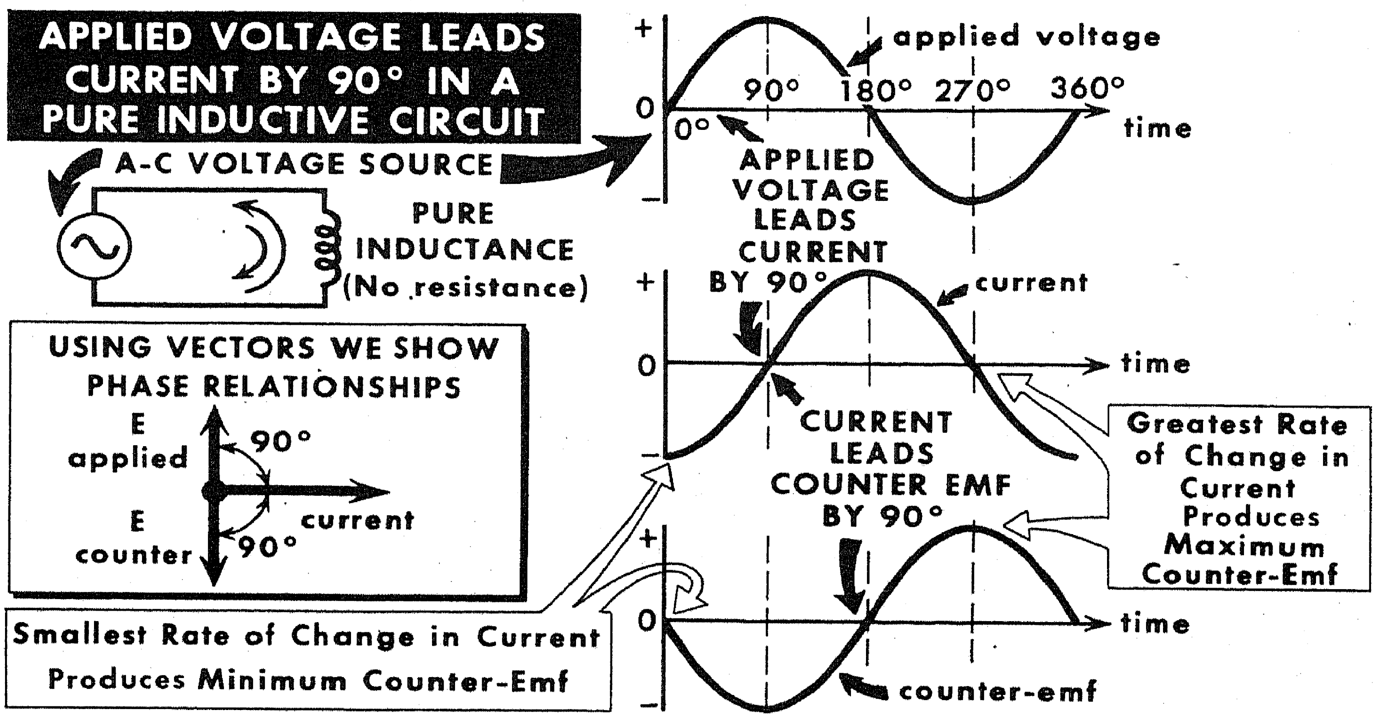
$$X_L = 2\pi fL = 6.28 \times 4{,}000{,}000 \times 0.00005 = 1256 \text{ ohms.}$$

Let us find the inductive reactance of a 1-millihenry coil at 10 kilocycles.

$$X_L = 2\pi fL = 6.28 \times 10{,}000 \times 0.001 = 62.8 \text{ ohms.}$$

Alternating Voltage and Current in an Inductive Circuit

In discussing voltage and current in an inductive circuit, we shall first assume an ideal inductance--one without any resistance. This will establish basic theory about a "pure" inductive circuit which, while it never occurs, enables us to understand practical inductive circuits. We learned that inductive reactance not only limits the amount of current flowing in an inductive a-c circuit, but also delays the increase or decrease of current in the circuit. The current in an inductive circuit takes the form of a sine wave if the applied voltage is of a sine waveform, except that the current is delayed or lags behind the voltage variations.



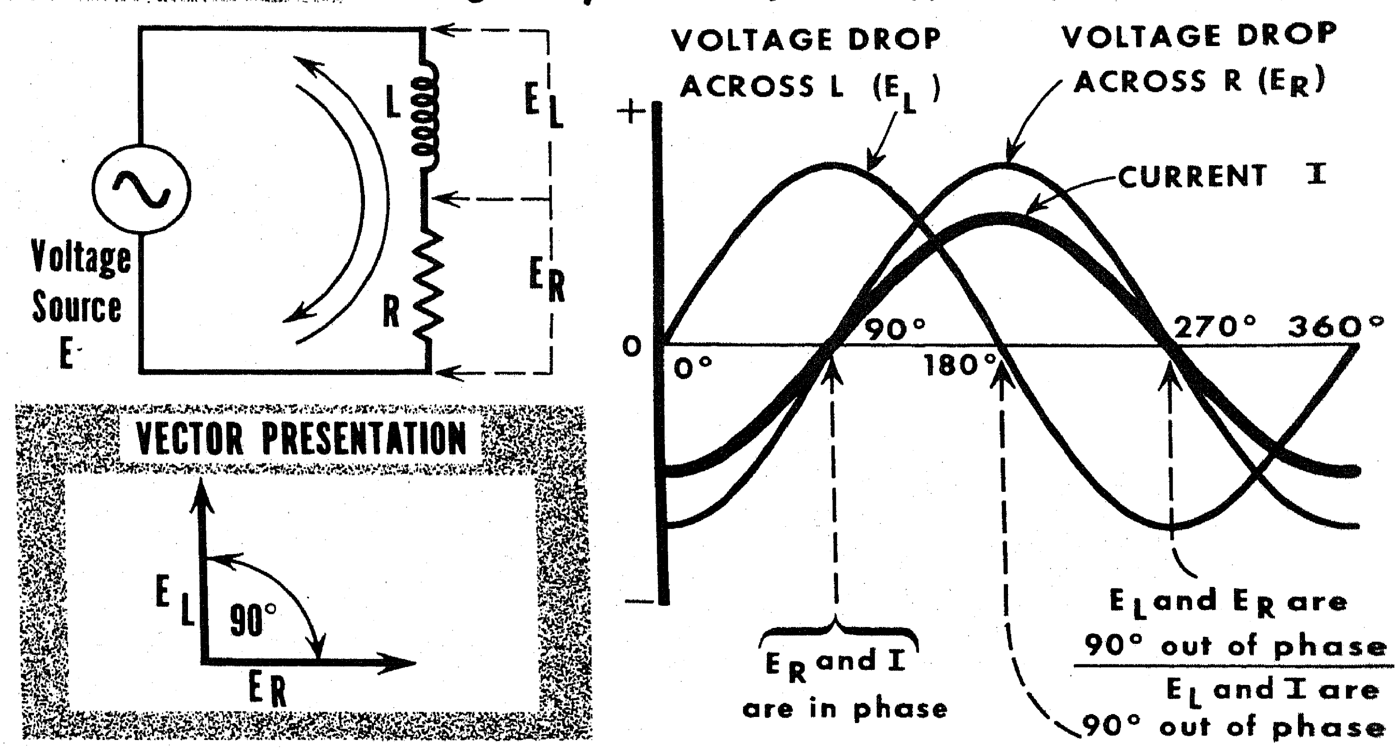
We learned that in an inductive circuit, a change in current produces a counter-emf which acts to oppose that change. It was established that the counter-emf was 180° out of phase with the applied voltage. We can now consider the relationship of the current in an inductive circuit to the applied voltage and the counter-emf. Since the counter-emf is induced by the changing current, it follows that the maximum counter-emf is induced when the current is changing at its greatest rate. We learned that the greatest rate of change in a sine waveform occurs when the waveform passes through its 0°, 180°, and 360° points; the least rate of change (but maximum value) occurs at its 90° and 270° points. Thus, when the current waveform is at maximum, for example, the counter-emf waveform will be zero; when the current waveform is at zero, the counter-emf is at maximum.

From this, we can see that there is a 90° phase difference between the current in an inductive circuit and the counter-emf it produces. Since the applied voltage is 180° out of phase with the counter-emf, there is a 90° phase difference between the applied voltage and the current. The applied voltage causes the current to flow, so we say that the applied voltage leads the current by 90°, or the current lags the applied voltage by 90°.

Alternating Voltage and Current in R-L Circuits

The practical coil consists of both inductance and resistance acting in series. We have just discussed the effect of inductance on voltage and current. Let us now review the effect of resistance on voltage and current in an a-c circuit. Since the property of resistance has no association with magnetic effects (actually, resistors contain some inductance), current flow through a resistance is assumed to be free of a magnetic field. The absence of a magnetic field prevents the self-induction of an emf; hence, a varying voltage applied to a resistance causes a simultaneously varying current. In other words, voltage and current are in phase in a resistance. We can use Ohm's law to find the current in an a-c resistive circuit just as in a d-c circuit, except that in an a-c circuit, we must think of I and E in the same terms--average, effective, or peak values.

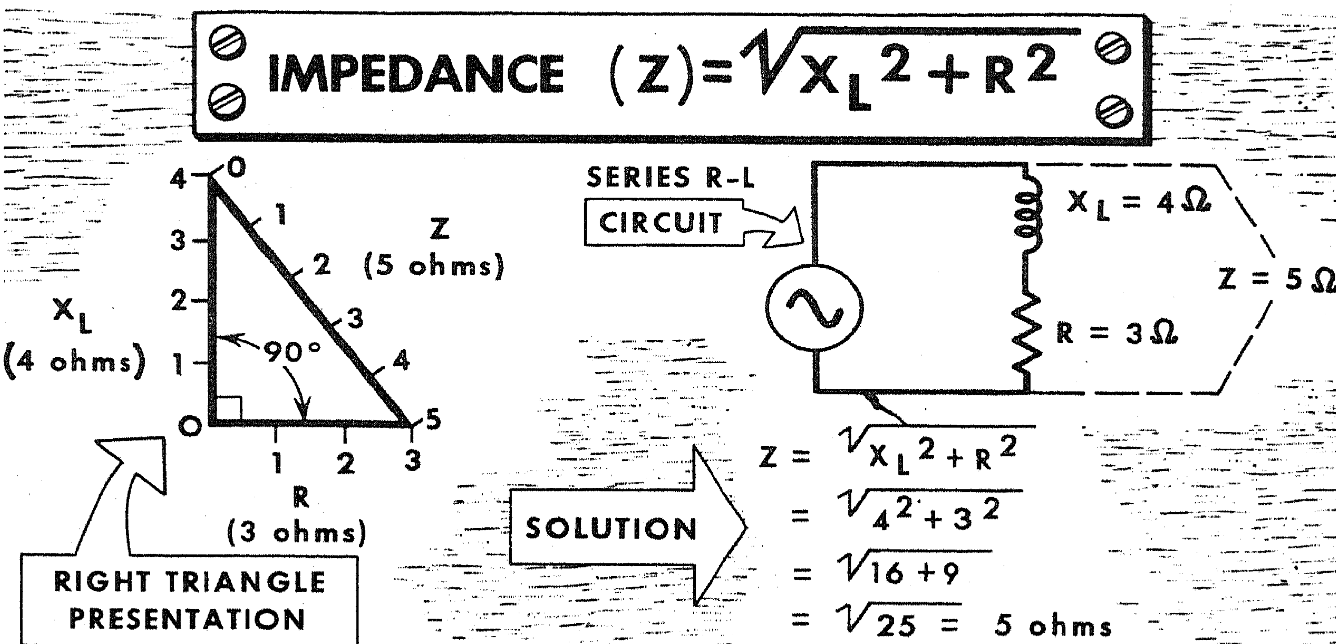
In a Series A-C Circuit Containing L and R, Voltage Drops across L and R are 90° Out of Phase



When an alternating voltage is applied to a practical coil, the same current (I) flows in the inductive and resistive parts of the coil. In flowing through the R-L circuit, the current produces two voltage drops--one across the inductance (E_L) and one across the resistance (E_R). The inductance voltage drop is equal to IX_L; the resistance voltage drop, to IR. With the same current flowing through the coil, E_L leads I by 90°, and E_R is in phase with I. Thus, voltage drops E_L and E_R are 90° apart, with E_L leading E_R. Since the effect of the inductance is to produce a voltage drop 90° out of phase with the current, and resistance produces a voltage drop in phase with the current, the net effect is that the resultant or applied voltage will lead the current in an R-L circuit by a phase angle 90° or less. Later, we will learn how to find the exact phase angle.

Impedance (Z)

Two sources of opposition to current flow exist in the practical inductor--one is inductive reactance (X_L), arising from the action of inductance (L); the other is resistance (R), arising from the nature of the conductor material. The combined actions of X_L and R constitute the total opposition to current flow known as impedance. Impedance is expressed in terms of ohms and is indicated by the letter Z.



THE IMPEDANCE OF A SERIES R-L CIRCUIT CAN NEVER BE EQUAL TO OR AS GREAT AS THE SUM OF X_L AND R, NOR CAN IT BE EQUAL TO OR LESS THAN EITHER X_L OR R.

The two current-opposition components--inductive reactance and resistance--are considered as being in series in an inductor. However, to find their impedance, we cannot add them arithmetically for our answer. To determine the impedance of a series R-L circuit, it is necessary to take into account the 90° phase difference between the voltage drops across the inductance and the resistance. This can be done in two ways: by using the right triangle equation; or by vectors (graphically). The equation method uses the right triangle as its basis. Early in this volume, we learned that if we squared the hypotenuse of a right triangle, that sum would be equal to the sum of the other two sides squared. When we consider the right triangle for calculating impedance, we make the vertical side, or altitude, represent the inductive reactance. The horizontal side, or base, represents the resistance. The hypotenuse which joins the ends of these two sides represents the impedance of the circuit. From this, we can see a basic formula for finding the impedance of a series R-L circuit: $Z^2 = X_L^2 + R^2$. To simplify this and find Z directly, we take the square root of both sides of the equation and get the highly usable formula: $Z = \sqrt{X_L^2 + R^2}$.

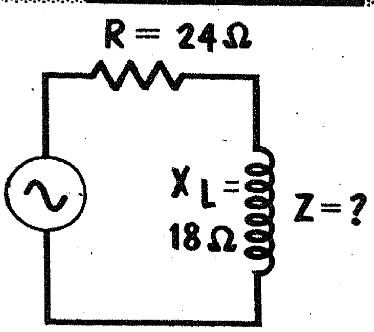
In a series R-L circuit, X_L and R must be considered 90° apart because the same current flows through R and L, but the voltage drops are 90° displaced.

Solving Impedance Problems

Let us now solve some impedance problems in order to familiarize ourselves with this procedure. In the first two, we will assume we already know the inductive reactance; in the third, we will work out the problem.

We have a series R-L circuit in which the inductive reactance is 18 ohms and the resistance is 24 ohms. Find the impedance of the circuit. Using the impedance formula $Z = \sqrt{X_L^2 + R^2}$, we first find the square of X_L. Since X_L is equal to 18 ohms, X_L^2 is equal to 18×18, or 324 ohms. R is equal to 24 ohms, so R^2 equals 24×24, or 576 ohms. We now add X_L^2 (324 ohms) and R^2 (576 ohms) and get a total of 900 ohms. Taking the square root of 900 gives us 30 (30×30 equals 900). Thus, this circuit's impedance is 30 ohms.

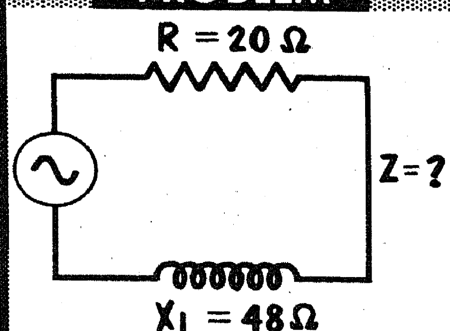
PROBLEM



SOLUTION

$$\begin{aligned} Z &= \sqrt{X_L^2 + R^2} \\ &= \sqrt{18^2 + 24^2} \\ &= \sqrt{324 + 576} \\ &= \sqrt{900} \\ &= \underline{30 \text{ ohms}} \end{aligned}$$

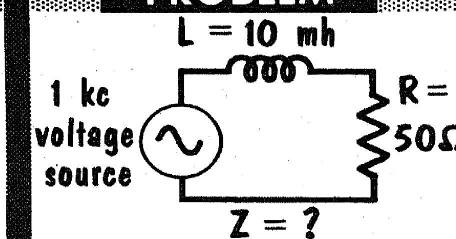
PROBLEM



SOLUTION

$$\begin{aligned} Z &= \sqrt{X_L^2 + R^2} \\ &= \sqrt{48^2 + 20^2} \\ &= \sqrt{2304 + 400} \\ &= \sqrt{2704} \\ &= \underline{52 \text{ ohms}} \end{aligned}$$

PROBLEM



SOLUTION

1st: FIND X_L

$$\begin{aligned} X_L &= 2\pi fL \\ &= 6.28 \times 1000 \times 0.01 \\ &= 62.8 \text{ ohms} \end{aligned}$$

2nd: FIND Z

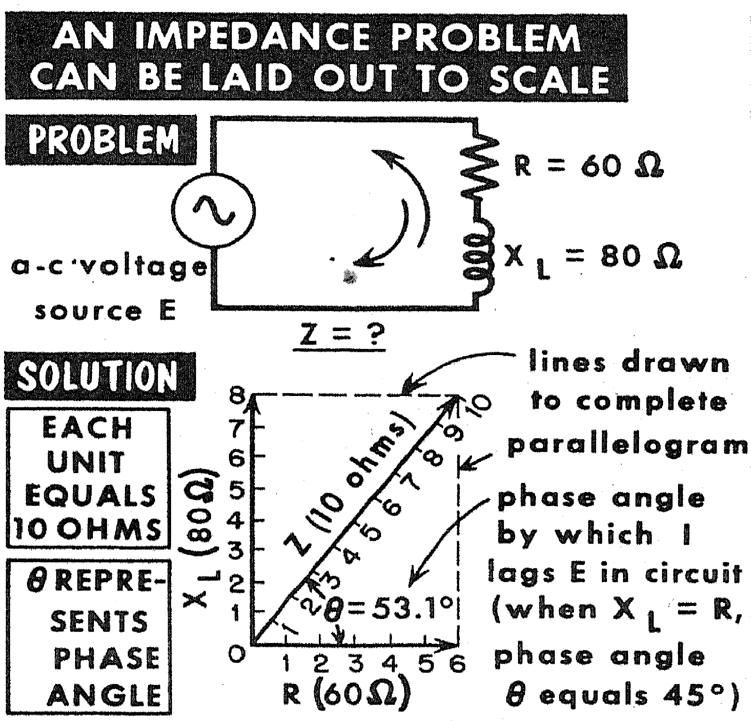
$$\begin{aligned} Z &= \sqrt{X_L^2 + R^2} \\ &= \sqrt{62.8^2 + 50^2} \\ &= \sqrt{3943.84 + 2500} \\ &= \sqrt{6443.84} \\ &= \underline{80.3 \text{ ohms}} \end{aligned}$$

We now have a series R-L circuit having an inductive reactance of 48 ohms and a resistance of 20 ohms. Find the impedance of the circuit. Once again, we first square X_L, and get 48×48, or 2304 ohms. We then square R, and get 20×20, or 400 ohms. Adding these two squared numbers, we get 2304 plus 400, or 2704. Taking the square root of 2704, we get 52 (52×52 equals 2704). The impedance is 52 ohms.

In our third problem, we have a series R-L circuit in which L is a 10 millihenry coil and R is a 50-ohm resistor. The frequency of the applied voltage is 1 kc (or 1000 cycles). Find the impedance of the circuit. We know the resistance is 50 ohms, so we must first find the inductive reactance. This is equal to: $X_L = 2\pi fL$. Filling in the formula, we get $X_L = 6.28 \times 1000 \times 0.01$, or 62.8 ohms. Knowing the inductive reactance is 62.8 ohms, we can now find the impedance of the circuit. We square 62.8 (62.8×62.8) and get 3943.84. We then square the resistance (50 ohms) and get 2500. Adding 3943.84 and 2500, we get 6443.84. Finally, taking the square root of 6443.84 we get 80.3 ohms, the impedance of this circuit.

Graphical Determination of Impedance (R and L in Series)

There is a simple graphical method that can be used to determine the impedance of an R-L circuit. It makes use of a parallelogram method that has certain advantages and disadvantages. To apply this method, let us first state a problem with which we can work. We will assume a series R-L circuit in which the inductive reactance is 80 ohms and the resistance is 60 ohms at the frequency of the voltage source. The problem--to find the impedance of the circuit. Instead of using the impedance formula, the problem is laid out to scale. We draw the vertical line which represents X_L to some exact length to represent the number "80". It could be 8-inches long, with each inch representing 10 ohms, or any other unit of measurement can be used. Then, with X_L being 8 units long, we draw the horizontal R axis exactly 6 units long to represent 60 ohms.



PRECISE METHOD FOR FINDING PHASE ANGLE (θ)

using triangle: Z, θ, $X_L = 8$, $R = 6$

$$\text{Tangent of angle } \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{X_L}{R} = \frac{8}{6} = 1.333$$

Looking in tangent column of trigonometry table, we find:
53.1° = 1.3319
53.2° = 1.3367

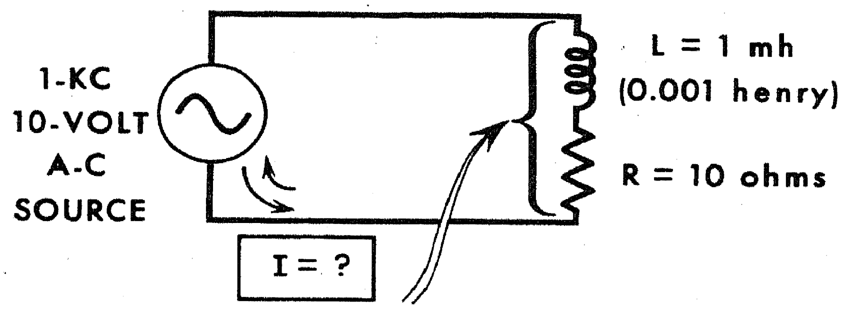
TO NEAREST TENTH OF DEGREE, θ = 53.1°

A parallelogram is now drawn, with one side parallel to the X_L axis and one side parallel to the R axis. Each new side is drawn from the end of the X_L and R axis. We now draw the resultant from the point where X_L and R meet to the diagonal corner of the parallelogram. This diagonal represents the resultant of the X_L and R vectors and indicates the impedance of the circuit. If the parallelogram were scaled and drawn properly, the resultant would be exactly 10 units long, representing 100 ohms impedance. Naturally, the same units of length would have to be applied to X_L, R, and Z in any given problem.

An advantage of this method is that it gives a quick, rough approximation. Its disadvantage is that it is somewhat clumsy and impractical where high precision is required. Another thing can be seen. The angle formed by the resultant and the R axis is the angle by which the current lags the voltage in an R-L circuit, and can be measured with a protractor. In our circuit, it is about 53.1°.

Alternating Current in an Inductor

When the impedance of a coil and the applied voltage are known, the alternating current flowing through the inductor can be readily calculated. Previously explained versions of Ohm's law for current are used except that R (resistance) in the equation is replaced by Z (impedance). Ohm's law as applied to a-c then reads: $I = E/Z$; $Z = E/I$; and $E = I \times Z$.



TO FIND CURRENT FLOW IN AN A-C CIRCUIT, WE USE OHM'S LAW FOR A-C CIRCUITS
$I = E/Z$

L and R are shown as separate components for the purpose of this problem only. Actually, in an inductor the two are inseparable since the same winding produces the resistance and the inductance.

1st FIND X_L:	2nd FIND Z:	3rd FIND I:
$X_L = 2\pi fL$	$Z = \sqrt{X_L^2 + R^2}$	$I = \frac{E}{Z}$
$= 6.28 \times 1000 \times 0.001$	$= \sqrt{6.28^2 + 10^2}$	$= \frac{10}{11.8}$
$= \underline{6.28 \text{ ohms}}$	$= \sqrt{39.4 + 100}$	$= \underline{0.847 \text{ ampere}}$
	$= \sqrt{139.4}$	
	$= \underline{11.8 \text{ ohms}}$	

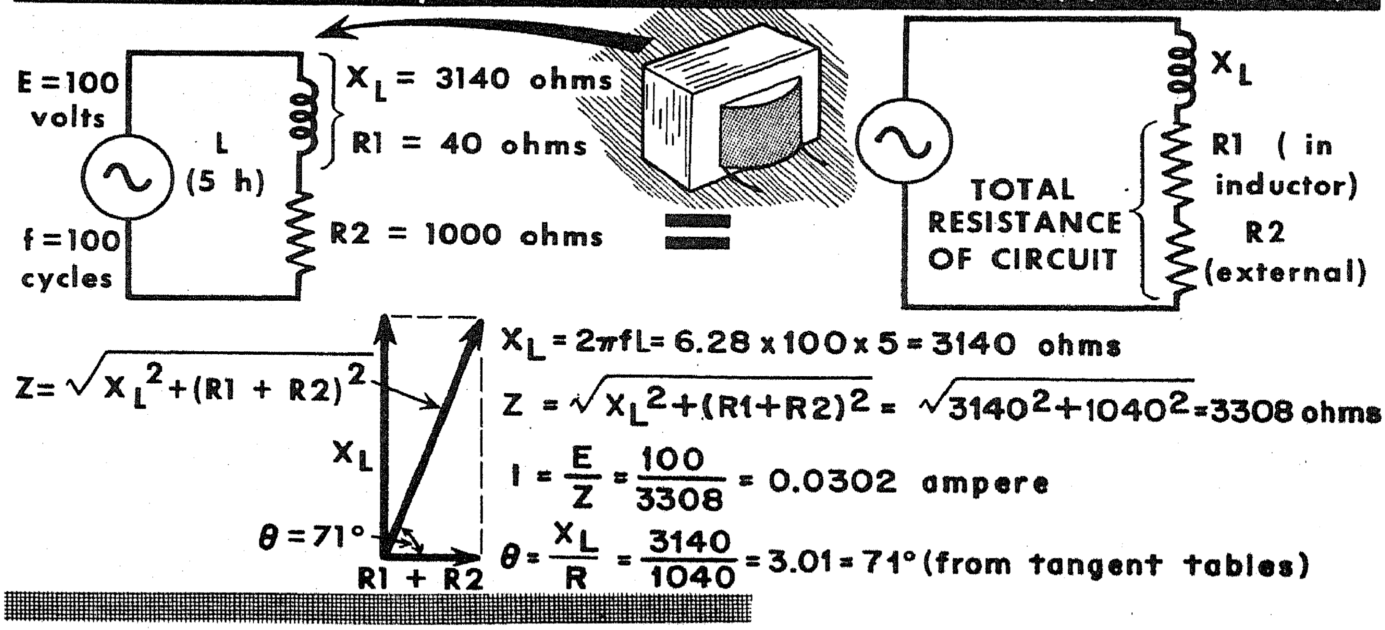
In each of these ratios, current and voltage must be expressed in the same terms. If we are considering the effective value of I, then we must consider the effective value of E. If we use peak values or average values of I, we must use like values of E. By so doing, it does not matter what values of E and I are being used. In virtually all instances, except where specifically noted, it is assumed that the effective value of E and I are being used. Note that nothing really new is being introduced--Ohm's law is still perfectly usable. The only thing new is that we must now consider other things such as X_L and Z since we are dealing with an inductive circuit.

The following problem illustrates a typical situation of an inductor in an a-c circuit. The resistance in the inductor is the actual d-c resistance of the copper wire that makes up the coil. Assume we have a 1-millihenry coil to which is applied a 1-kilocycle 10-volt a-c source. The d-c resistance of the inductor is 10 ohms. How much current flows through the coil? To solve the problem, we must first determine the inductive reactance and then the impedance. We find the inductive reactance using the formula $X_L = 2\pi fL = 6.28 \times 1000 \times .001 = 6.28$ ohms. We can now find the impedance: $Z = \sqrt{X_L^2 + R^2} = \sqrt{39.4 + 100} = \sqrt{139.4} = 11.8$ ohms. Now using Ohm's law for a-c circuits, we find the current: $I = E/Z = 10/11.8 = .847$ ampere. We can go one step further and find the phase angle Θ by which current lags the voltage: Tangent $\Theta = X_L/R = 6.28/10 = .628$. From the tangent table, $.6273 = 32.1^{\circ}$.

Determining the Current in an R-L Series Circuit

The theory you have learned about the series-connected R and L components of an inductor applies equally when an external resistance is series connected with the inductor. Now, two values of resistance are involved--the d-c resistance of the coil winding, and the external resistance. Assume a series-connected circuit in which inductance L = 5 henries, coil resistance R1 = 40 ohms, external resistor R2 = 1000 ohms, and the applied voltage is 100 volts at 100 cycles. Let us find the current in this circuit.

PRACTICAL INDUCTOR presents both INDUCTANCE and RESISTANCE to the circuit.



Before any calculating is done, let us examine the circuit. The coil resistance of R1 (40 ohms) is negligible (less than 1/10) relative to the external resistance of R2 (1000 ohms), but we shall take R1 into account just the same. Being series connected, we can visualize the resistance elements as a single sum R1 + R2. Then, the equation for impedance Z reads:

$$Z = \sqrt{X_L^2 + (R1 + R2)^2}$$

To calculate the impedance, we must first solve for the inductive reactance (X_L).

$$X_L = 2\pi fL = 6.28 \times 100 \times 5 = 3140 \text{ ohms}$$

Then:

$$Z = \sqrt{3140^2 + (40 + 1000)^2}$$
$$= \sqrt{3140^2 + 1040^2}$$
$$= \sqrt{10,941,200}$$
$$= 3308 \text{ ohms}$$

(If the coil resistance of R1 (40 ohms) is neglected, Z = 3296 ohms.)

$$\text{Current } I = \frac{E}{Z} = \frac{100}{3308} = .0302 \text{ ampere, or } 30.2 \text{ milliamperes.}$$

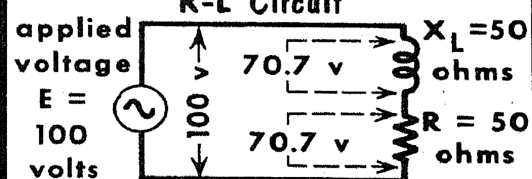
The phase angle of the current is: tangent $\theta = \frac{X_L}{R} = \frac{3140}{1040} = 3.01 = 71°$

Voltage Distribution in a Series R-L Circuit

When we studied d-c electricity, it was established that the sum of all the voltage drops in a series circuit was equal to the battery or applied voltage. We discussed this further in Kirchhoff's laws. The same is true of the voltage drops in an R-L series a-c circuit, with one single exception. The voltage drops across R and L are not simply added together. The reason for this is that there is a 90° phase difference between the inductive voltage drop (E_L) and the resistive voltage drop (E_R). The 90° phase difference between these voltage drops is caused by the fact that while the same current flows through R and L in a series circuit, the current through R is in phase with the voltage, but through L, the current lags behind the voltage by 90°.

The Sum of the Voltage Drops in a Series R-L Circuit is Equal to the Applied Voltage -- However, these Voltage Drops must be Added Vectorially.

R-L Circuit



Circuit Impedance

$$Z = \sqrt{50^2 + 50^2} = \sqrt{2500 + 2500} = \sqrt{5000} = 70.711 \text{ ohms}$$

Circuit Voltage Drops

$$E_L = I \times X_L = 1.414 \times 50 = 70.7 \text{ volts}$$

$$E_R = I \times R = 1.414 \times 50 = 70.7 \text{ volts}$$

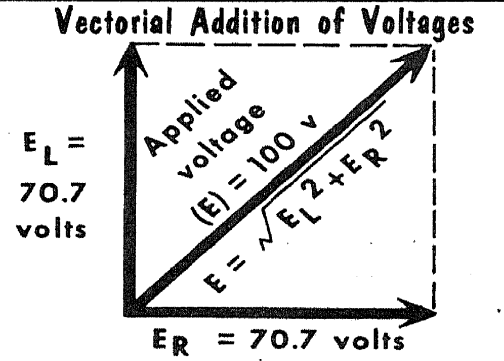
Circuit Current

$$I = \frac{E}{Z} = \frac{100}{70.711} = 1.414 \text{ amperes}$$

Applied Voltage = Sum of Voltage Drops

$$E \text{ applied} = \sqrt{E_L^2 + E_R^2} = \sqrt{70.7^2 + 70.7^2} = \sqrt{5000 + 5000} = \sqrt{10{,}000} = 100 \text{ volts}$$

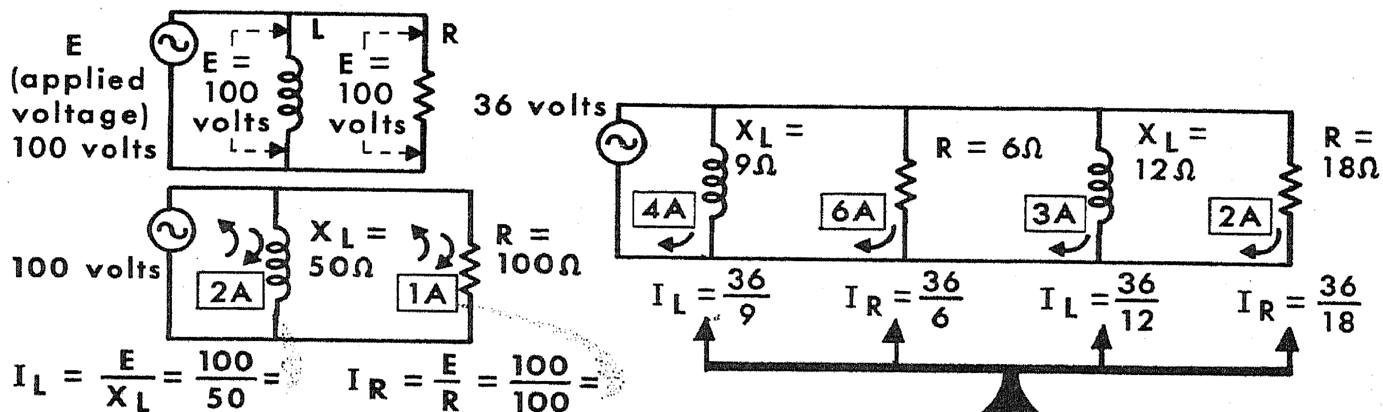
Vectorial Addition of Voltages



This does not present any new problem. Our fundamental rule that the sum of all the voltage drops in a series circuit is equal to the applied voltage still holds true. The only difference is that in order to get the sum of an inductive and a resistive voltage drop, it is necessary to add them vectorially. Since the same current flows through R and L, we can find the sum of their voltage drops in the same manner that we found the sum of their resistance and reactance, because the voltage drops are equal to IR and IX_L. We found the impedance of an R-L series circuit using the formula $Z = \sqrt{X_L^2 + R^2}$. By a very simple substitution, we can find the sum of two voltage drops 90° out of phase: E (applied) = $\sqrt{E_L^2 + E_R^2}$. Thus, while measuring the voltage drop across R and L separately, it would seem to give a ridiculous answer (greater than the applied voltage); vectorial addition of these voltages would give an answer equal to the exact applied voltage. Thus, Kirchhoff's laws for d-c circuits holds up equally well with a-c circuits. In an inductor, it is impossible to measure separately the voltage drop across R and L; we get one voltage drop across the R-L impedance.

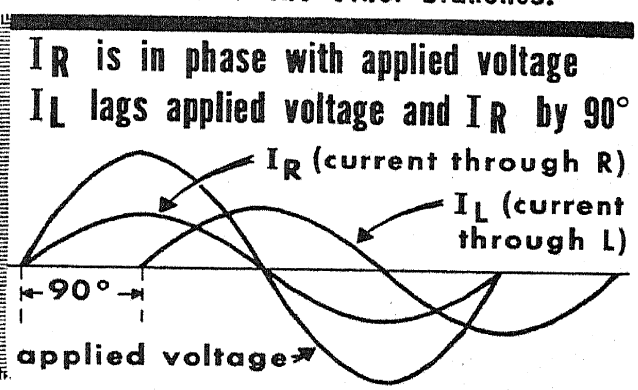
The Parallel R-L Circuit--Voltage Distribution

The parallel R-L circuit consists of a voltage source across which an inductive element and a resistive element are connected. By definition, there must be one or more of each of these elements in this type of circuit. Once again, we assume that the inductive element has zero resistance. Of course, in practice this is impossible. Later, we will discuss parallel circuits in which a particular branch contains both R and L. First, let us analyze the voltage distribution of a parallel R-L circuit.



The current in each branch is independent of the current in the other branches.

The Voltage Applied across Every Branch of a Parallel R-L Circuit is the Same



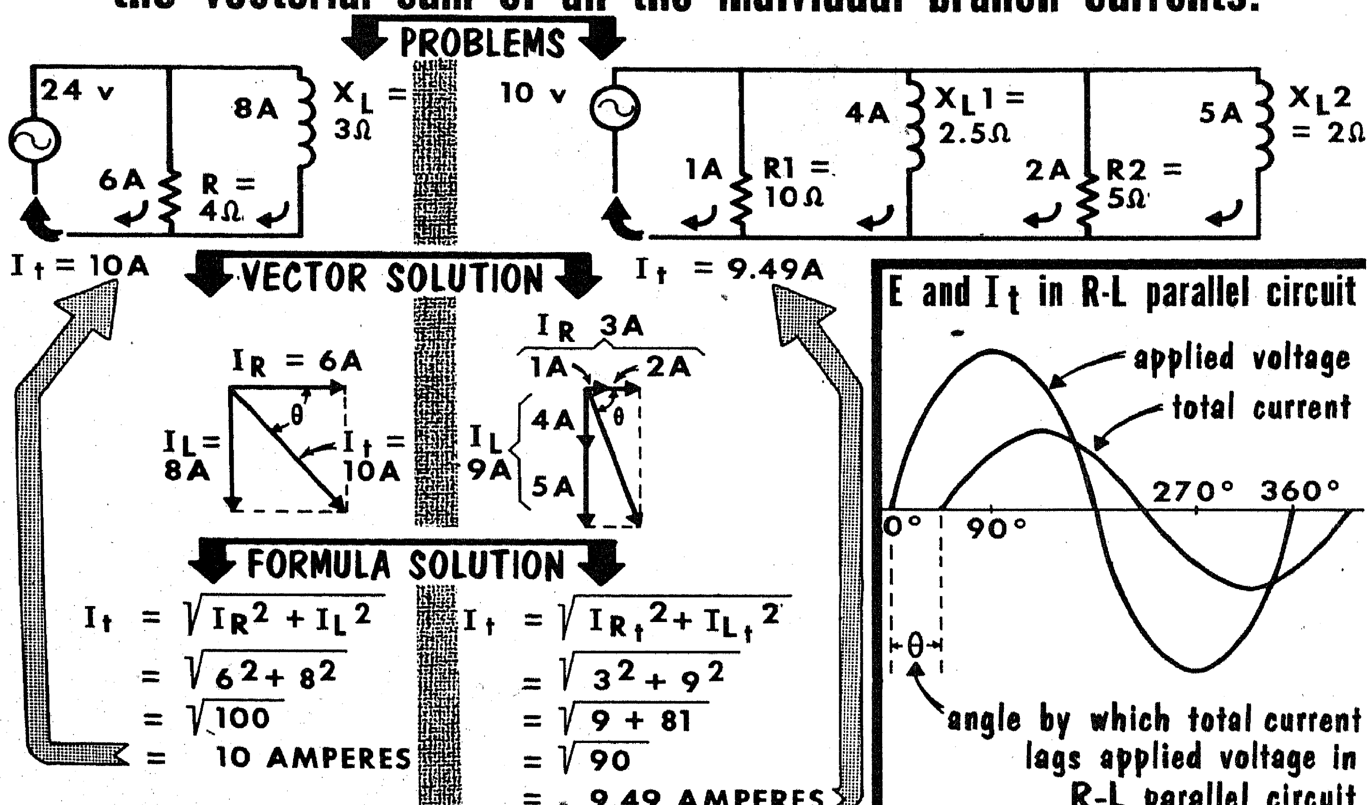
As in the case of the parallel circuit in our study of d-c circuits, a parallel circuit contains two or more branches. The applied voltage is across each and every branch of this circuit. Thus, a voltage or difference of potential equal to the full applied voltage is across each branch. In this respect, the current flow through each branch acts independently of the current flow in every other branch. Should one branch of the parallel circuit be opened, the stoppage of current flow in that branch would not affect the operation of any of the other branches; only the total current (I_t) would be affected.

The amount of current in each branch of a parallel circuit is determined by the voltage applied to that branch and the R or X_L of that branch. In short, the current in each branch would be equal to $I_R = E/R$ or $I_L = E/X_L$, as the case may be. The current flow in each branch must be treated separately. However, there is one important new consideration. The current through a resistive branch is in phase with the applied voltage; the current through an inductive branch lags the applied voltage by 90°. This is an important consideration when computing the total current.

The Parallel R-L Circuit--Current Distribution

We have stated that the current flow in each branch of a parallel circuit is completely independent of the current flow in every other branch. The important difference between a purely resistive parallel circuit and an R-L parallel circuit is in finding the total current. In a purely resistive parallel circuit, we simply find the total of all the individual branch currents, and this sum equals the total current. But in the R-L parallel circuit, the currents in the inductive branches are 90° out of phase with the current in the resistive branches. Thus, once again we are faced with vectorial addition. We must first add up all the inductive currents and all the resistive currents, and then add them vectorially.

The total current in a parallel R-L circuit is equal to the vectorial sum of all the individual branch currents.

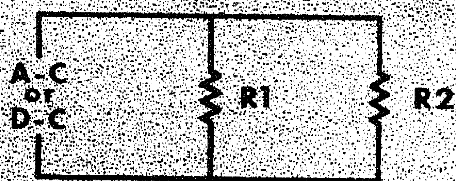


The total current in an R-L circuit can be found in two ways: by a graphical layout of the current vectors; and by direct formula. In the graphical representation, the current vectors for the inductive current (I_L) and the resistive current (I_R) are placed at right angles. Since the resistive current is in phase with the applied voltage, I_R is the reference point and is located on the horizontal axis. With the inductive current lagging 90° behind it, we place the inductive current vector straight down, representing a 90° lag behind I_R. Using the parallelogram, the resultant represents the total current and the angle of lag between the applied voltage and the total current. Using the formula for impedance, $Z = \sqrt{X_L^2 + R^2}$, which we developed for right triangle we simply substitute and get: total current (I_t) $= \sqrt{I_L^2 + I_R^2}$. The ırrent lag behind the applied voltage is equal to: tangent $\Theta = I_L/I_R$.

Impedance of the Parallel R-L A-C Circuit

To Find the Total Impedance

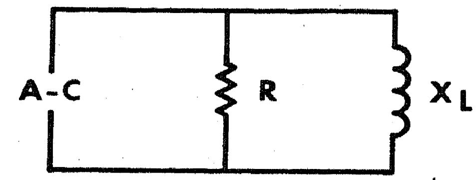
In a purely resistive parallel circuit



use the formula

$$R_t \text{ (or } Z) = \frac{R1 \times R2}{R1 + R2}$$

In an R-L parallel circuit



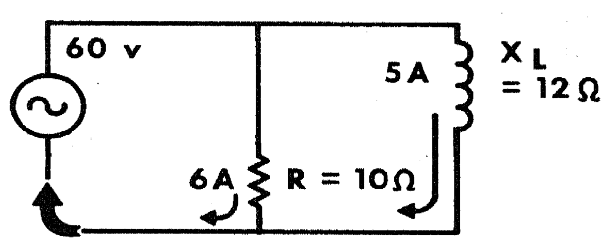
use the formula

$$Z = \frac{R \times X_L}{\sqrt{R^2 + X_L^2}}$$

The impedance of the parallel R-L a-c circuit is computed by a method very much like that used for calculating the total resistance of resistors connected in parallel. We learned that to find the total resistance of two resistances in parallel, we used the formula $R_t = (R1 \times R2)/(R1 + R2)$. We can now substitute in this formula to bring our R-L circuit into play. To find impedance, we say $Z = (R \times X_L)/(R + X_L)$. However, the addition of two vector quantities, as we have seen, cannot be made by simple addition. Therefore, to take into consideration the fact that R and X_L must be added vectorially, we change the formula to: $Z = R \times X_L/\sqrt{R^2 + X_L^2}$. Using this formula, we would be accounting for the 90° phase difference between the currents in the resistive and inductive branches.

PROBLEM

$Z = ?$



PROOF

$$I_t = \sqrt{36 + 25} = \sqrt{61} = 7.81 \text{ AMPERES}$$

$$Z = \frac{E}{I_t} = \frac{60}{7.81} = 7.6 \text{ OHMS}$$

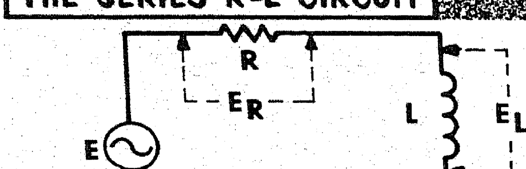
SOLUTION

$$Z = \frac{R \times X_L}{\sqrt{R^2 + X_L^2}} = \frac{10 \times 12}{\sqrt{100 + 144}} = \frac{120}{\sqrt{244}} = \frac{120}{15.62} = 7.6 \text{ OHMS}$$

COMPARISON BETWEEN SERIES AND PARALLEL R-L CIRCUITS

SERIES R-L CIRCUIT	PARALLEL R-L CIRCUIT
The current is the same everywhere.	The current divides between the branches; each branch current is a function of the branch resistance or reactance.
The current is in phase throughout the circuit.	The current in the inductive branch lags the current in the resistive branch by 90°.
The voltage across the inductance leads the voltage across the resistance by 90°.	The voltage across the inductance is in phase with the voltage across the resistance.
The angle of lag between the total circuit current and the applied circuit voltage is determined by the amount of reactance and resistance.	The angle of lag between the line current and the applied voltage is determined by which circuit component is smaller--the reactance or the resistance.
Increasing the frequency makes the circuit more inductive because the inductive reactance exerts greater control on the circuit current. The angle of lag increases.	Increasing the frequency makes the circuit more resistive because the line current is predominantly the resistive branch current.
The applied voltage divides vectorially between the series reactance and resistance.	The applied voltage is the same across all parallel-connected elements.
Increasing resistances makes the circuit more resistive. The angle of lag of the circuit current approaches 0° more closely.	Increasing resistances makes the circuit more inductive. The angle of lag of the line current approaches 90° more closely.
Increasing the inductance makes the circuit more inductive. The angle of lag of the circuit current approaches 90° more closely.	Increasing the inductance makes the circuit more resistive. The angle of lag of the line current approaches 0° more closely.

THE SERIES R-L CIRCUIT

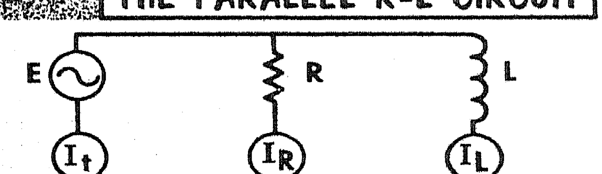


$E = E_R + E_L$ (added vectorially)

SAME CURRENT THROUGH R AND L

$$Z = \sqrt{R^2 + X_L^2}$$

THE PARALLEL R-L CIRCUIT



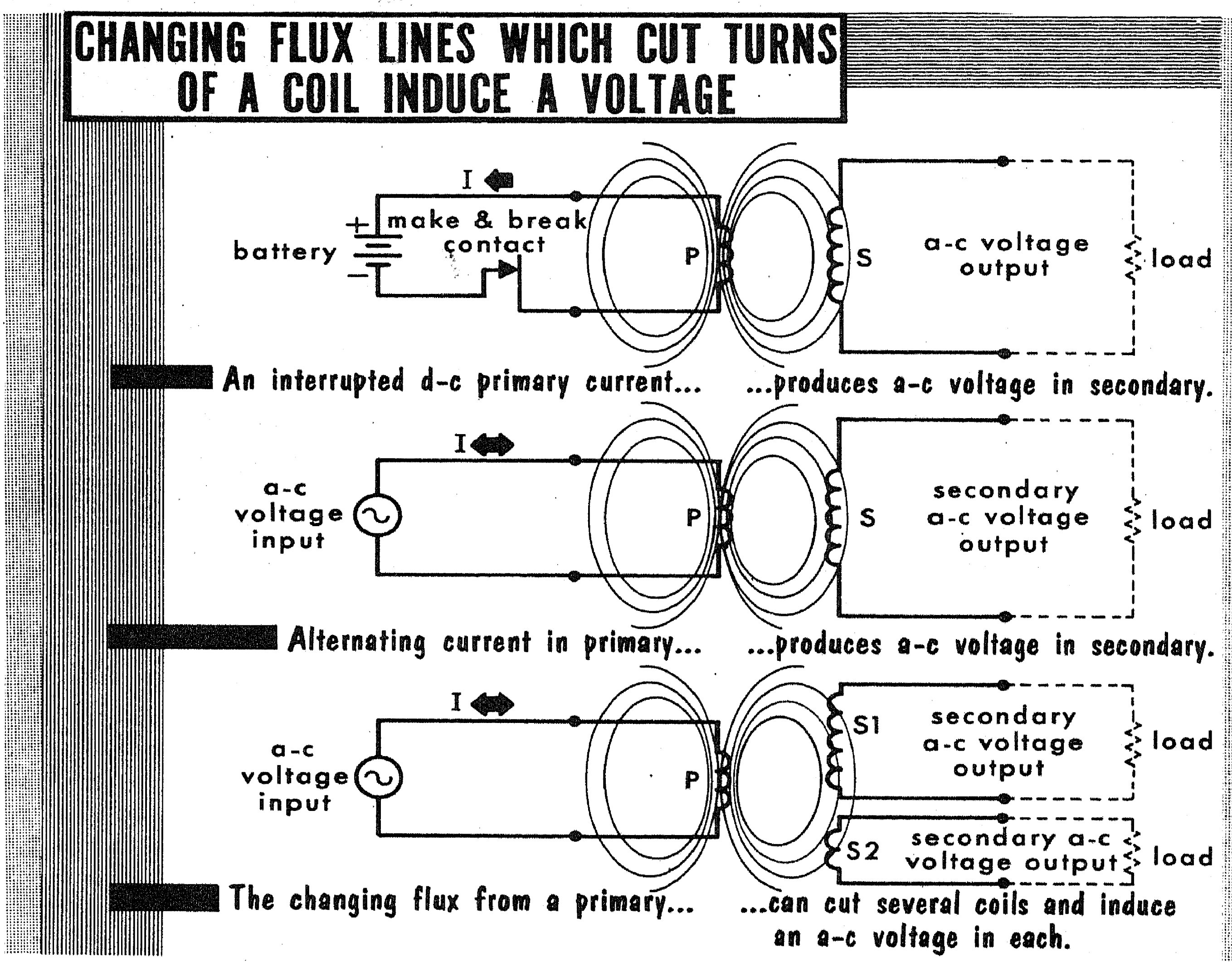
$I_t = I_R + I_L$ (added vectorially)

SAME VOLTAGES ACROSS R AND L

$$Z = \frac{R \times X_L}{\sqrt{R^2 + X_L^2}}$$

The Transformer

You have learned that an alternating current or a varying d-c current flowing in one coil can induce a varying voltage in a neighboring coil. The changing magnetic lines of force from the varying current in one coil (which we call the primary) cuts the turns of the other coil (called the secondary) and induces a changing voltage in each of the turns of the secondary. When two coil windings are arranged so that a changing current in one induces a voltage in the other, the combination of windings constitutes a transformer. Every transformer has a primary winding and one or more secondary windings. The primary winding (usually labeled P) receives the input electrical energy from a voltage source, whereas the secondary winding or windings (usually labeled S) delivers the induced output voltage to a load.



The transformer serves many functions. It enables the transfer of electrical energy from one electrical circuit to another by using changing magnetic lines of force as the link between the two. In this way, it behaves as a coupling device. Also, it provides a means whereby an alternating voltage of a given amount can be changed (transformed) to higher or lower amounts, making electrical power distribution practical. Such transformation also can be applied to current and impedance. These functions are explained later.

Transformer Action (Unloaded Secondary)

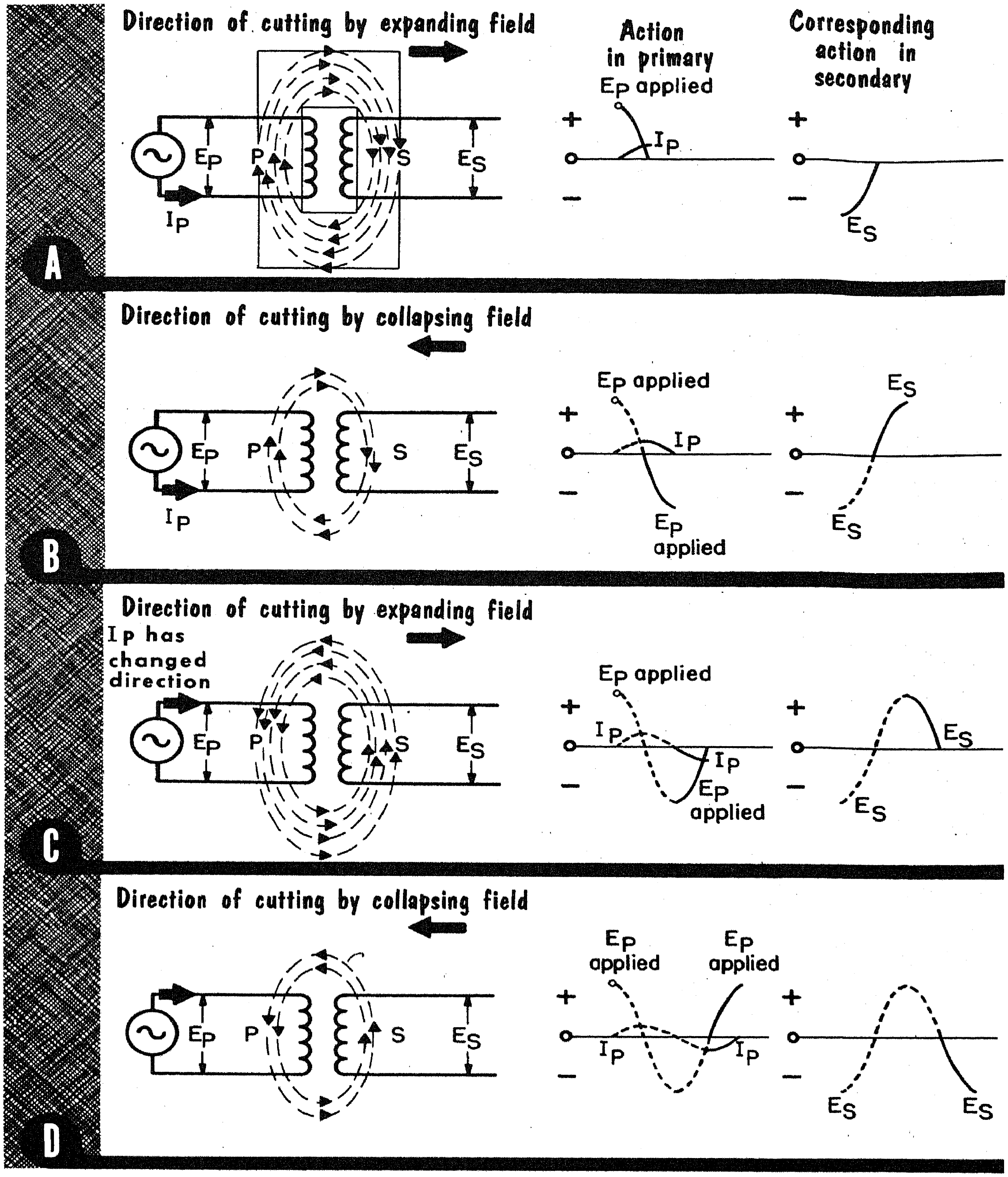
Assume a two-winding iron-core transformer with a primary (P) and a secondary (S), both of equal number of turns and of very low resistance. There is no load connected to the secondary winding. The a-c voltage applied to the primary is shown as a single cycle starting at maximum positive.

As the primary voltage E_p starts decreasing from maximum positive value, the primary current I_p starts increasing from zero in a positive direction. (See A on facing page.) The 90^o lag of primary current relative to primary voltage is due to the inductance of the primary winding. As the primary current starts increasing from zero, its associated magnetic field starts expanding. At this instant, the rate of change of the current, and of the flux, is maximum. The flux lines cut the turns of the primary winding and generate a self-induced emf of maximum value in the primary winding. This emf acts in opposition to the applied primary voltage E_p. Since there is nothing to prevent the generation of a maximum number of flux linkages, the self-induced emf is high, thereby causing the primary current I_p to be very low in value. Stated another way, the primary current is held low by the high inductive reactance of the iron-core primary.

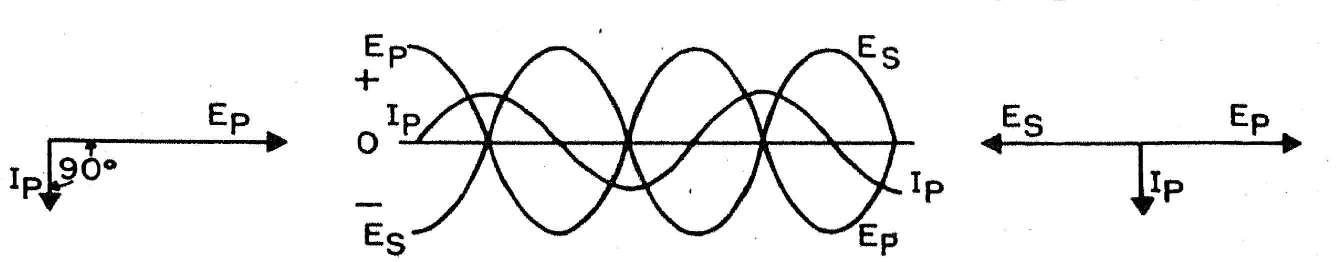
At the same instant in time (still A on the facing page), the expanding field produced by the primary current cuts the turns of the secondary (S), where it induces the secondary voltage, E_s. Inasmuch as the rate of change of the magnetic field is maximum, the voltage E_s is maximum. This secondary voltage has a polarity that is opposite to that of the primary voltage. It appears across the secondary, but since the secondary is unloaded (open) there is no secondary current. Hence, the action in the secondary has no effect on the action in the primary circuit. When the primary current reaches its maximum positive value, the rate of change of its field is theoretically zero; hence, the voltage induced in the secondary is zero. This coincides with the instant in time when the primary voltage E_p is zero.

As the primary voltage E_p passes through zero, changes polarity, and starts increasing towards its negative peak, the primary current I_p (still of the same direction) starts decreasing from maximum to zero, accompanied by the collapse of the magnetic field back into the primary. (See B.) The flux lines again cut the turns of the secondary winding, but now, in a direction opposite to that when the field was expanding. The result is a secondary voltage opposite in polarity to the previous voltage, and opposite to that of the applied primary voltage. As E_s increases towards its maximum positive value, the applied primary voltage increases towards its maximum negative value, both peaks being reached at the same instant. Also, at the same moment, the primary current I_p passes through zero.

The action of the transformer during the remainder of the primary voltage and current cycle is shown in C and D. It is the same as previously described except for the reversal in direction of the primary current. The expansion and collapse of the field is as before, during which time the remainder of the secondary voltage cycle is generated. At each instant of time, the secondary voltage is 180^o out of phase with the primary voltage. The primary current is 90^o behind the primary voltage, but 90^o ahead of the secondary voltage E_s.

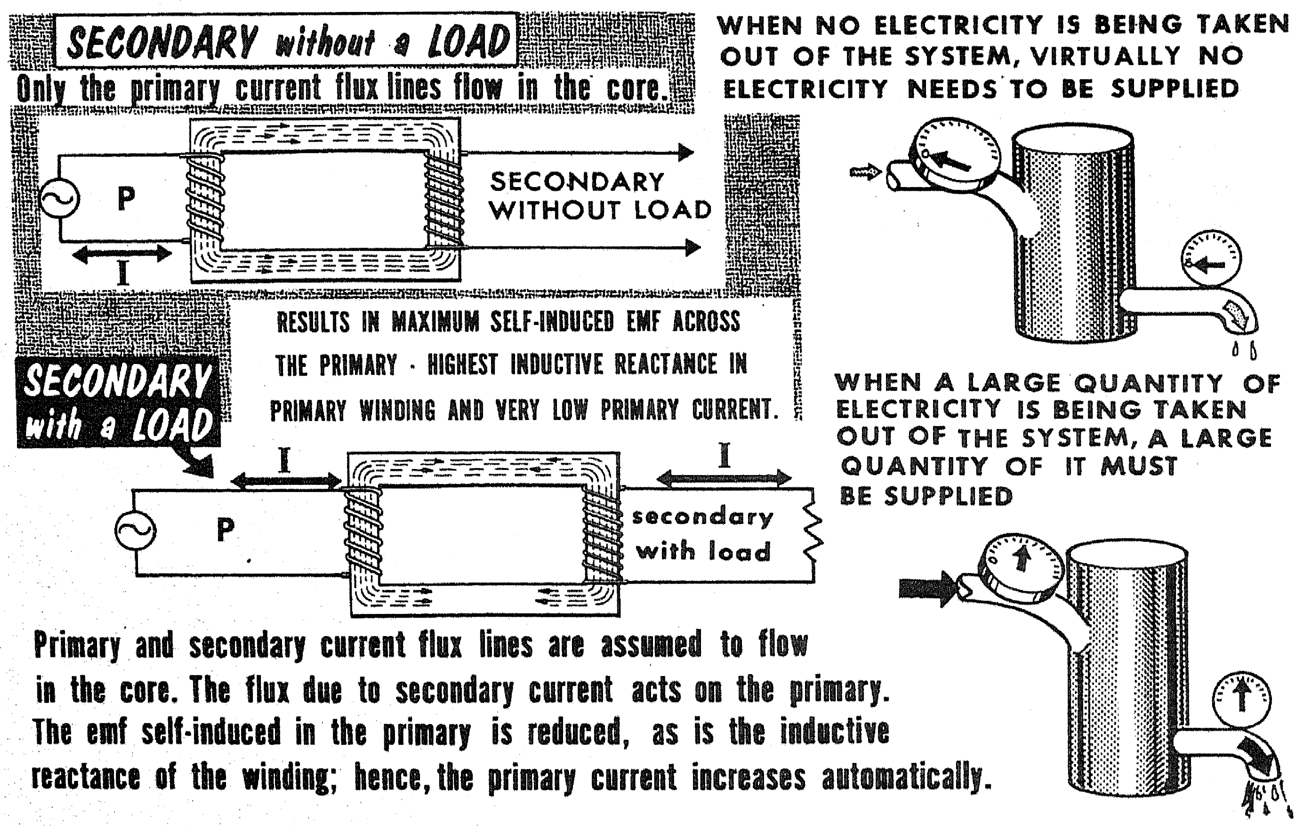


SUMMARY OF VOLTAGE-CURRENT CONDITIONS IN UNLOADED SECONDARY TRANSFORMER



Transformer Action (Loaded Secondary)

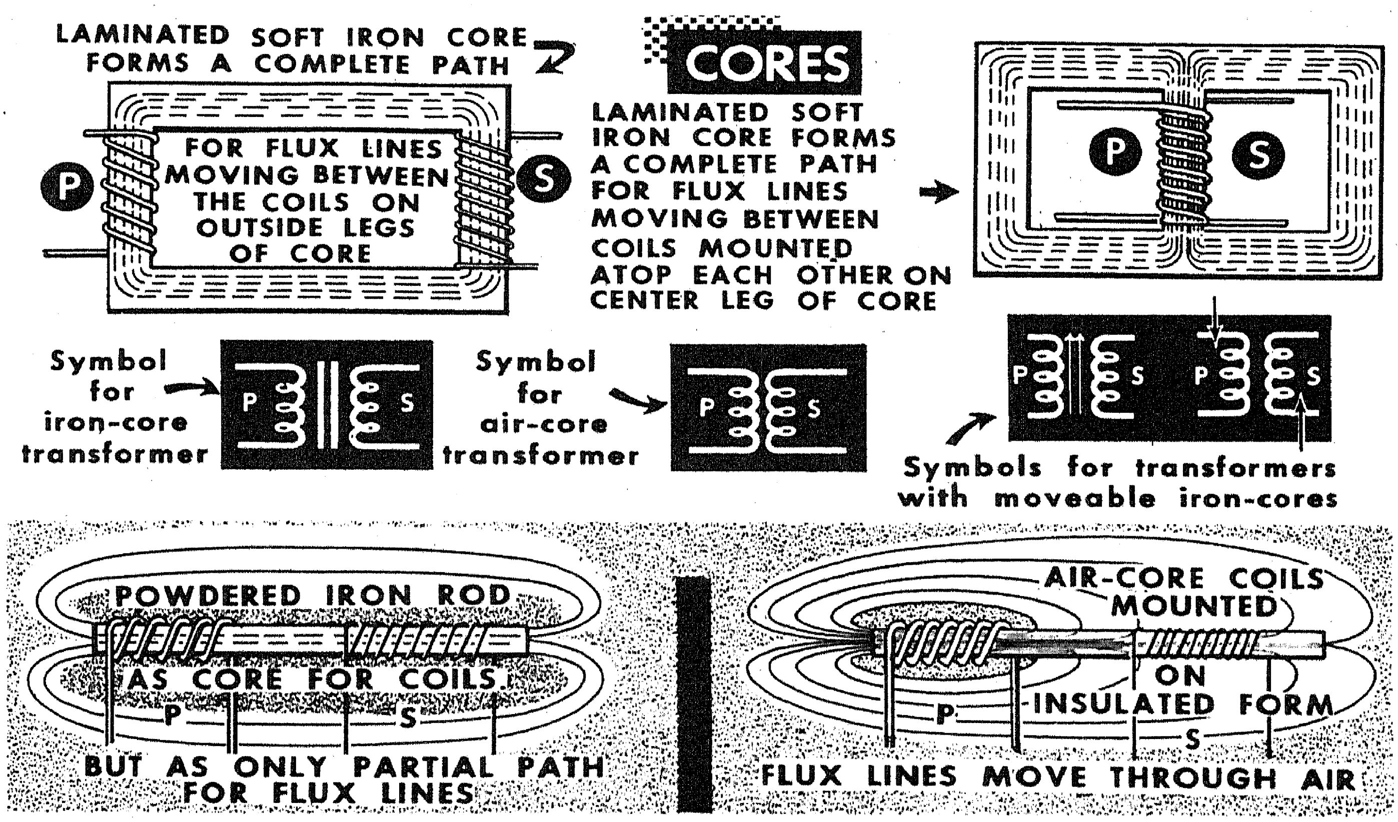
When the transformer is delivering voltage to a load, current flows in the load and in the secondary winding. This current affects the primary current. We will assume that the load connected to the transformer is a resistance R. When a voltage is induced in the secondary winding, a current flows through the load. This current also flows through the secondary winding. The load current is a current drain on the transformer. Like any other alternating current, the current in the secondary is accompanied by changing flux lines. The path for these flux lines is the transformer core, but the direction of these lines is opposite to that of the flux lines associated with the primary current. So, in effect, two sets of flux lines flow in the core--one due to the primary current, and the other due to the secondary current.



By virtue of its direction, the secondary current flux opposes the primary current flux. In doing so, the number of linkages which occur between the primary current flux and the primary turns, and which accounts for the self-induced emf in the primary, is reduced. This action is like a self-regulating valve that permits the primary current to increase above the small amount which flows when there is no load on the secondary. The amount of increase in primary current is determined by the amount of current drawn from the secondary winding. In other words, when the secondary winding delivers power (E × I) to a load, the primary winding draws more power from the voltage source than when the secondary winding is not delivering power to a load. With the primary voltage being fixed in value, the increase in power required by the primary appears as an increase in primary current. Of course, the
rue--if the secondary current drain decreases, the primary cur-
ically decreases to adjust to the new situation.

Iron-Core and Air-Core Transformers

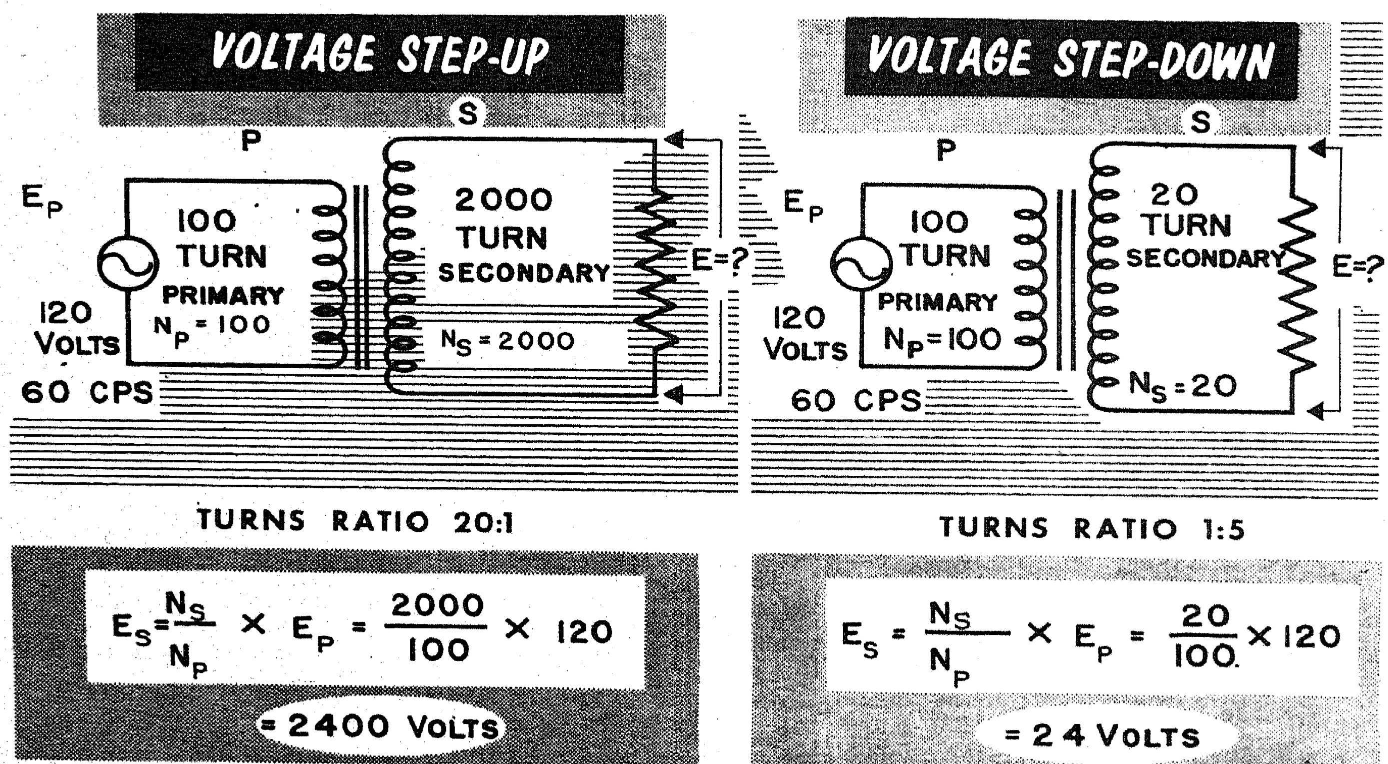
There are many kinds of transformers. In a broad sense, they fall into two categories--iron-core and air-core. Each category has numerous subdivisions relating to its particular uses. The two types mentioned state the kind of material that serves as the path over which the magnetic lines of force travel from the primary to the secondary and in the reverse direction. Since soft iron is a much better path for magnetic lines of force than air, it is used as the core for the transformer windings, except at very high frequencies. The iron core conducts most of the flux lines originating from the primary current in the primary winding to the turns of the secondary winding, thereby allowing the maximum number of flux linkages (tight coupling between the windings), or the transfer of the greatest amount of electrical energy from the primary to the secondary. To improve the action in many iron-core transformers, the primary and secondary windings are wound on top of each other. Some iron-core transformers contain a powdered-iron core in the form of a rod on which the primary and secondary coils are located side by side. The flux linkage is reduced; hence, the coupling is not as tight. Transformers of this type may have iron cores that are movable for "tuning" (varying inductance) purposes. The presence of a ferrite (iron) material as the core in a transformer is symbolized by two or more straight lines located in the space between the symbols for the coils, or above or below them.



The air-core transformer has its coils wound on insulated forms that use air as the path for the flux lines moving between the windings. The absence of the iron core provides low values of inductance and limited flux linkages; hence, very loose coupling between the coils. Such transformers are used at very high frequencies. They are discussed at greater length later in this course.

Voltage Step-Up and Step-Down in Transformers (Turns Ratio)

One of the fundamental considerations in transformers is the amount of voltage derived from the secondary winding relative to the amount of voltage that is applied to the primary winding. If the voltage output from the secondary winding is higher than the voltage applied to the primary winding, a voltage step-up has taken place; if the secondary voltage is less than the primary voltage, a voltage step-down has taken place. Some transformers are designed to furnish both voltage step-up as well as voltage step-down. When a transformer is desired with a single secondary winding intended to furnish a voltage equal to the primary voltage, the voltage transformation is 1-to-1 (1:1) and the device is known as an "isolation" transformer. Its only function is to isolate one circuit from the other.



Whether the secondary voltage exceeds the primary voltage or is less than the primary voltage is determined by the turns ratio between the secondary winding and the primary winding. This is expressed as an equation as follows:

$$\frac{\text{secondary voltage}}{\text{primary voltage}} = \frac{\text{number of turns in secondary winding}}{\text{number of turns in primary winding}}; \quad \frac{E_S}{E_P} = \frac{N_S}{N_P}$$

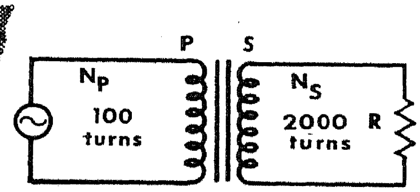
As you can see, the secondary-primary turns ratio equals the secondary-primary voltage ratio. When the number of turns in the secondary (N_S) exceeds the number of turns in the primary (N_P), or N_S is greater than N_P, a voltage step-up occurs. When the reverse is true, a voltage step-down occurs. The actual voltage derived from the secondary winding is equal to the product of the secondary-primary turns ratio and the voltage applied to the primary. The above assumes perfect (100%) coupling between primary and secondary. This is seldom the case. However, in some power transformers, coupling is almost perfect.

Current Turns Ratio

The secondary-primary turns ratio determines the amount of primary current that will flow for a given secondary current. You have learned that the more the number of turns (N) in a coil through which a current (I) in amperes is flowing, the greater the number of flux lines that are established by the current. The product of the number of turns and the current (or $N \times I$) was identified as the ampere-turns. In the ideal transformer, the number of ampere-turns in the primary equals the number of ampere-turns in the secondary. Imagine that you are working with a loaded transformer in which the primary winding has 100 turns and the secondary winding has 2000 turns. The secondary-primary turns ratio then is $2000/100 = 20$. If the secondary load current is 0.2 ampere, the secondary ampere-turns are $2000 \times .2 = 400$. For the same number of ampere-turns to exist in the primary of 100 turns, the primary current must be increased in the same proportion as the secondary-primary turns ratio. This ratio is 20; hence, the primary current must be 20 times greater than the secondary current. The self-regulating action of the primary winding establishes the primary current at the required value.

The Primary-Secondary Current Relationship

IN A VOLTAGE STEP-UP TRANSFORMER



Assume I_S = 0.2 amp load current

TURNS RATIO = N_S/N_P = 2000/100 = 20

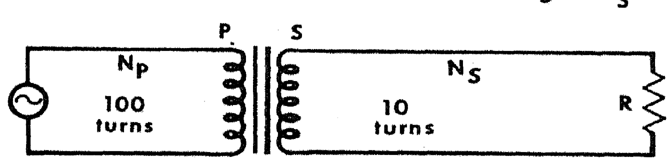
then

$$I_P = \frac{N_S}{N_P} \times I_S = \frac{2000}{100} \times 0.2 = 4 \text{ amperes}$$

PRIMARY AMPERE-TURNS = $N_P \times I_P = 100 \times 4 = 400$

SECONDARY AMPERE-TURNS = $N_S \times I_S = 2000 \times 0.2 = 400$

IN A VOLTAGE STEP-DOWN TRANSFORMER



Assume I_S = 2 amps load current

TURNS RATIO = N_S / N_P = 10/100 = 0.1

then

$$I_P = \frac{N_S}{N_P} \times I_S = \frac{10}{100} \times 2 = 0.1 \times 2 = 0.2 \text{ ampere}$$

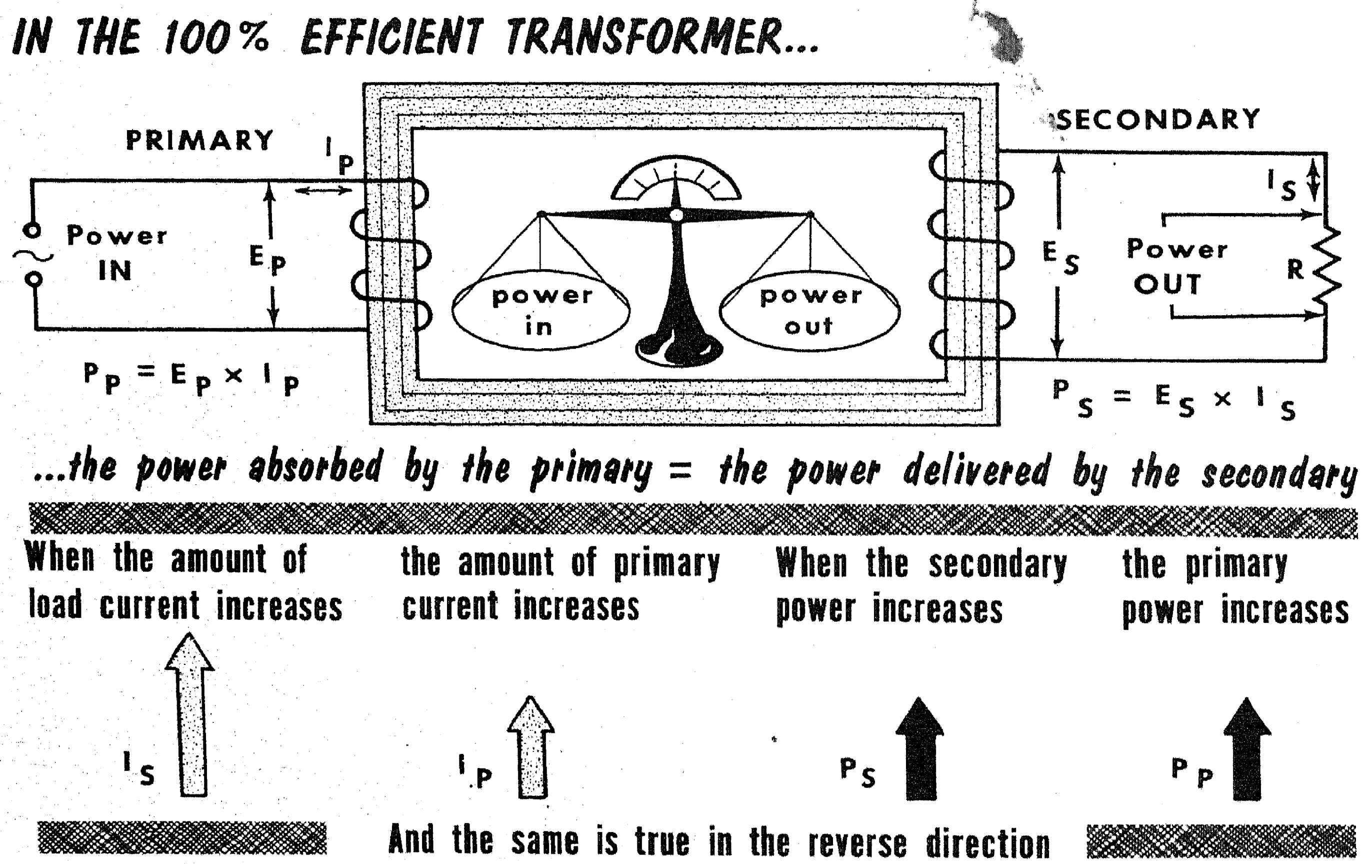
PRIMARY AMPERE-TURNS = $N_P \times I_P = 100 \times 0.2 = 20$

SECONDARY AMPERE-TURNS = $N_S \times I_S = 10 \times 2 = 20$

The same conditions hold true when the secondary winding has fewer turns than the primary winding. In this case, the equality of ampere-turns is gained by the higher current in the secondary of fewer turns, and the lower current in the primary of more turns. Thus, the primary-secondary current ratio is opposite to that of the primary-secondary voltage ratio. A 1:10 voltage step-up transformer will exhibit a 10:1 current step-down characteristic. We see that $E \times I$ in the primary will equal $E \times I$ in the secondary. In short, primary power equals secondary power.

Current Transformation (Power)

A transformer is not a generator of electrical power; its primary circuit absorbs power from a voltage source and its secondary circuit delivers power to a load. Assuming an ideal transformer with a resistive load in the secondary, the power absorbed by the primary equals the power consumed by the secondary. (In practice, the power consumed by the secondary is slightly less than that absorbed by the primary, the difference being due to electrical losses in the transformer, as will be explained later.)

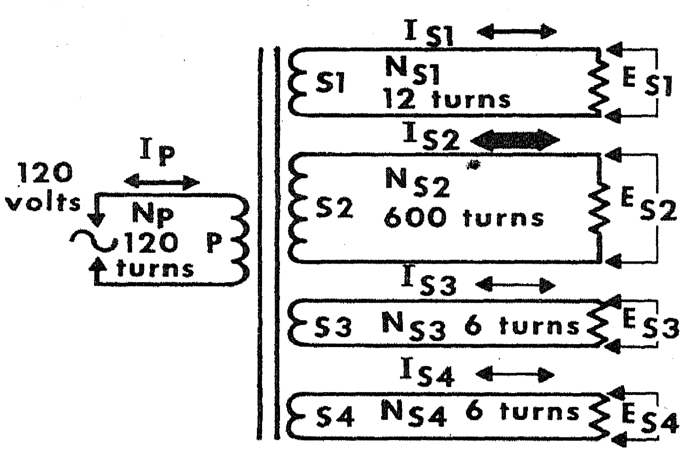


The ideal input-output power relationship is stated as: power (in watts) in primary = power (in watts) in secondary, or P_P watts = P_S watts. Since power is equal to voltage times current ($E \times I$), the power relationship can be restated as: $E_P \times I_P = E_S \times I_S$. Assume a loaded transformer with a 1:1 turns ratio. Then, the primary voltage E_P and the secondary voltage E_S will be equal. Whatever the secondary load current I_S may be, the primary current I_P will adjust itself to the same value so as to satisfy the condition $E_P I_P = E_S I_S$, and the power is the same in both circuits. If the load on the secondary is changed, thereby changing secondary current I_S, the primary current I_P will readjust itself to be equal to I_S. For any amount of power delivered by the secondary within the capabilities of the transformer, the primary circuit behaves as a self-regulating system in which the current changes in value so that the primary circuit power equals the secondary circuit power. The primary current change is the result of the increased or decreased action of the secondary current flux lines on the primary current flux lines; hence, on the emf self-induced in the primary.

Multi-Secondary Winding Transformers

A broad category of iron-core transformers is known as "power transformers." They have several secondary windings and a single primary winding which is common to all the secondaries. The purpose of these transformers is to supply a number of operating voltages required by radio communications equipment. Sometimes, as many as two voltage step-up secondaries and three voltage step-down secondaries are part of the same transformer. The voltage derived from each secondary winding is independent of the others, its amount being determined by the individual secondary-common primary turns ratio.

VOLTAGE RATIOS in MULTI-SECONDARY WINDINGS



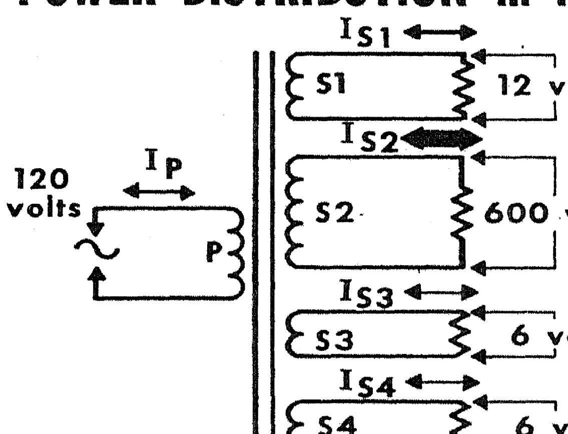
$$E_{S1} = \frac{N_{S1}}{N_P} \times E_P = \frac{12}{120} \times 120 = 12 \text{ volts}$$

$$E_{S2} = \frac{N_{S2}}{N_P} \times E_P = \frac{600}{120} \times 120 = 600 \text{ volts}$$

$$E_{S3} = \frac{N_{S3}}{N_P} \times E_P = \frac{6}{120} \times 120 = 6 \text{ volts}$$

$$E_{S4} = \frac{N_{S4}}{N_P} \times E_P = \frac{6}{120} \times 120 = 6 \text{ volts}$$

POWER DISTRIBUTION in MULTI-SECONDARY WINDINGS



12 volts @ 2 amps = P_{watts} = 12 × 2 = 24 watts

600 volts @ 0.1 amp = P_{watts} = 600 × 0.1 = 60 watts

6 volts @ 4 amps = P_{watts} = 6 × 4 = 24 watts

6 volts @ 1 amp = P_{watts} = 6 × 1 = 6 watts

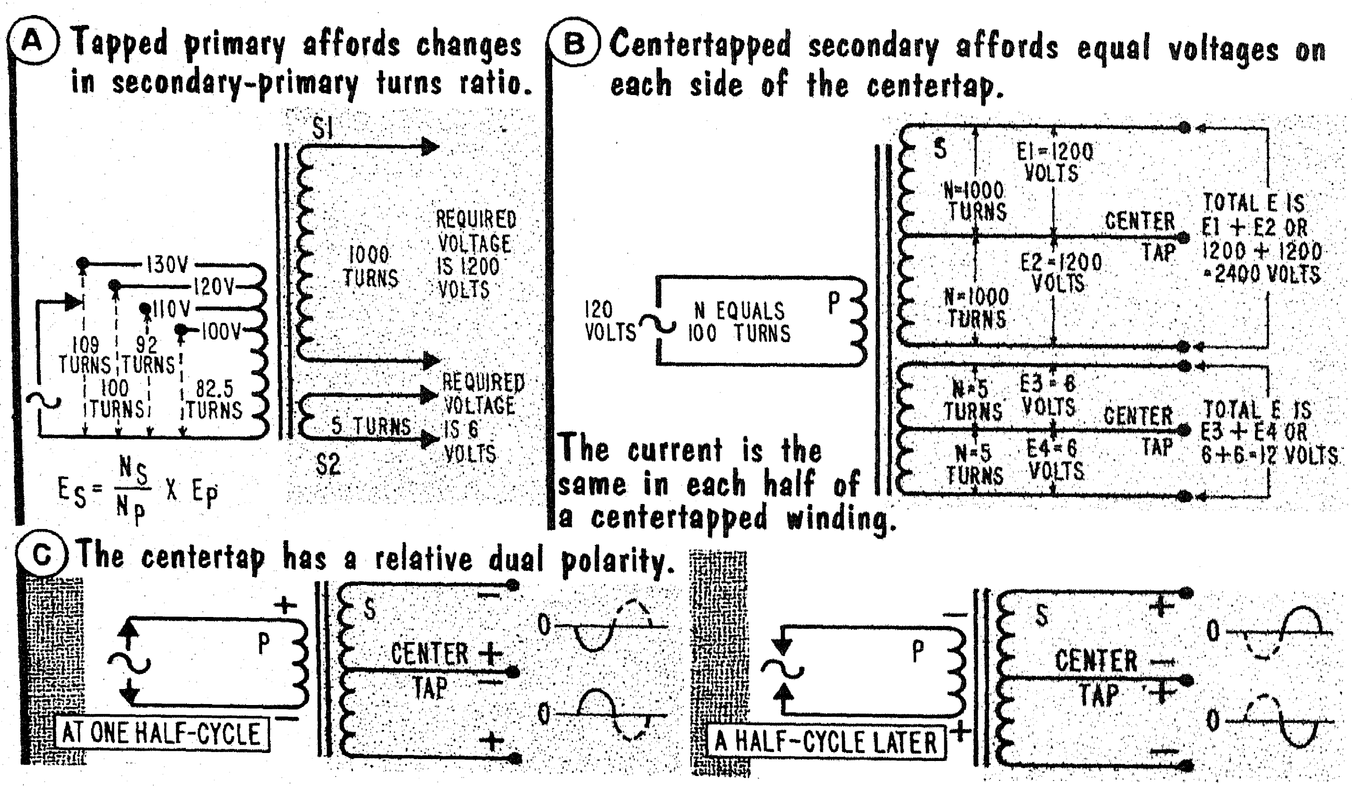
Secondary winding powers are additive and equal to primary power.

$$P_{primary} = P_{S1} + P_{S2} + P_{S3} + P_{S4} = 24 + 60 + 24 + 6 = 114 \text{ watts}$$

Concerning the power consumed by the secondary windings relative to the power absorbed by the primary winding, if we assume the ideal case, the power in the primary equals the arithmetical sum of the power delivered by each of the secondary windings, or $P_P = P_{S1} + P_{S2} + P_{S3}$. In the practical case, the primary power may be from 5 to 10% higher than the sum of the power delivered by the secondary windings, this much being wasted as electrical losses in the transformer.

Tapped Primary and Secondary Windings

Some types of iron-core transformers have a multi-tapped primary winding and a centertapped secondary winding. A tap is simply a wire connection to the winding. Usually, for convenience, it is joined to a terminal. The tapped primary is a continuously wound coil that affords a selection of the number of primary turns which are active during the operation of the transformer. It permits the use of the transformer over a range of primary voltages for a given secondary voltage. Assume a transformer that is rated 1200 volts from one secondary winding (S1) and 6 volts from another secondary winding (S2) with 120 volts a-c applied to the primary. Winding S1 has 1000 turns and S2 has 5 turns. To satisfy the above voltage conditions, the primary must have 100 active turns. Now suppose that the available primary voltage is only 110 volts. If this voltage were applied to the transformer, the two secondaries would deliver less than the required 1200 and 6 volts. But if we increased the secondary-primary turns ratio by the correct amount, it would compensate for the reduced primary voltage. The taps on the primary permit such a change in turns ratio to allow for the condition when the primary voltage is either lower or higher than the rated optimum voltage. (See A).



(A) Tapped primary affords changes in secondary-primary turns ratio.

(B) Centertapped secondary affords equal voltages on each side of the centertap.

The current is the same in each half of a centertapped winding.

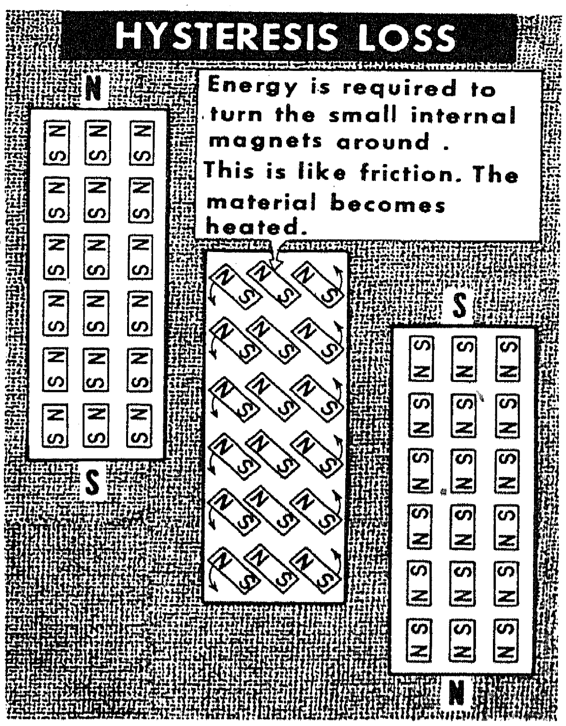
(C) The centertap has a relative dual polarity.

The centertapped secondary (B) is simply a means of achieving equal voltages on both sides of a common reference point--the centertap--to suit certain operating conditions. The voltage available from each half of the winding relative to the primary voltage is determined by the turns ratio between each half of the winding and the primary. At any one instant, the polarity of the voltage available from the whole centertapped winding is 180° out of phase with the primary polarity, but the centertap behaves as if it has a dual polarity (C). With respect to the centertap, the secondary provides two equal voltages 180° out of phase with each other.

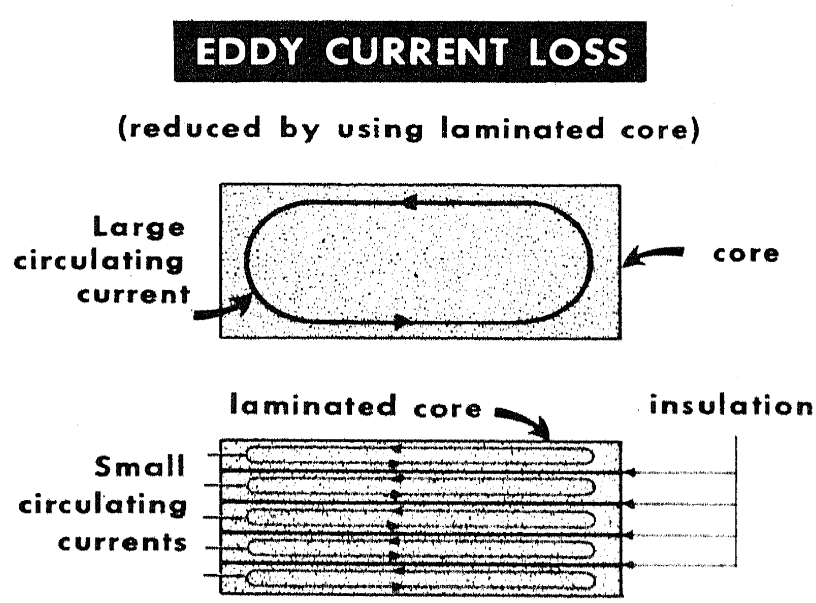
Transformer Losses

We have stated that the transformer can be made almost a 100% efficient device. There are, however, certain inherent losses in a transformer that can be minimized but never completely eliminated. The most apparent losses are called copper losses. Since the primary and secondary are wound with many turns of copper wire, there will be wasted I^2R losses. This accounts for the secondary voltage being slightly lower under load than when unloaded. These losses are reduced by using the largest practical cross-sectional area wire.

Hysteresis losses are due to the lagging of the magnetization and demagnetization of the soft steel core behind the alternating current in the circuit. The atoms of the core material must keep changing polarity, and a sort of frictional loss is developed. The use of material such as soft silicon steel for the core greatly reduces hysteresis losses.



TRANSFORMER LOSSES

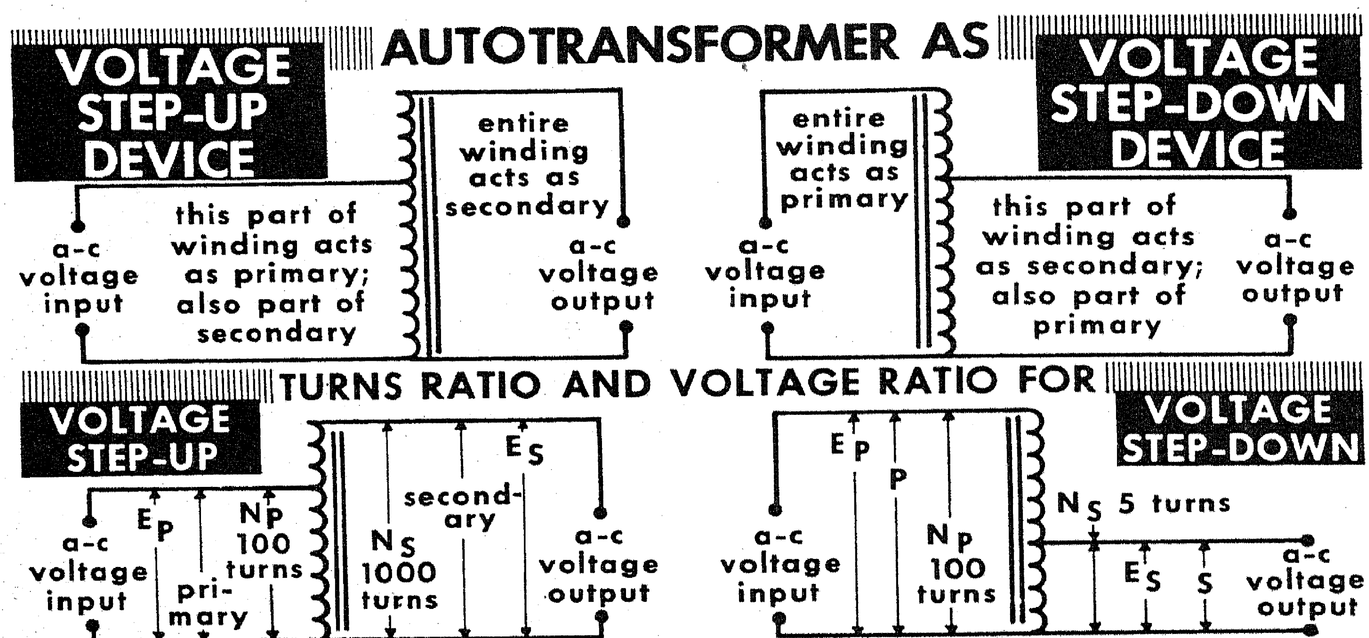


Magnetic core materials cause additional transformer losses because, as conductors, small short-circuited currents called eddy currents are induced in them. To reduce these currents, transformer cores are laminated. Each lamination (strip) is sprayed with an insulated coating so that the d-c resistance between them is very high. The strips are then pressed together to form the core.

A source of inefficiency stems from the fact that all the lines of flux produced by the primary and secondary windings do not move through the iron core--some leak directly out of the windings into space and do not link the windings. This is known as flux leakage. Another core inefficiency occurs during core saturation. Above a certain point, an increase in magnetizing force causes no additional magnetization. Thus, more magnetizing current is being used than required, resulting in a loss in efficiency.

The Autotransformer

The iron-core autotransformer differs from the multi-coil device we have been studying. Its operation, as before, depends on flux lines being produced by the primary current, cutting the turns of the secondary and inducing a voltage. However, its advantage over the conventional transformer is that the secondary voltage remains substantially constant when the load on the secondary is changed. However, the disadvantage of the autotransformer is that its primary and secondary are part of the source coil; hence, there is no isolation of the external circuits that are connected to the primary and the secondary. This does not prevent the use of the autotransformer in many circuits.



Turns ratio $= \frac{N_S}{N_P} = \frac{1000}{100} = 10$

Secondary voltage $E_S = \frac{N_S}{N_P} \times E_P$

If $E_P = 120$ volts, $E_S = \frac{1000}{100} \times 120$
$= 1200$ volts

Turns ratio $= \frac{N_S}{N_P} = \frac{5}{100} = 0.05$

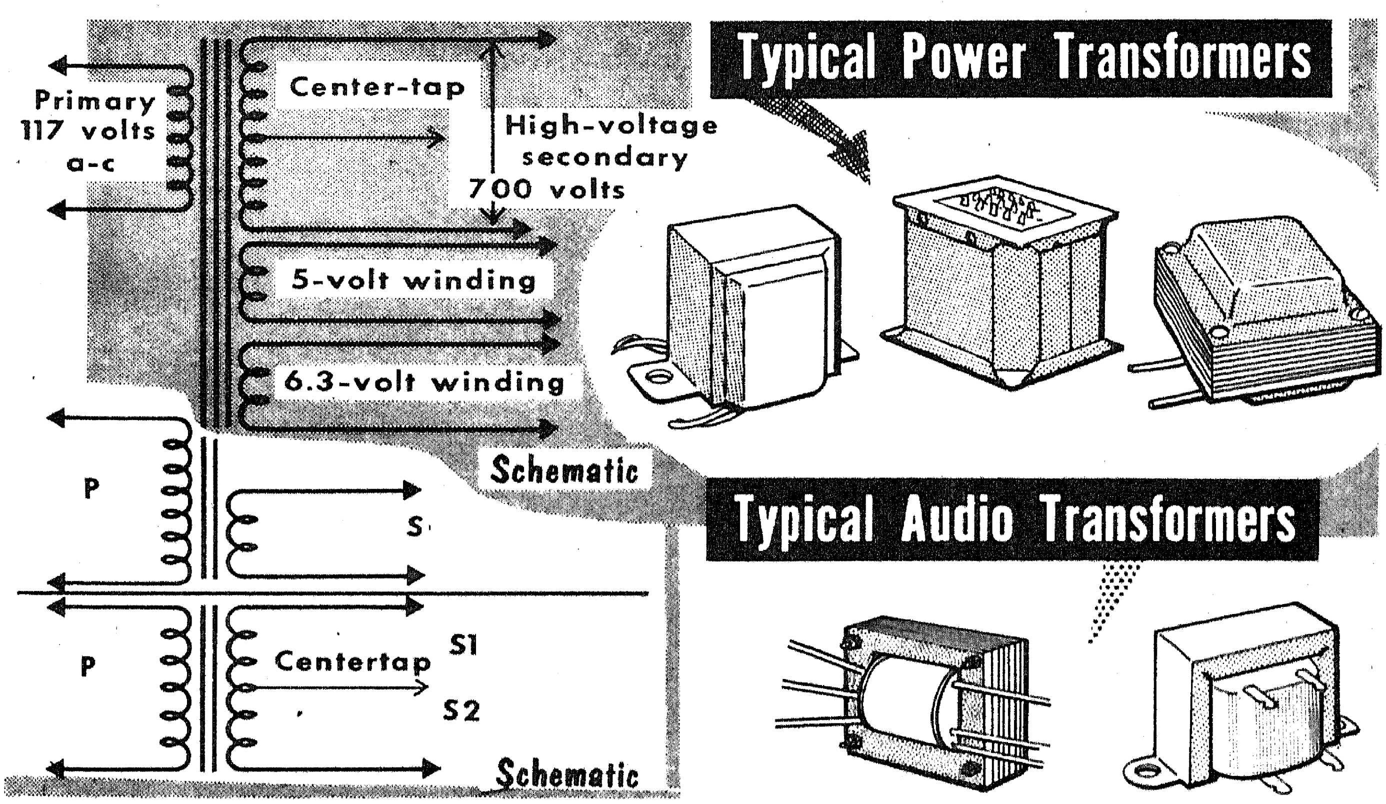
Secondary voltage $E_S = \frac{N_S}{N_P} \times E_P$

If $E_P = 120$ volts, $E_S = \frac{5}{100} \times 120$
$= 6$ volts

When used as a voltage step-up transformer, the entire winding is the secondary and a part of the winding is the primary. When used as a voltage step-down transformer, the entire winding is the primary and part of the winding is the secondary. Obviously, one part of the winding is always common to both the primary and secondary functions. The turns ratio between the secondary and primary portions of the winding determines the output (secondary) voltage relative to a given input (primary) voltage, just as in the case of the conventional transformer. In this regard, note that a certain number of turns is common to both the so-called secondary and primary windings. This does not change the usual manner of determining the turns ratio--the secondary and primary are considered as if each were separate and individual. Usually, taps on the winding permit changing the secondary-primary turns ratio; hence, the output voltage.

Transformer Applications

The transformer truly has an unusual variety of applications in radio communications. We will discuss a few here, and many others will be covered during this course when applicable. The most commonly used is the power-transformer type. This transformer is used in the power supply of electronic equipment to furnish the various a-c voltages necessary for the production of d-c voltages, and for the operation of particular circuits. The basic power transformer has a single primary winding, with two or more secondary windings. One secondary winding usually provides high-voltage a-c for the rectifier, and one or more others provide various filament voltages for tubes (these terms will be discussed later in the course). The high-voltage winding commonly provides from 400 to 800 volts a-c at currents from 25 to 400 milliamperes, and is usually center-tapped. The filament windings usually provide 6.3 or 12.6-volts at 1 to 5 amperes, and 5 volts at 2 or 3 amperes.

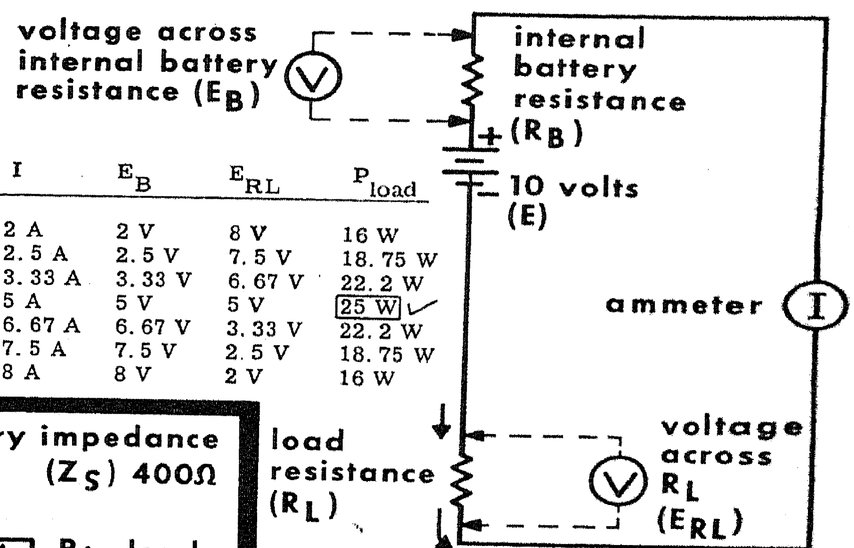


Another commonly used transformer is the audio type. Designed to operate at audio frequencies (20-16,000 cycles), it is usually smaller than the power transformer and has a wide variety of functions. These transformers are used primarily for impedance matching and, in some instances, for voltage amplification. Audio transformers are usually designated by their particular application--input transformer, output transformer, microphone transformer, modulation transformer, interstage transformer, driver transformer, etc. Usually, they are rated by their primary and secondary impedances and current-handling capabilities.

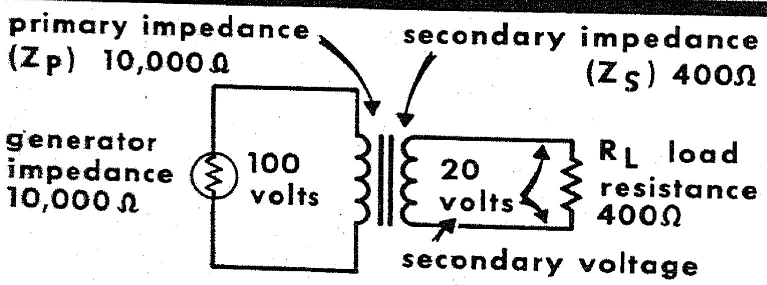
Transformers designed to operate at high frequencies (above audio range) are referred to as intermediate-frequency and radio-frequency types, and will be discussed where applicable in this course.

Impedance Matching

In the transfer of power from any electrical source of its load, the impedance of the load must be equal to or match the internal impedance of the source for maximum transfer of power. From our table, we see how this is so. Assuming a 10-volt battery having an internal resistance of 1 ohm, we connect various loads ranging from 0.25 ohm to 4 ohms. From the calculations, it is seen that the greatest amount of power is delivered to the load when the load has a resistance or impedance of 1 ohm--the same as the internal impedance of the battery or voltage source.



$E_{battery}$	$R_{battery}$		R_{load}	R_t	I	E_B	E_{RL}	P_{load}
10 V	1 Ω		4 Ω	5 Ω	2 A	2 V	8 V	16 W
10 V	1 Ω		3 Ω	4 Ω	2.5 A	2.5 V	7.5 V	18.75 W
10 V	1 Ω		2 Ω	3 Ω	3.33 A	3.33 V	6.67 V	22.2 W
10 V	$\boxed{1\ \Omega}$	=	$\boxed{1\ \Omega}$	2 Ω	5 A	5 V	5 V	$\boxed{25\ W}$ ✓
10 V	1 Ω		0.50 Ω	1.5 Ω	6.67 A	6.67 V	3.33 V	22.2 W
10 V	1 Ω		0.33 Ω	1.33 Ω	7.5 A	7.5 V	2.5 V	18.75 W
10 V	1 Ω		0.25 Ω	1.25 Ω	8 A	8 V	2 V	16 W



$$\frac{N_P}{N_S} = \sqrt{\frac{Z_P}{Z_S}} \quad \text{or} \quad \frac{Z_P}{Z_S} = \left(\frac{N_P}{N_S}\right)^2$$

The transformer is a useful device for <u>matching</u> the impedance of a generator to that of its load. This is important because in radio work, it is often necessary to connect a low-impedance load to a high-impedance generator, and vice versa. Unless there is an impedance match, there will not be maximum transfer of power. Assuming a source or generator impedance of 10,000 ohms, we will match it to a load of 400 ohms. Using a transformer, the primary impedance must match the generator impedance, and the secondary impedance must match the load impedance. The turns ratio of the transformer must be:

$$\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{10,000}{400}} = \sqrt{\frac{25}{1}} \quad \text{Then, } \frac{N_p}{N_s} = 5:1$$

If 100 volts are applied to the primary, the secondary voltage is 20 volts. Secondary current is 20/400, or .05 ampere. The primary current is 100/10,000, or .01 ampere. Since the primary power (1 watt) is equal to the secondary power, the transformer has matched a 400-ohm load to a 10,000-ohm source with maximum transfer of power. We can say that the source "sees" the primary impedance as a matching impedance, and the secondary, which by transformer action receives the primary power, "sees" the load impedance as a matching impedance.

Any change of current flowing in a circuit containing inductance produces a counter emf which opposes the change taking place. This self-induced emf tends to prevent an increasing current from increasing and a decreasing current from decreasing.

Basically, Lenz's law states, "A changing current induces an emf whose polarity is such as to oppose the change in current."

The greater the coil inductance, the higher the induced emf, and the greater the opposition to the increase and decrease of current in the coil.

Mutual induction occurs when a changing magnetic field produced by one coil cuts the windings of a second coil and induces an emf in the second coil.

Inductive reactance (X_L) is the opposition presented by an inductance to an alternating current. $X_L = 2\pi fL$.

In a series R-L circuit, the voltage drops across R and L are 90° out of phase.

In a circuit containing both inductance and resistance, impedance (Z) is the total opposition to the flow of alternating current, and is a combination of X_L and R. It is expressed in ohms. $Z = \sqrt{X_L^2 + R^2}$.

Ohm's law for a-c circuits is: $E = IZ$; $I = E/Z$; and $Z = E/I$.

The vectorial sum of all the voltage drops in a series R-L circuit is equal to the applied voltage.

The primary winding of a transformer absorbs the input electrical energy from a voltage source; the secondary winding delivers the induced output voltage to a load.

The turns ratio between the secondary and primary windings of a transformer determines whether the secondary voltage is greater or less than the primary voltage.

The primary-secondary current ratio is opposite to that of the primary-secondary voltage ratio.

In the transfer of power from an electrical source to a load, the load impedance must be equal to, or match, the internal impedance of the source for maximum transfer of power.

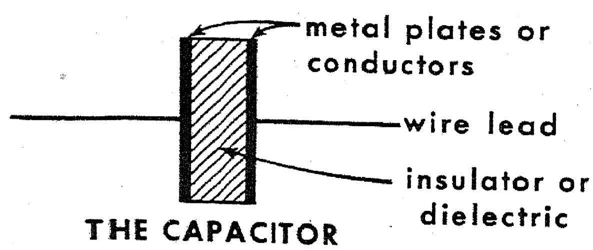
REVIEW QUESTIONS

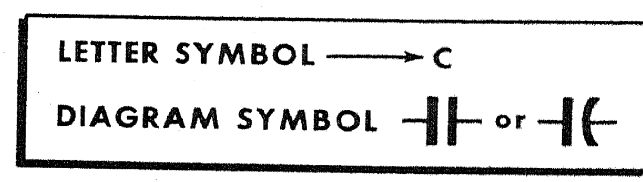
1. What is self-induction of emf? Describe its action.
2. Define inductive reactance and give the formula for calculating the X_L of a circuit.
3. Describe the action of mutual induction and tell what its purpose is.
4. In an inductive circuit, what are the phase relationships between: (1) the current and the counter emf it produces; (2) the applied voltage and the counter emf; (3) the applied voltage and the current?
5. What is impedance? How is it calculated in series R-L circuits?
6. What two methods can be used to determine the impedance of a series R-L circuit?
7. State Ohm's law for a-c circuits.
8. What is the total current in a parallel R-L circuit equal to?
9. What formula is used to find the impedance of a parallel R-L circuit?
10. Give two main functions of a transformer.
11. What determines whether a voltage step-up or step-down takes place in a transformer?
12. Name three types of losses inherent in a transformer.

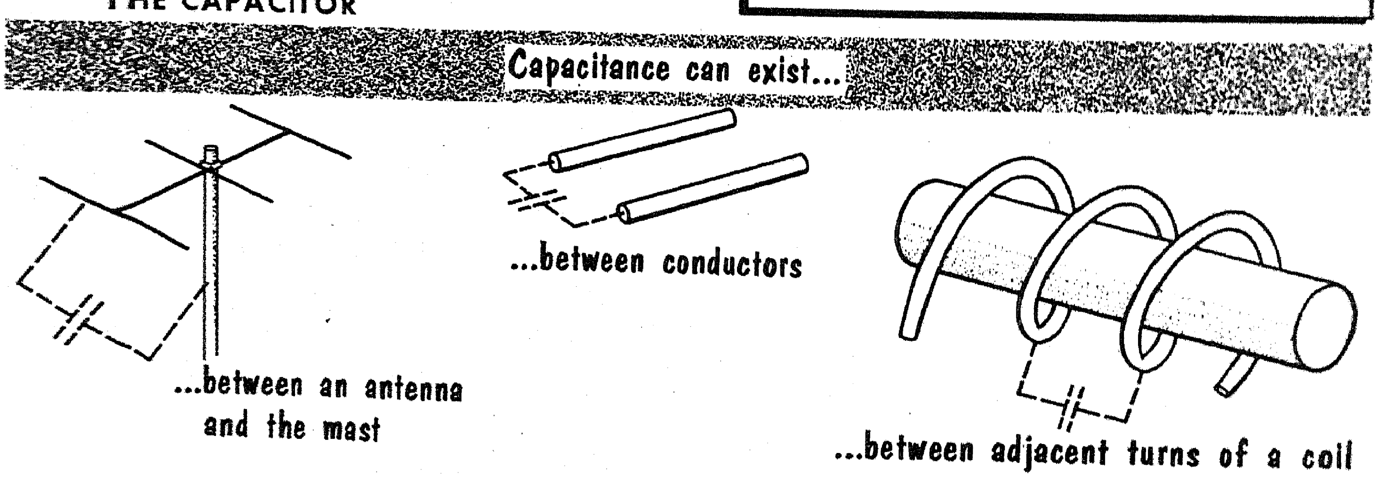
Definition, Function, Construction

An electrical capacitor (also known as a condenser) stores electricity by accumulating free electrons on a metal surface, and then releases them as a current into the circuit of which the capacitor is a part. It can be said that capacitance is a property of a circuit in which energy may be stored in the form of an electric field. This capability that we call capacitance is generally designated by the letter C. Although electrical capacitors are used in many different ways (as you shall see), and seemingly for different purposes, every use entails the storage and release of electrical energy. This action underlies the definition of capacitance — that property of a circuit which opposes a change in voltage. This property differs from that of an inductance, which opposes any change in current.

CAPACITANCE IS A PROPERTY OF AN ELECTRIC CIRCUIT THAT TENDS TO OPPOSE A CHANGE IN VOLTAGE





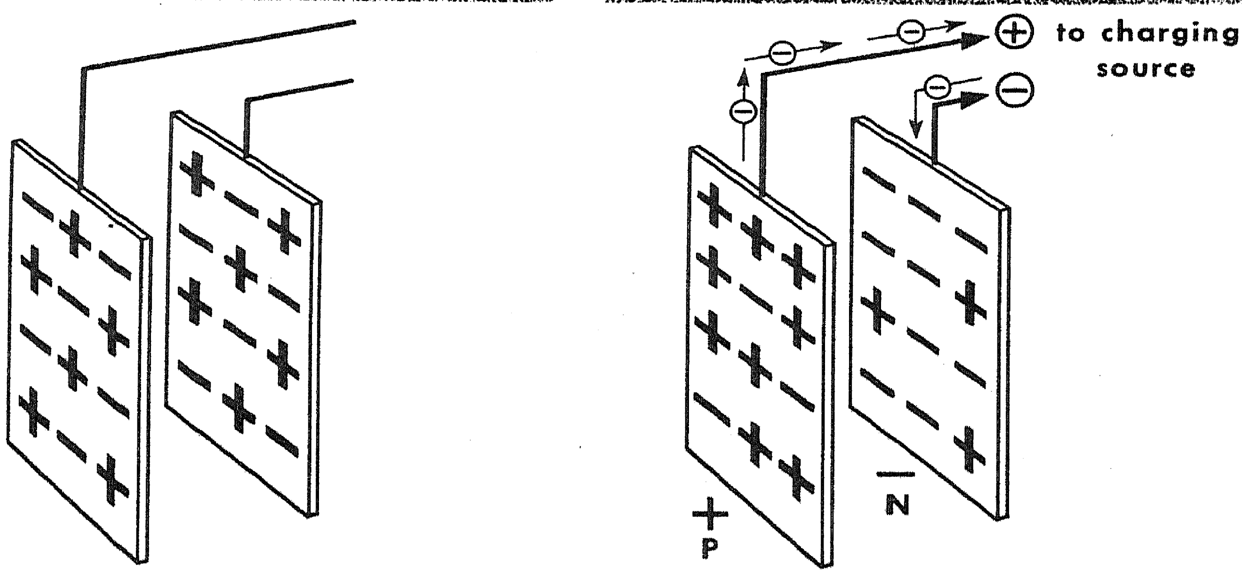


Any two conductors separated by an insulator (called a dielectric) can behave as a capacitor. The conductors may be long or short lengths of wire, large or small pieces of metal or metal foil, or any other conducting material. The dielectric that separates the conducting materials may be the insulation around the wires, a thin insulating film chemically deposited on the metal, ceramics, mica, oil, or wax-impregnated paper to mention just a few. In some cases, the dielectric is air, and in still others, a vacuum. As a rule, capacitors used in electrical and radio circuits are specially manufactured items but, as will be explained later, the metal parts of electrical and radio systems often behave as capacitors.

Charging a Capacitor

The action of storing electricity in a capacitor is called charging. The electricity stored in the capacitor is called the charge. As used here, "charge" refers to an amount of electricity being stored, rather than to the fundamental particles: the electron and the proton. The action of releasing the stored electricity is called discharging. For purposes of explanation, let us assemble a basic capacitor consisting of two thin sheets of aluminum 10 inches square separated by air and positioned 1/2 inch apart. Because air separates the two active plates (conductors) of the capacitor, the unit is called an air-dielectric capacitor.

IN THE UNCHARGED CAPACITOR | **IN THE CHARGED CAPACITOR**



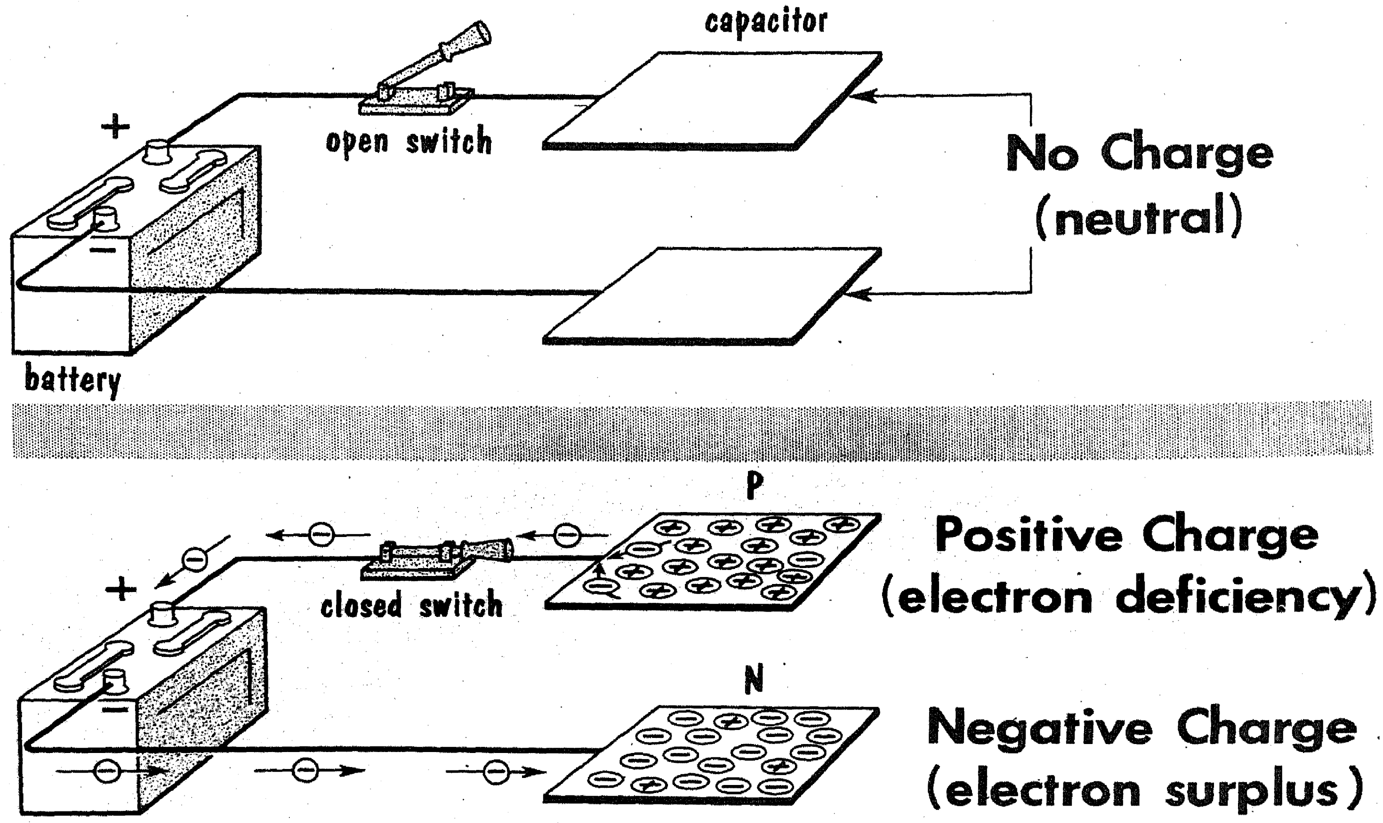
there are equal amounts of positive and negative electricity on each plate

one plate has an excess of positive electricity; the other plate has an excess of negative electricity

PRODUCED BY FORCING FREE ELECTRONS FROM ON PLATE ONTO ANOTHER

We begin by assuming that the two metal plates are electrically neutral — each plate contains equal amounts of positive and negative electricity. The capacitor is therefore in an uncharged state. In the process of charging, one plate (plate P) of the capacitor is made to give up free electrons and be left with a preponderance of positive electricity. The other plate (plate N), is made to accept as many free electrons as were released by plate P, and now has a surplus of negative electricity. Both electrical conditions are created simultaneously. When this electrical condition prevails, the capacitor is said to contain a charge, or be charged.

Charging a Capacitor (Cont'd)



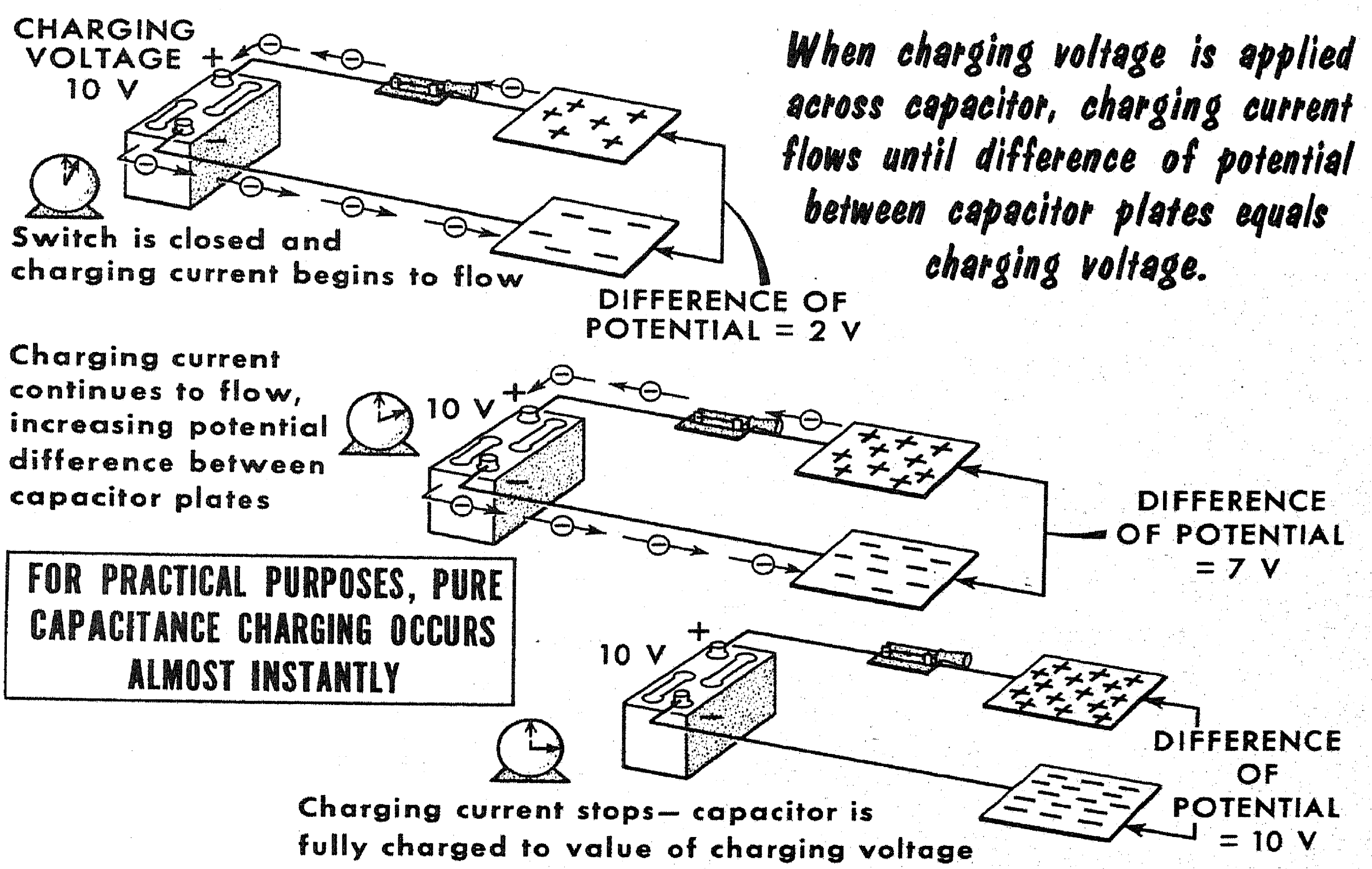
A CAPACITOR IS CHARGED BY PLACING A DIFFERENCE OF POTENTIAL ACROSS PLATES

To charge a capacitor, it is necessary to apply a voltage (or difference of potential) to its plates (across its terminals). In other words, the electrical energy stored in the capacitor must come from a source of voltage (we show a battery). Assume the circuit elements shown. The open switch isolates the battery from the capacitor. Then, the switch is closed and the capacitor becomes charged from the voltage applied to its plates by the battery.

The positive terminal of a voltage source is always deficient in free electrons. It draws these electrons from one plate of the capacitor and leaves that plate (P) with a deficiency of free electrons; hence, with a positive charge. At the same time, the negative terminal of the voltage source releases an equal number of free electrons into the wire connected to it, thereby forcing free electrons onto the other plate (N). These added free electrons create a surplus of negative charge on this plate, thus giving the N plate a negative charge. The creation of such an electrical condition on the plates of the capacitor is known as charging. The free electrons that are pulled from the P plate of the capacitor to the positive terminal of the battery and the free electrons that move from the negative terminal of the battery to the N plate constitute a momentary current usually referred to as a charging current.

Building Up Voltage in the Capacitor – Charging Current

From the instant that free electrons leave one plate and begin accumulating on the other, charging current flows, and a difference of potential (voltage) appears between the plates of the capacitor. (Note that the charging current flows first; then, the voltage buildup occurs.) Unless deliberately prevented from doing so, the charging (applied) voltage will continue attracting free electrons from one plate and forcing them onto the other. This process builds up the capacitor voltage until it becomes equal to the maximum value of the charging voltage. The voltage built up across the capacitor has the same polarity as the charging voltage; consequently, it acts in opposition to the charging voltage. When the capacitor voltage equals the charging voltage, the two voltages offset each other, and there is no further movement of free electrons (no charging current).



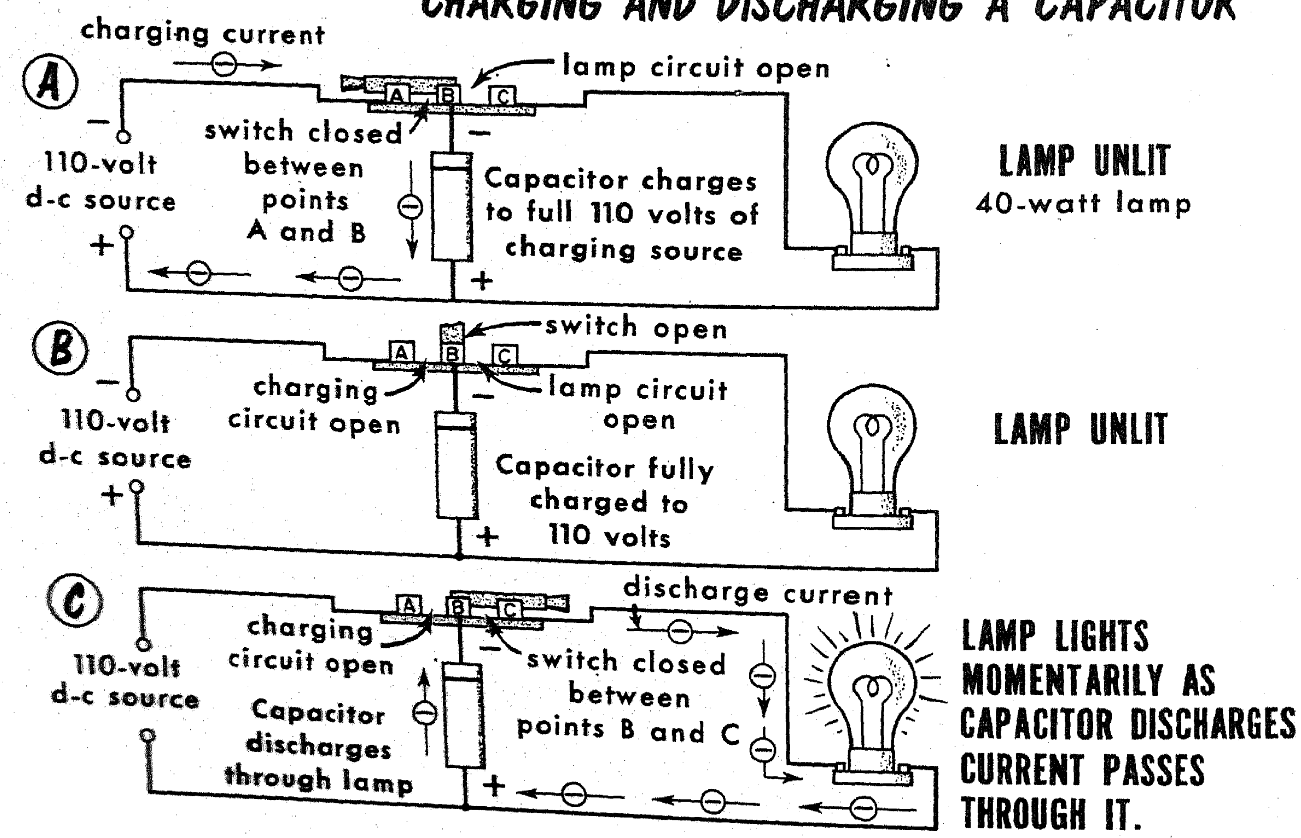
There is a limit to how much voltage can be built up across a given capacitor by a given charging voltage. The capacitor voltage cannot exceed the maximum value of the charging voltage at any time. But it is possible to subject a capacitor which has been charged by a lower value of charging voltage to an increased value of charging voltage. The capacitor voltage then rises to the higher value. The ability to charge a capacitor to higher and higher values of voltage is not without limitations. This limitation arises from a constructional characteristic of the capacitor. Every capacitor has a maximum d-c working voltage rating (which will be explained later). For the present, let us say that the capacitor voltage rating sets the limit on the highest value of chargir voltage that may be applied to the capacitor.

Demonstrating the Voltage across a Charged Capacitor

When a charged capacitor is disconnected from the charging voltage source, the charge remains in the capacitor. If the capacitor is very "good," it retains its charge for a long period of time, during which time a voltage will be present across its terminals. This is not the usual way in which capacitors are used in electrical and communication systems, but this capability does have its uses.

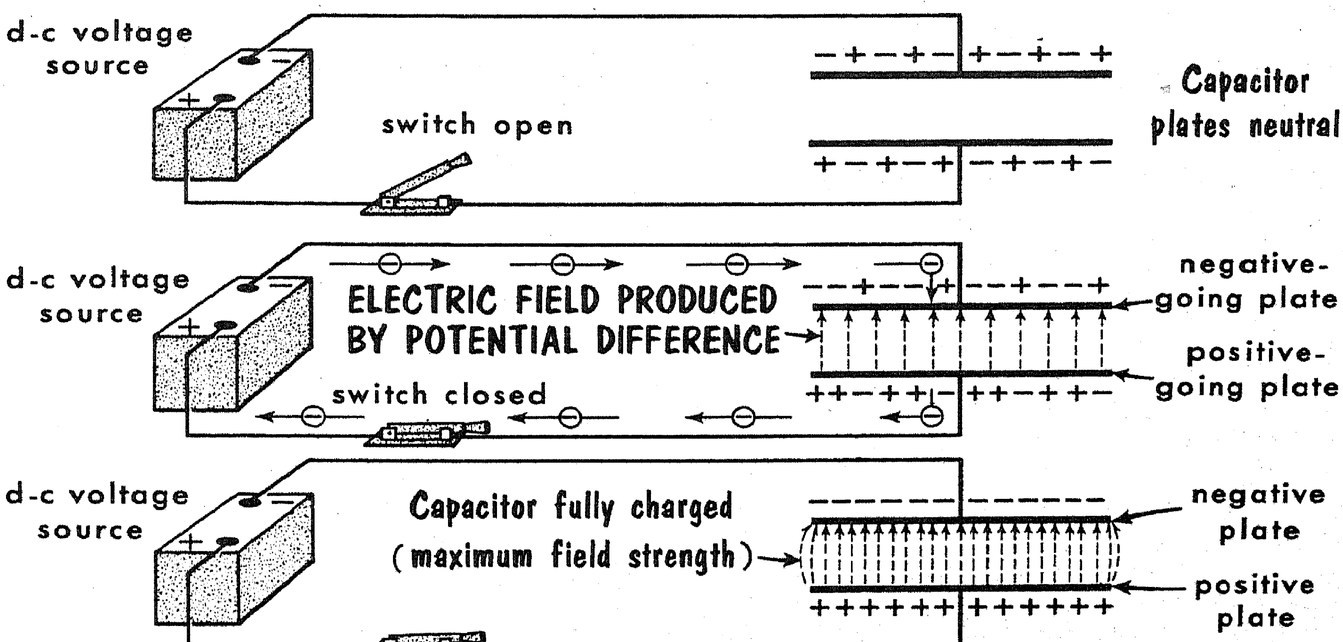
Assume that a capacitor is connected to a 110-volt d-c charging source by means of a single-pole double throw (spdt) switch. After the lapse of sufficient time (a short interval) to charge the capacitor fully, the switch is opened, thus removing the capacitor from contact with the charging circuit. To prove that the capacitor is charged and that a voltage exists across its terminals, we connect an ordinary 40-watt, 110-volt household electric light across the capacitor by closing the switch. The instant the switch is closed, the lamp glows brightly for a moment. The electrical energy required to light the lamp was obtained from the charged capacitor. The voltage across the terminals of the charged capacitor overcomes the resistance of the lamp filament (and the connecting wires), and allows the surplus free electrons on the negative plate of the capacitor to behave as current and move to the positively charged plate of the capacitor through the filament. This action <u>discharges</u> the capacitor. The movement of the free electrons from the negatively charged plate of the charged capacitor constitutes a <u>discharge current</u>; this is accompanied by a fall of voltage across the capacitor to zero. After this discharge, the filament no longer glows, because there is no flow of electrons (current) through the lamp filament.

CHARGING AND DISCHARGING A CAPACITOR



The Electric Field between the Capacitor Plates

You have learned that every fundamental particle of electricity – each electron and each proton – is inseparably associated with an invisible region of energy that exists all around the charge. The zone of energy is referred to as an electric field made up of electric lines of force.

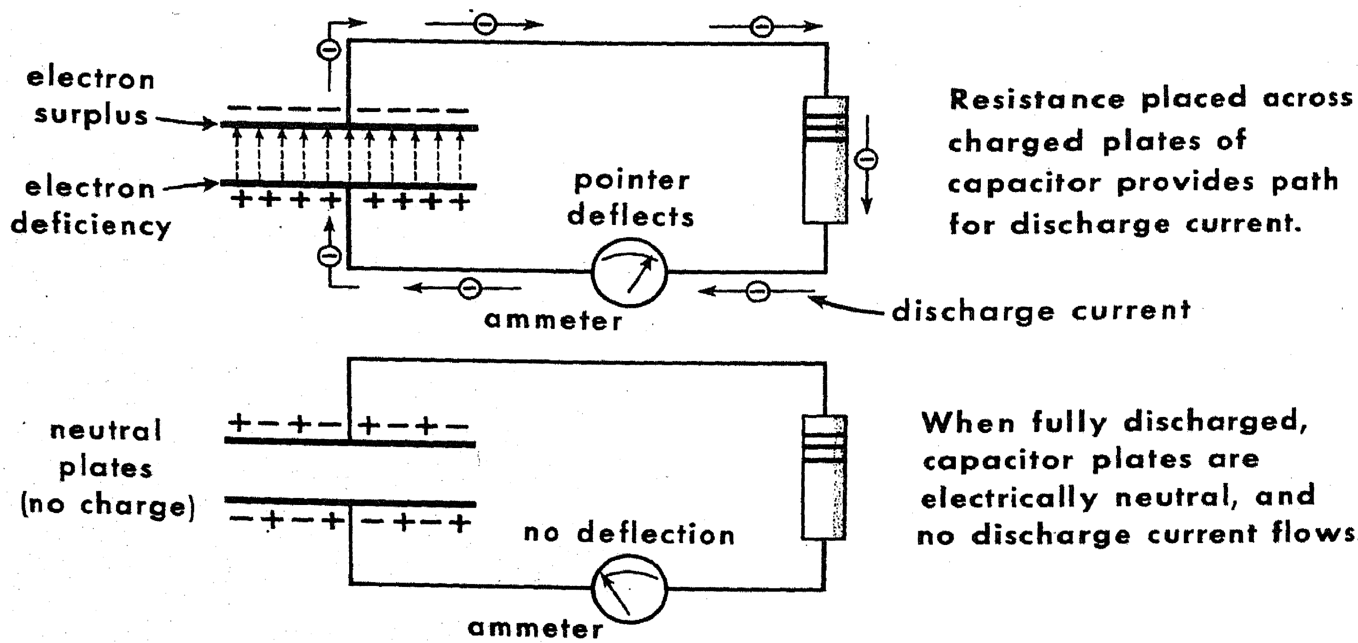


Arrows between plates represent ELECTROSTATIC LINES OF FORCE which make up ELECTROSTATIC FIELD between plates. FIELD STRENGTH is proportional to POTENTIAL DIFFERENCE between plates.

Beginning with the first instant after the flow of charging current (the appearance of a positive charge on one plate of the capacitor and a negative charge on the other), an electric field is developed between the two charged plates within the space between the plates. This field is between the positive charges on the positive plate and the negative charges on the negative plate. As more and more electrons are removed from the plate and more and more electrons are added to the negative plate, the lines of force increase in number, indicating increased field intensity. This action accompanies the rise in voltage. When the capacitor voltage has reached its maximum value, the field intensity has reached a maximum, and remains as long as the charge given the capacitor remains unaltered. The energy stored in the capacitor is in the electric field.

Discharging a Capacitor

All of the electricity stored in a theoretically perfect capacitor can be recovered from it by providing a suitable electrical conducting path between the terminals of the capacitor. Such a current path is called a discharge path. The electric light filament referred to previously formed such a path. Whether the energy taken out of the charged capacitor is used or wasted is a function of the electrical makeup of the discharge path. The usual discharge path is the circuit connected between the terminals of the capacitor. It is never the battery which serves as the charging voltage source.

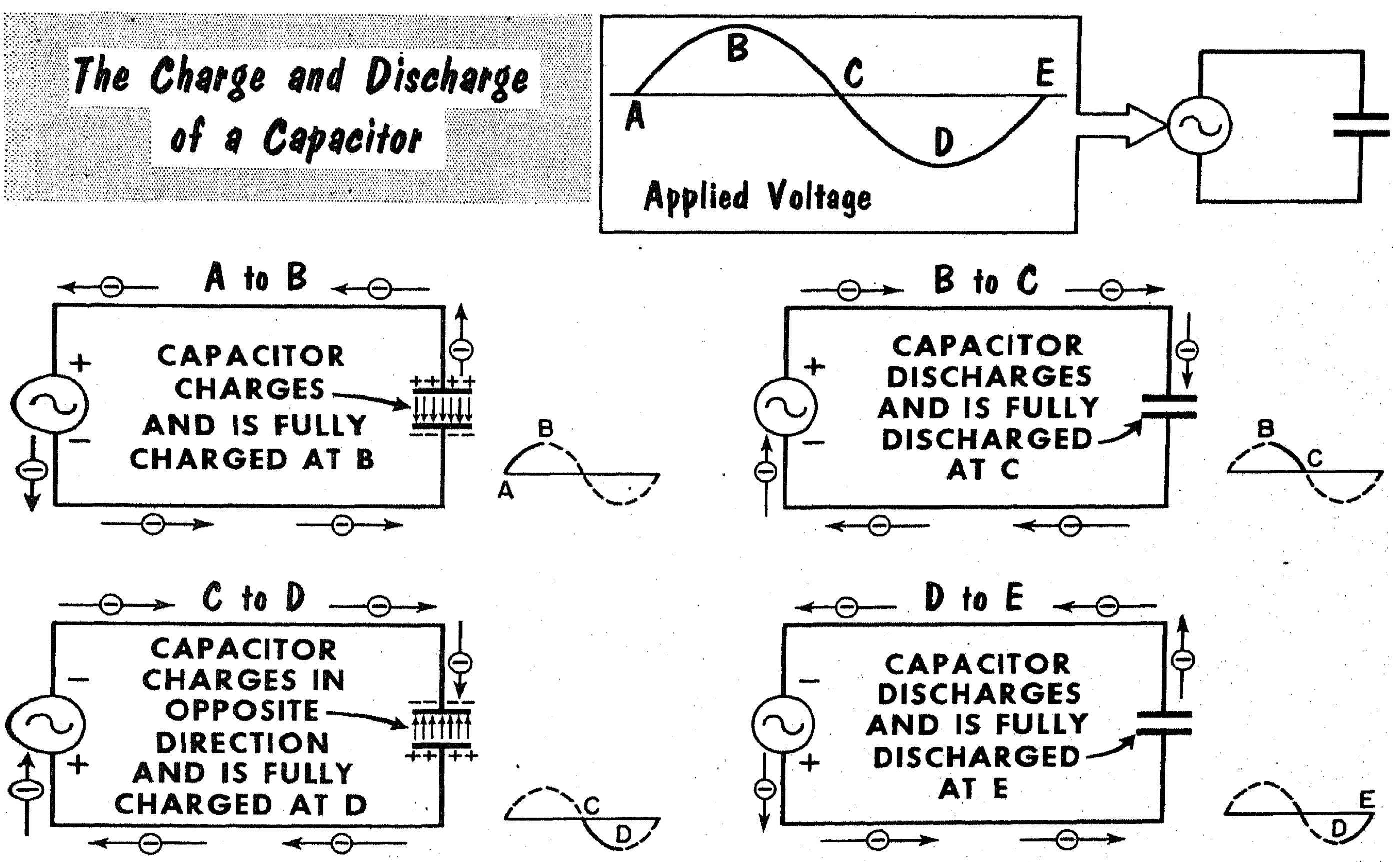


CAPACITOR DISCHARGES WHEN THERE IS A DIFFERENCE OF POTENTIAL BETWEEN PLATES AND A COMPLETE EXTERNAL PATH BETWEEN CAPACITOR TERMINALS

The action of recovering the energy stored in a charged capacitor is referred to as discharging the capacitor. During discharge, the surplus free electrons on the negatively charged plate move toward the positive charges on the positively charged plate via the discharge path. This movement of surplus electrons reduces the negative charge on the negative plate and the positive charge on the positive plate. Since the movement of free electrons during discharge is a directed motion, it is actually a current, and is referred to as the discharge current. The loss of charge on the negatively and positively charged plates by the flow of the discharge current causes the voltage originally built up across the capacitor plates to decrease. When all the surplus electrons have moved from the negatively charged plate to the positively charged plate, thus making both plates electrically neutral, there is no further charge in the capacitor, and the voltage across its terminals is zero. The capacitor is then fully discharged.

Charging a Capacitor from an A-C Voltage Source

A capacitor can be charged as readily by an a-c voltage as by a d-c voltage. However, the constantly changing amplitude and periodic polarity reversal of the a-c voltage point up many interesting capacitor characteristics. Assume a sine waveform voltage from an a-c source. The charging voltage starts at zero amplitude and increases in a positive direction. The first increase in charging voltage results in the flow of maximum charging current. The instant charge is applied to the capacitor, a voltage, or potential difference, starts building up across the capacitor. As the charging voltage increases in amplitude, more and more charge is added to the capacitor by progressively decreasing amounts of current. The decrease in current is due to the increased bucking action by the voltage (sometimes called counter-voltage) building up in the capacitor. When the charging voltage reaches its peak value, there is zero charging current and maximum voltage built up in the capacitor.

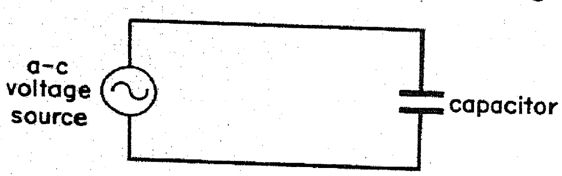


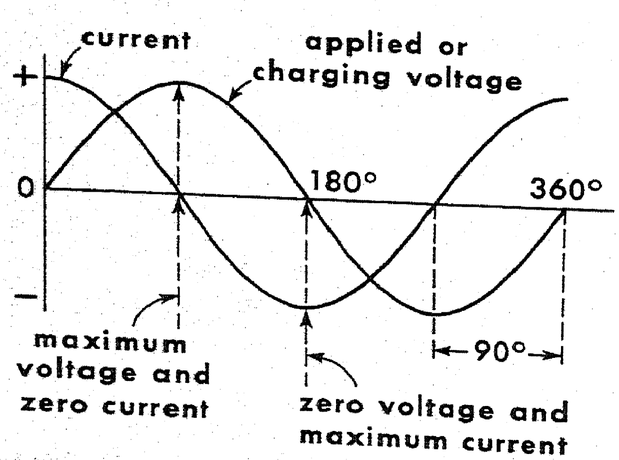
Having reached its peak positive value at the end of the first quarter-cycle (90°), the charging voltage begins to decrease in amplitude. At the first instant of decrease, the voltage built up across the capacitor exceeds the value of the charging voltage source. The voltage in the capacitor begins to fall as charge decreases. Note that the flow of discharge current began first, followed by a fall in capacitor voltage. Similarly, the charging current flow was ahead of the rise in capacitor voltage. The time sequence between the charging current and the rise in capacitor voltage, and between the discharge current and the fall in capacitor voltage, is described by saying that the capacitor current leads the capacitor voltage by a quarter-cycle, or 90°

Voltage and Current Phase Shift in a Capacitor

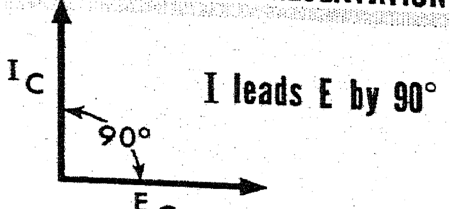
Current and voltage variations in the capacitor during the positive half-cycle of the charging voltage repeat themselves during the negative half-cycle. Of course, the polarity change in charging voltage produces changes in the direction of current flow and in the polarity of the voltage across the capacitor, but the 90° phase difference between I and E is constant throughout the cycle. (This is an important point to remember.) It is also important to remember that the capacitor charge is zero when the current in the circuit is maximum, and maximum when the current is zero. Therefore, the charge on the plate of a capacitor is said to lag the current through it by 90°. However, since the building up and falling off of charge is the building up and falling off of voltage, the voltage across the capacitor is said to lag the current through it by 90°, or the current is said to lead the voltage by 90°.

From Kirchhoff's law, we know that the sum of the voltage drops in a series circuit is equal to the applied voltage. Therefore, the voltage across the capacitor is, by definition, a voltage opposite to the applied voltage, or 180° out of phase with the applied voltage. Thus, when the applied or charging voltage is zero, there is no opposition, and when the applied voltage is maximum, there is a maximum opposition – a voltage produced by the charge on the capacitor. The variation in charge on the plates of the capacitor also follows the form of the sine wave and is in phase with the voltage, since for any given capacitor, the voltage across it depends directly on the charge.

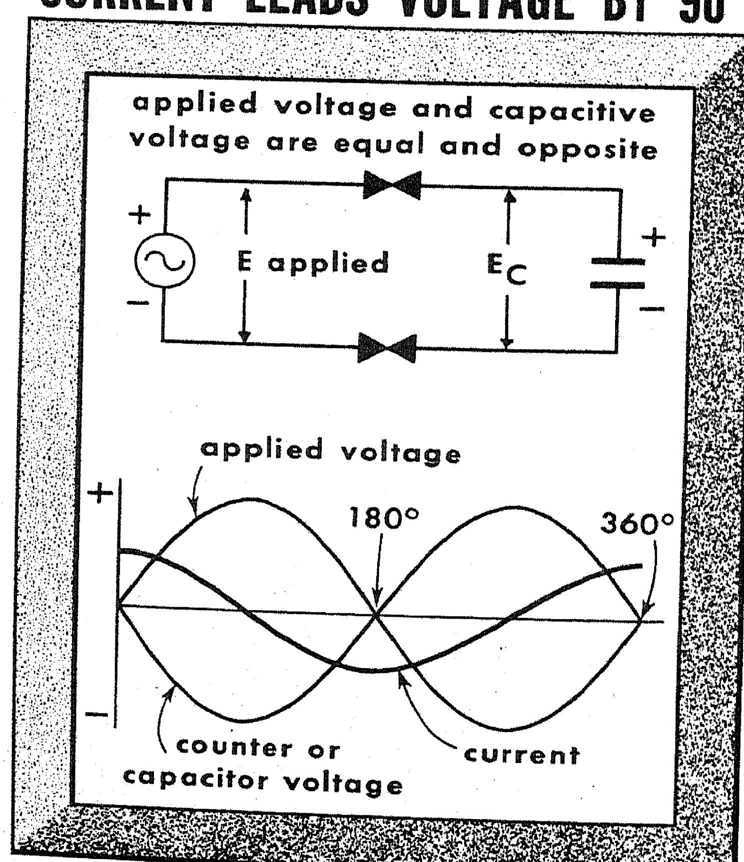




VECTOR PRESENTATION



IN A CAPACITIVE CIRCUIT CURRENT LEADS VOLTAGE BY 90°



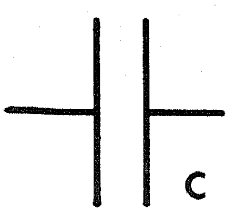
Unit of Capacitance — the Farad

The unit of capacitance is the farad, named in honor of Michael Faraday, the scientist who advanced the concept of electromagnetic induction. The number of electrons entering and leaving the capacitor plates depends upon the free electrons available and on the applied voltage. If the applied voltage is high, the forces of attraction and repulsion are great, and the charge deposited on the plates is also great. It was discovered that for a given capacitor, the ratio between the amount of this charge and the voltage causing it is always a constant. Therefore, the ratio of the charge (Q) to the voltage (E) is considered to be a measure of the capacitor action, called capacitance (C). The formula for capacitance is: $C = Q/E$.

By definition, a capacitor has a capacitance of 1 farad if a 1-volt difference in potential results in the storage of 1 coulomb of charge. One coulomb represents a quantity of 6.28×10^{18} electrons (6,280,000,000,000,000,000). For practical purposes, a capacitance of 1 farad represents a fantastically large capacitance. As a practical matter, capacitors used in radio communications are measured in terms of microfarads (one-millionth of a farad, μf) and micromicrofarads (one-millionth of a millionth of a farad, μμf).

Suppose we find the capacitance of a capacitor when the charge (Q) stored is 0.001 coulomb and the voltage is 1000 volts. By applying the above formula, $C = Q/E$, we obtain 0.001/1000, or 0.000001 farad. We call this 1 microfarad.

THE FARAD -- A measure of the Storage Ability of a Capacitor



$$C \text{ (capacitance in farads)} = \frac{Q \text{ (charge in coulombs)}}{E \text{ (voltage in volts)}}$$

CAPACITANCE CONVERSIONS

1,000,000 MICROFARADS (μf) = 1 FARAD (f)
1,000,000 MICROMICROFARADS (μμf) = 1 MICROFARAD
1,000,000,000,000 MICROMICROFARADS (μμf) = 1 FARAD
1,000,000,000,000 PICOFARADS (pf) = 1 FARAD

$\frac{1}{1,000,000}$ f OR 0.000001 f = 1 MICROFARAD

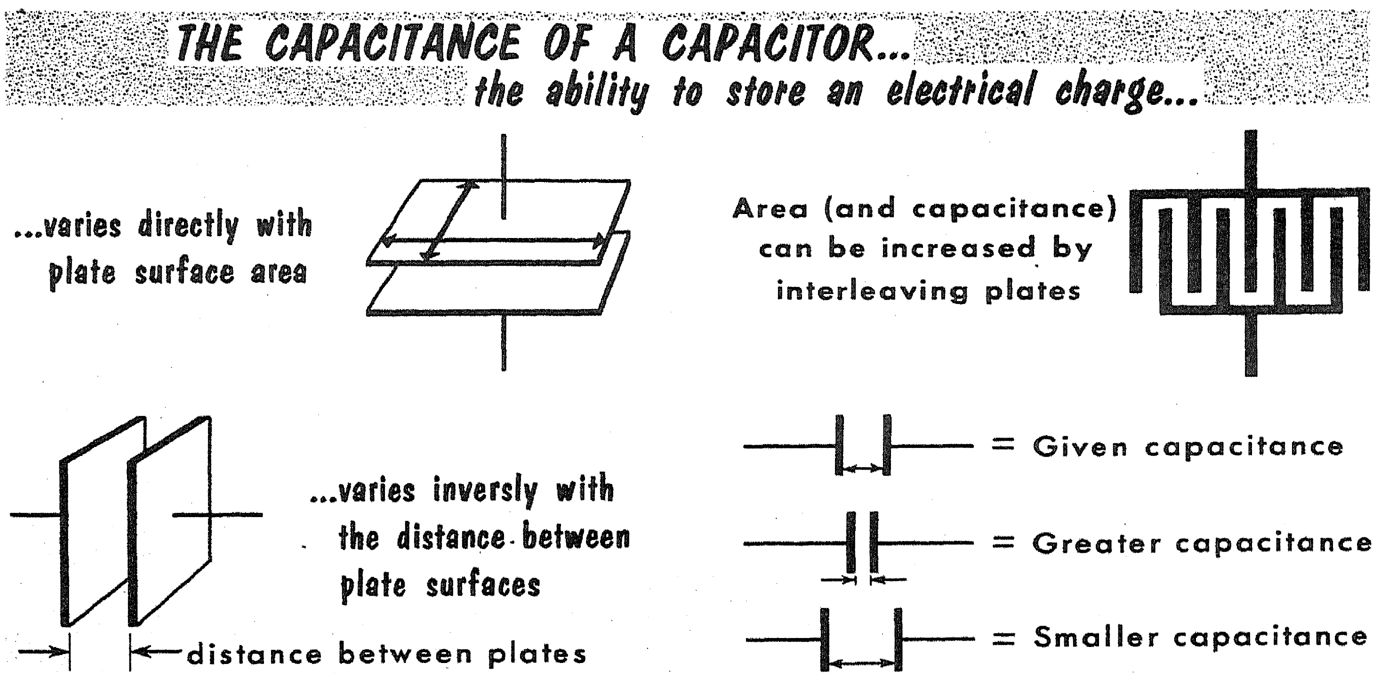
$\frac{1}{1,000,000}$ μf OR 0.000001 μf = 1 MICROMICROFARAD

$\frac{1}{1,000,000,000,000}$ f OR 0.000000000001 f = 1 MICROMICROFARAD

$\frac{1}{1,000,000,000,000}$ f OR 0.000000000001 f = 1 PICOFARAD

500μμf	= 0.0005μf	50μf	= 0.00005 f	500μf	= 0.0005 f
10 μμf	= 0.000010μf	0.01μf	= 0.00000001 f	0.001μf	= 1000μμf
3000μμf	= 0.003μf	0.0047μf	= 0.0000000047f	0.0047μf	= 4700μμf
50 pf	= 0.000050μf	0.05μf	= 0.00000005 f	0.005μf	= 5000 p

Factors Determining Capacitance Area and Plate Separation



Three factors determine the capacitance of a capacitor: the area of plate surfaces; the distance between the plates; and the material used as the insulation or dielectric between the plates. The capacitance of a capacitor is related directly to the surface area of the plates. Given a fixed dielectric material and distance separating the plates, the capacitance is directly proportional to the area of the plates. The greater the area, the greater the amount of electricity (charge) which can be stored in the capacitor. This follows from the condition that the greater the area, the more the number of free electrons available for charging. Doubling the surface area, with everything else being fixed, doubles the capacitance; halving the area produces half the capacitance. We shall see that different capacitor designs give various capacitance values in the same physical space.

Assuming a capacitor with a given plate area and dielectric material, the closer the facing plate surfaces are to each other, the greater the capacitance. This is an inverse proportion. Halving the area of separation doubles the capacitance; doubling the area of separation halves the capacitance. The reason for this is that the closer the facing surfaces are to each other, the more strongly the unlike charges on the surfaces are attracted towards each other. This tends to concentrate the free electrons on the negatively charged surface nearest to the positively charged surface, thus allowing more negative charges to be crowded onto a plate or plates of a given area.

There is a limitation to the permissible closeness of the active surfaces to each other, regardless of the separating medium. If the voltage built up across the capacitor exceeds the voltage rating of the capacitor (as explained later), electrons may be pulled away from the negatively charged surfaces and leap to the positively charged surfaces inside the capacitor. This action discharges the capacitor and may destroy it.

Factors that Determine Capacitance Dielectric Constant

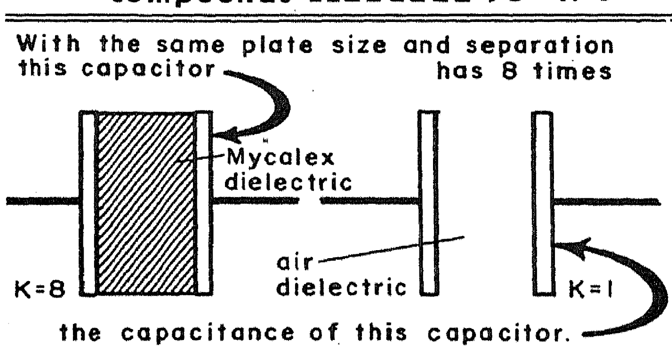
Given a capacitor with plates of a certain area and separated from each other by a specific distance, its capacitance is a function of the kind of material used for the dielectric. The standard of comparison is dry air, which is considered as having a dielectric constant (K) of 1. The dielectric constant of a vacuum differs so little from air that both are considered equal to unity. Dielectric constant is the ability of a material or medium to permit the establishment of electric lines of force between oppositely charged plates. Many materials will support more electric lines of force in a given space than air; these are said to have a dielectric constant greater than 1. Dielectric constants vary considerably. As examples, various types of mica have dielectric constants of from 5 to 9, and some forms of titanium dioxide have dielectric constants of up to 120. Some special kinds of chemical film deposits may have dielectric constants as high as 1000 or more. A dielectric (other than air) makes the positively charged surface of a capacitor repel more free electrons and the negatively charged surface accept more electrons than when air is the dielectric, thus increasing the capacitance.

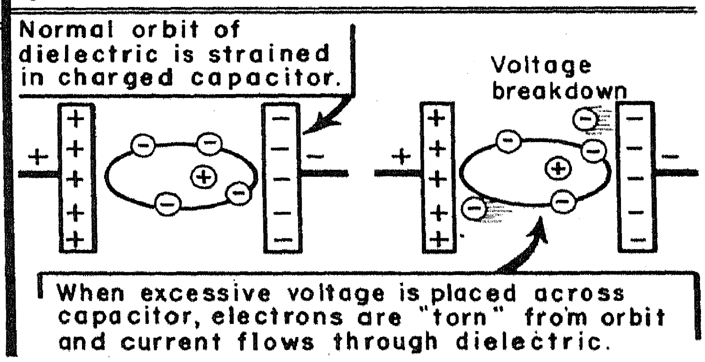
DIELECTRIC CONSTANTS

Material	K
air	1
resin	2.5
hard rubber	2.8
dry paper	3.5
glass	4.2
mica	5–9
porcelain	5.5
Bakelite	4.5–7.5
Mycalex	8
titanium dioxide compounds	90–170

VOLTAGE BREAKDOWNS

Material	Dielectric Strength (volts per 0.001 inch)
air	80
fiber	50
Bakelite	500
glass	200
mica	2000
castor oil	380
paper (beeswaxed)	1800
paper (paraffined)	1200
porcelain	750

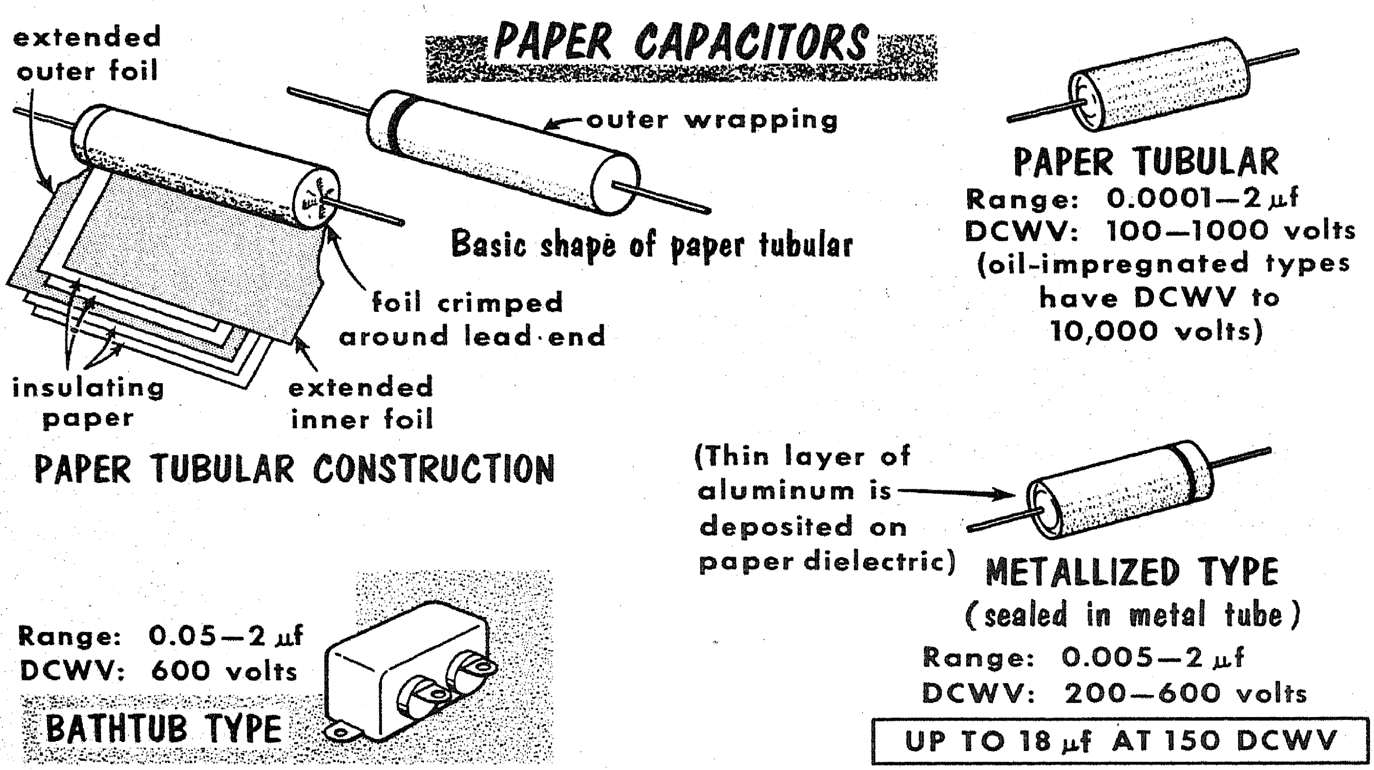




The dielectric material deserves one further consideration – that of breakdown voltage. While the dielectric is an insulator, voltages across the plates of a capacitor may be sufficiently high to "tear" electrons out of the atomic orbits of the dielectric. When this occurs, the dielectric "breaks down," and arcing occurs between the plates through the dielectric. In many instances, this destroys the capacitor, as it is short-circuited. Thus, it is important to observe the dielectric strength of a material. A high voltage would be needed to break down a vacuum dielectric, but lower voltages could break down certain other substances. Thus, the breakdown voltage of a dielectric must be considered as well as its dielectric constant.

Fixed Capacitors – Paper Type

The simplest and most widely used form of paper capacitor consists of two strips of metal foil rolled up, with strips of paper which have been impregnated with an insulating material (dielectric) placed between them. Impregnating materials generally include various types of oils, waxes, and plastics. The type used determines the voltage, temperature, and insulation-resistance characteristics of the capacitor. When the capacitor is to be used at high working voltages, several layers of insulating paper are used.

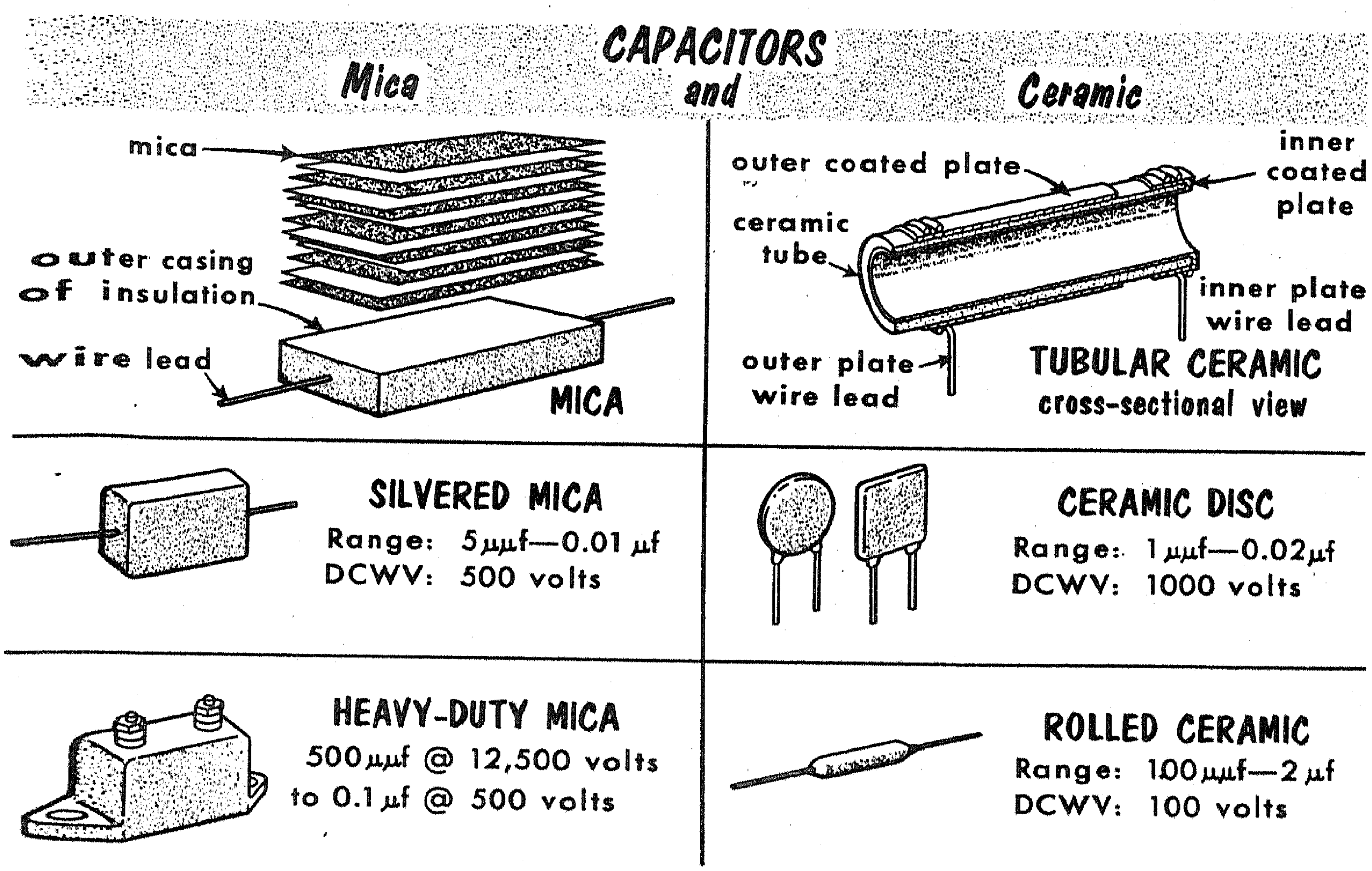


After the foil and paper strips are rolled up, the protruding ends of the foil are crimped over so that the individual layers of each strip are in electrical contact with each other. A lead is attached to each end, and an outer cover of insulating material is added. The cover is marked with the capacitance and working voltage, and a black ring is usually printed around one end to mark the terminal which is connected to the outermost layer of foil. In paper capacitors, the total capacitance is predetermined by the thickness and dielectric constant of the paper and the total of the foil plates. Capacitors are usually marked with a d-c working voltage (DCWV) which must be observed. It should be remembered that a-c voltages are spoken of in terms of rms (effective) value, and that the peak value is 1.41 times the rms value. Thus, when connecting a capacitor in an a-c circuit, we must always keep in mind the peak a-c voltage that will be impressed across the capacitor plates.

When paper capacitors are required to have a capacitance of over 1 μf, their physical size generally becomes too large for convenient mounting. Under such conditions, the capacitor is placed in a metal case filled with insulating material and then hermetically sealed. Units of this type are known as potted, or bathtub capacitors.

Fixed Capacitors – Mica and Ceramic

The mica capacitor consists of a number of flat strips of metal foil separated by similarly shaped strips of mica. The foil strips serve as the capacitor plates, and the mica acts as the dielectric. Alternate plates are connected together. An electrode is attached to each set of plates, and a terminal or lead wire is connected to each electrode. The entire unit is then encased in a container of plastic insulating material. An alternate construction is that of the "silvered" mica capacitor. In this unit, very thin layers of silver are deposited directly on one side of the mica, and the plates are stacked together so that alternate layers of silver are separated by alternate layers of mica. The result is the equivalent of the foil construction. Mica capacitors are available in three basic types: molded, molded-case potted, and ceramic-case potted. In addition, the "button" type mica unit is very popular.

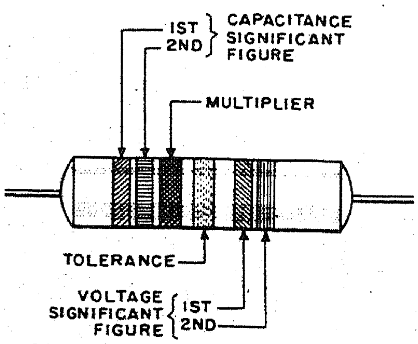


The basic construction of the ceramic capacitor consists of a ceramic disc or tube with silver or copper plates deposited on the opposite faces of the ceramic material. In the manufacturing process, electrodes are attached to the plates, leads or terminals are fastened to the electrodes, and a moisture-proof coating of plastic or ceramic is added. Ceramic capacitors are available in a number of basic shapes. The outstanding characteristic, however, is the high dielectric constant of ceramics. Steatite ceramics have a dielectric constant of 6, magnesium titanate has a K in the region of 16, and barium titanate has a K of approximately 1200. Ceramic capacitors have good stability with regard to temperature and voltage changes. The high-K ceramics provide increased capacitance without increased size.

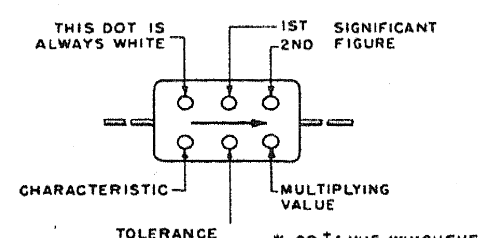
Capacitor Color Coding and Temperature Coefficient

As in the case of resistors, capacitors are also frequently color coded to indicate various capacitor characteristics. The points usually covered by color coding of capacitors are: capacitance, capacitance tolerance, and temperature coefficient. Some capacitors use color coding to indicate the d-c working voltage.

MOLDED PAPER
CAPACITOR COLOR CODE



CAPACITANCE IN MICROMICROFARADS



* OR ± 1 μμF WHICHEVER IS GREATER

EIA COLOR CODE
(FORMERLY RETMA)

CAPACITANCE IN MICROMICROFARADS

COLOR	CAPACITANCE SIGNIFICANT FIGURE	CAPACITANCE MULTIPLIER	* TOLERANCE ± %	CHARACTERISTIC
BLACK	0	1	20 (M)	A
BROWN	1	10	1 (F)	B
RED	2	100	2 (G)	C
ORANGE	3	1,000	3 (H)	D
YELLOW	4	10,000	-	E
GREEN	5	-	5 (J)	F
BLUE	6	-	-	-
VIOLET	7	-	-	-
GRAY	8	-	-	-
WHITE	9	-	-	-
GOLD	-	0.1	5 (J)	-
SILVER	-	.01	10 (K)	-

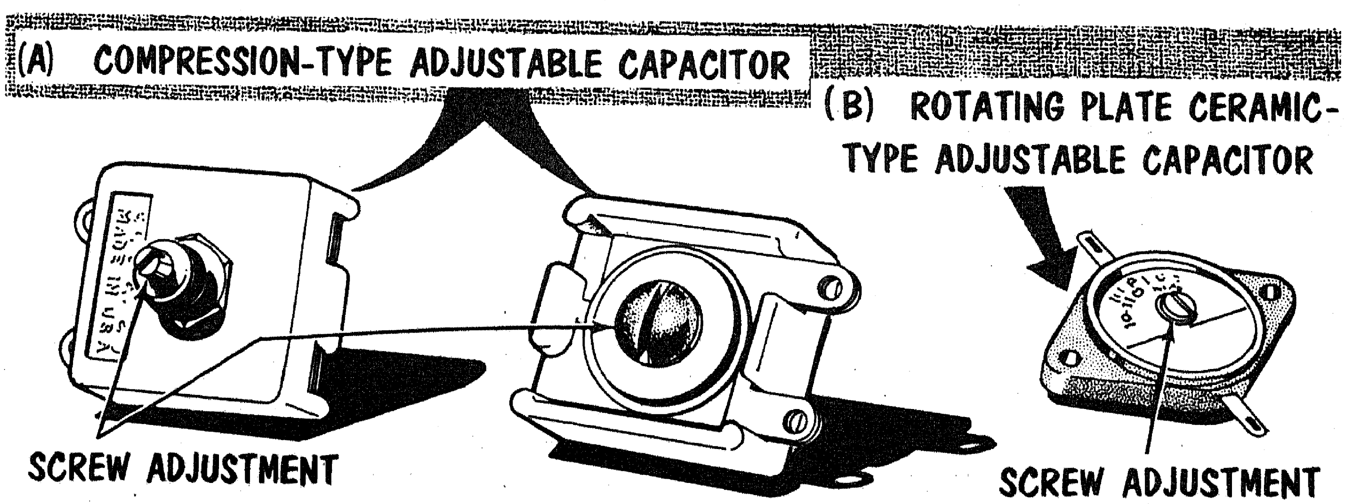
LETTER DESIGNATION FOR CHARACTERISTIC

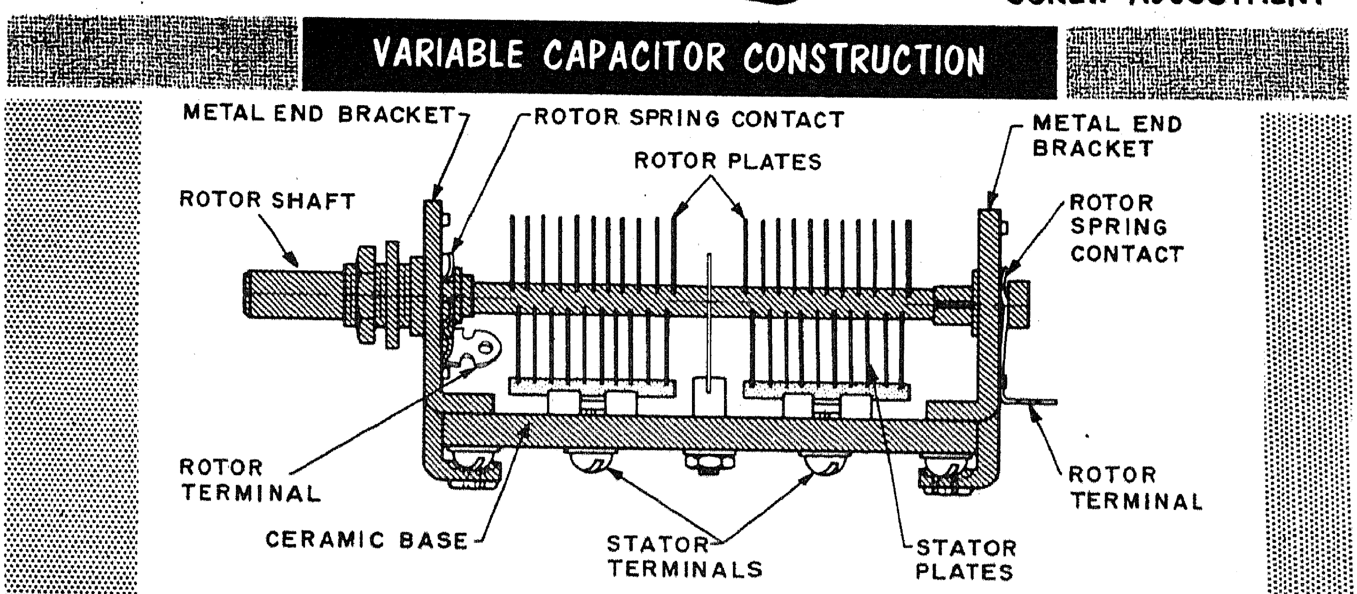
CHARACTERISTIC	TEMP. COEFF. PARTS PER MILLION PER °C	CAPACITANCE DRIFT (MAXIMUM)
A	± 1000	(5% OF NOM. CAP. + 1) μμF
B	± 500	(3% OF NOM. CAP. + 1) μμF
C	± 200	(0.5% OF NOM. CAP. + 0.5) μμF
D	± 100	(0.3% OF NOM. CAP. + 0.1) μμF
E	+ 100 TO - 20	(0.1% OF NOM. CAP. + 0.1) μμF

Mica- and ceramic-dielectric capacitors bear an electrical rating known as temperature coefficient. It is expressed by a number between 0 and 1300, prefixed by a minus or a plus sign (e.g., -220 or + 30). These designations state the change in capacitance (in μμf) from the nominal rating per million parts of capacitance (per μf) per degree Centigrade (ppm/°C) rise in temperature. The reference temperature is 20°C, which is the same as 68°F. The minus symbol indicates that the capacitance decreases; the plus symbol indicates that the capacitance increases. The prefix letter N indicates the same as the minus symbol, whereas the prefix NPO stands for negative-positive-zero, and indicates that the capacitance change is substantially zero over a wide range of temperature increase and decrease. For example, an N750 0.0001 = μf capacitor would have a decrease in capacitance of 750 × 0.0001, or 0.075 μμf per degree rise in temperature.

Variable Capacitors

A variable capacitor affords a continuous variation in capacitance between a fixed minimum and a fixed maximum value. While capacitance can be varied by varying plate area, distance between plates, and dielectric material, the most popular method is the varying of plate area. In this capacitor, there is a fixed set of metallic plates (called stators) mounted on some insulated base. Interleaved with these stator plates are the rotor (rotating) plates, controlled by a shaft. As the shaft is turned, the rotor plates mesh with (but do not touch) the stator plates, providing a variation in capacitor plate surface area. Usually, the entire frame is connected to the stator which serves as the grounded or common plates. Where more than one circuit is to be controlled, variable capacitors may be ganged to give simultaneous control.

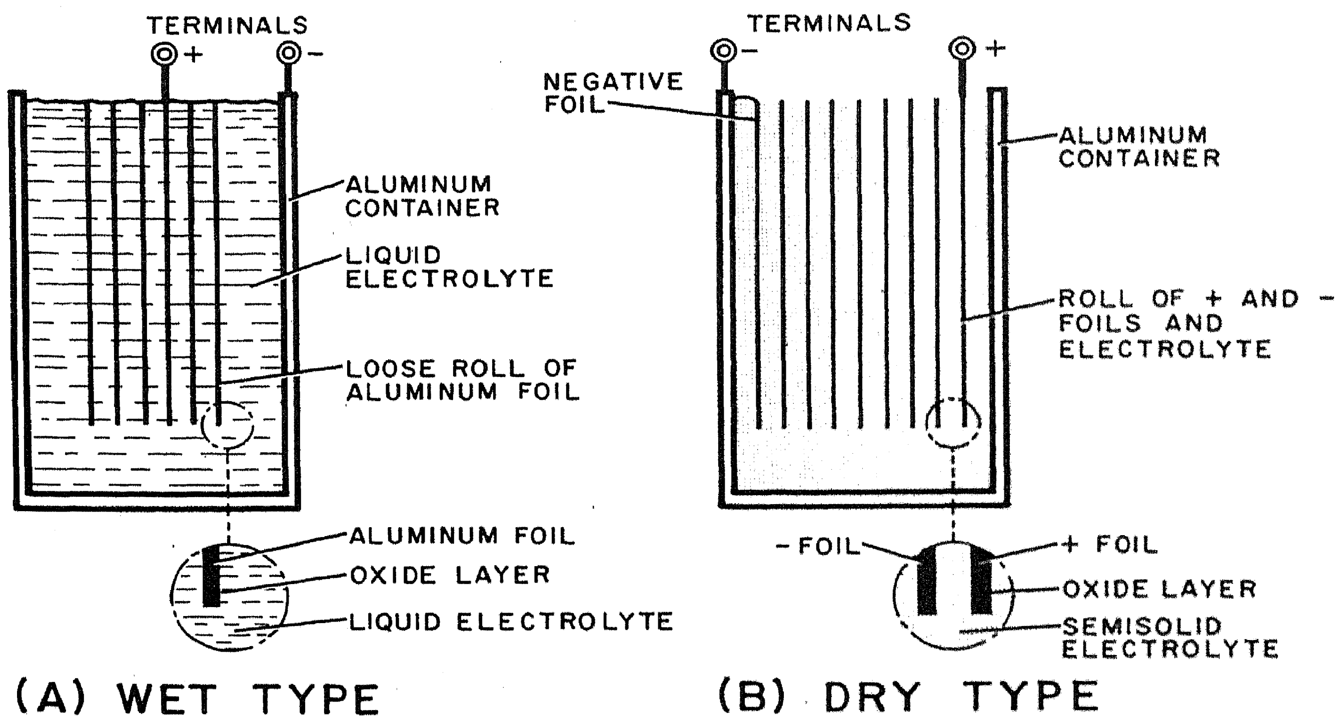




Two other types of variable capacitors are the compression type and the rotating plate. The compression type uses two flexible plates which can be compressed by a screw. The compression "squeezes" the mica dielectric, varying the distance between the plates. In the rotating-plate type, facing plates are turned so as to vary the facing plate area, and, therefore, the capacitance.

The Electrolytic Capacitor

ELECTROLYTIC CAPACITOR CONSTRUCTION



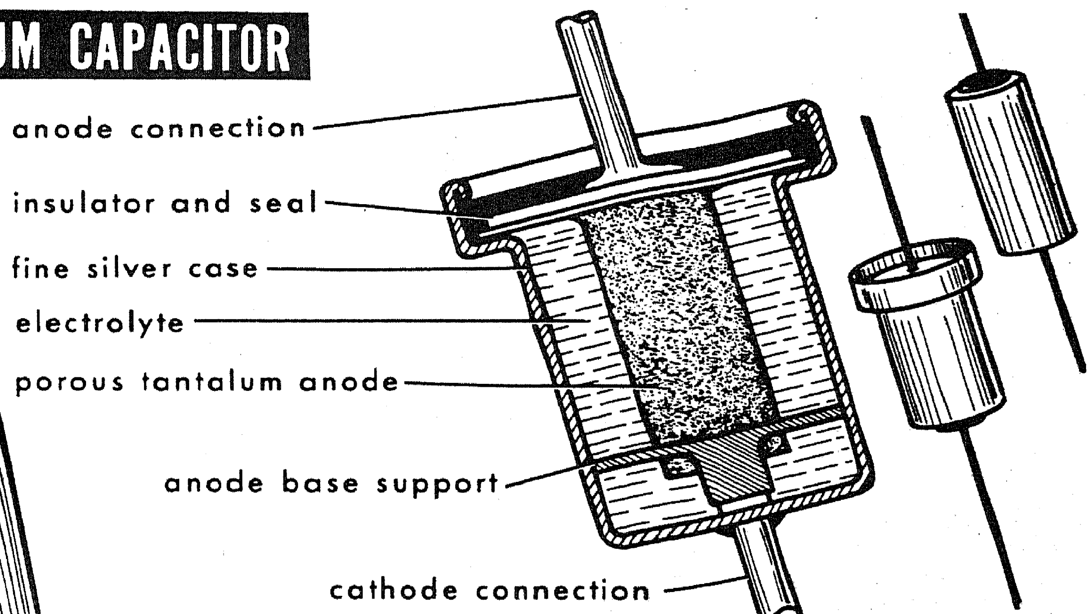
A very prominent type of capacitor used in electronic equipment is the electrolytic variety. In essence, it consists of a positive (anode) metal plate (aluminum foil or metal sprayed on cotton gauze, or a porous tantalum-oxide powder core) that is immersed in a liquid (wet type) or paste (dry type) bath known as the electrolyte. The entire capacitor is contained within a metal housing which usually serves as the negative (or cathode) terminal and as means of contact with the electrolyte, the other active surface in the capacitor. The dielectric is a very thin film (usually aluminum or tantalum oxide) which is "forced" on the metal plate.

At the time the electrolytic capacitor is being made, a d-c voltage is applied between the metal container (the negative terminal) and the metal plate (the positive, or anode electrode). It causes a relatively high current to flow in one direction inside the unit. As a result, a very thin dielectric film is formed on the outside of the positive (anode) electrode. As this film forms, the current decreases, eventually reaching a minimum. This minimum current is referred to as the leakage current. When the forming is completed, the capacitor is polarized; the metal plate is positive and the electrolyte is negative. The capacitor functions properly as long as the charging voltage has a polarity corresponding to that of the capacitor electrodes. This is a very important condition; hence, the conventional electrolytic capacitor bears polarity designations. The d-c type of unit is not suitable for charging by an a-c voltage. Another form of construction is used in a-c electrolytic capacitors. These capacitors contain two formed positive electrodes both of which act as either positive or negative electrodes, thus permitting the periodic reversal of polarity of the applied voltage.

The Electrolytic Capacitor (Cont'd)

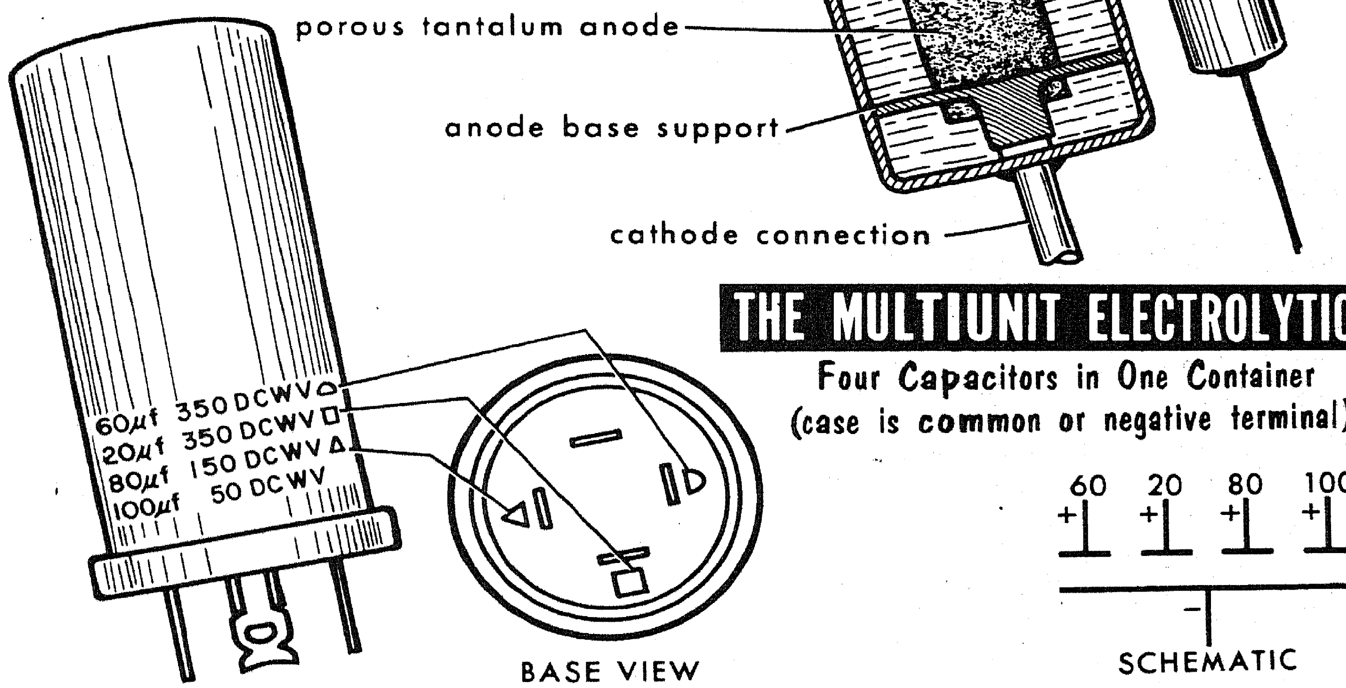
The electrolytic capacitor offers the advantages of a very high capacitance in a small space, and at low cost. Capacitance values available in "electrolytics" extend from as low as 1 microfarad to several thousand microfarads. This huge capacitance arises from the extreme thinness of the dielectric film which, in effect, means that the active surfaces of the capacitor – the metal plate and the electrolyte – are very close to each other. Film thicknesses of as little as 0.00001 inch are commonplace. However, the thin film introduces a limitation in working voltage. D-c electrolytic capacitors of up to perhaps 100 µf have working voltage ratings up to 450 volts d-c. As the capacitance ratings increase, the working voltage ratings decrease because the dielectric film is thinner; hence, the working voltage must be reduced to prevent puncture of the dielectric film.

THE TANTALUM CAPACITOR



THE MULTIUNIT ELECTROLYTIC

Four Capacitors in One Container
(case is common or negative terminal)



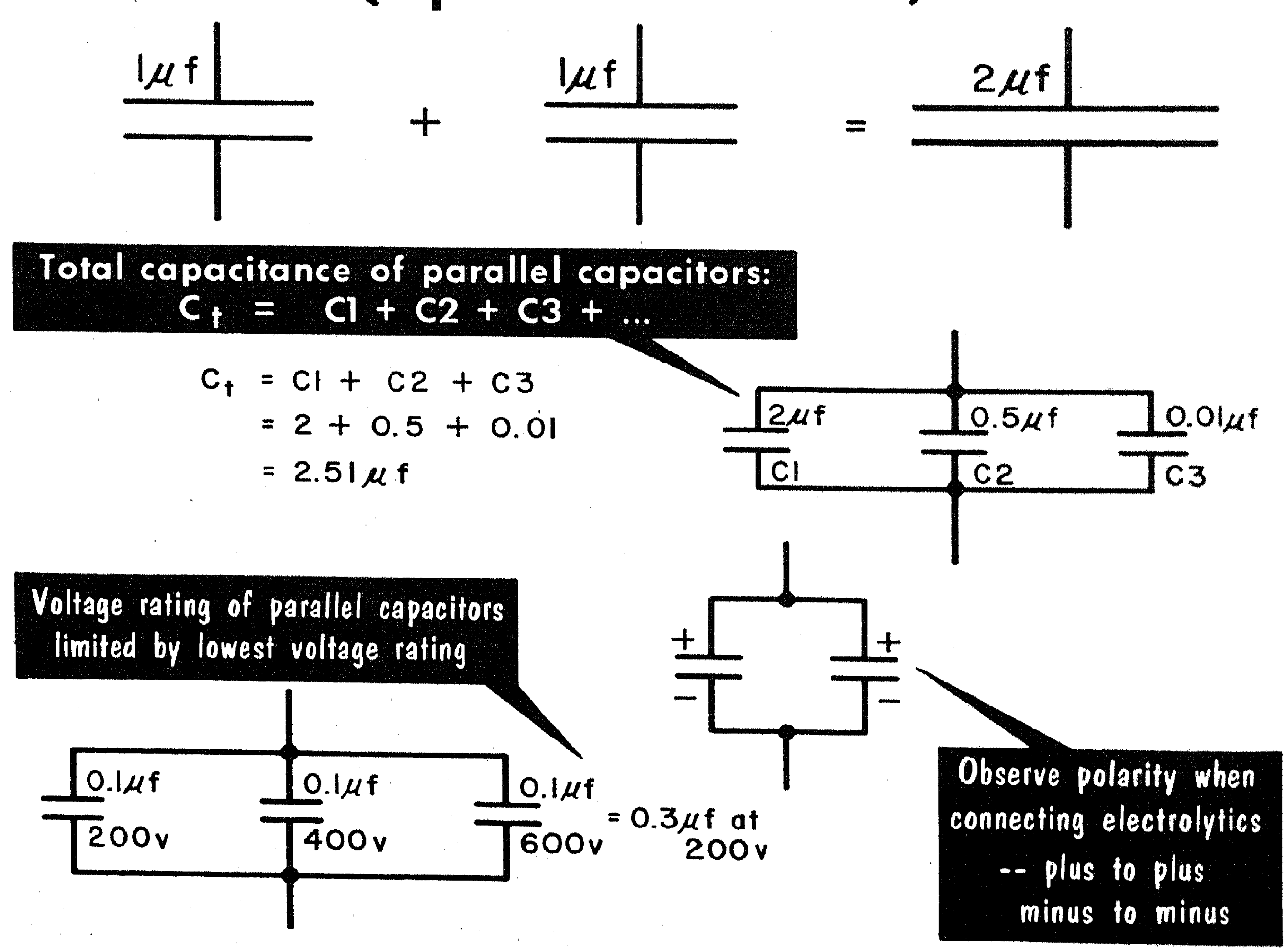
Capacitors rated at several thousand microfarads have working voltage ratings of from 6 to about 25 volts d-c. Interestingly enough, electrolytic capacitors sometimes are self-healing. Proper polarity of the applied voltage reforms the puncture in the film and "heals" the capacitor, providing the puncture is small. Operation at voltages in excess of the rating working voltage is not recommended.

Connecting Capacitors in Parallel

When capacitors are connected in parallel, the effect is to produce a total capacitance equal to the sum of all the individual capacitances. The reason for this is that, effectively, the total plate surface area of each capacitor is added, providing a larger total plate area. Since plate area is one of the factors that determines the capacitance of a capacitor, connecting capacitors in parallel increases the total capacitance. The formula for parallel capacitances is: $C_t = C1 + C2 + C3 + \ldots$. Thus, 5-µf, 10-µf, and 15-µf capacitors connected in parallel would provide a total capacitance of 30 µf.

When capacitors are connected in parallel, the total voltage of the circuit is applied across each capacitor. Therefore, no matter how high the voltage rating of each capacitor in the parallel hookup, the unit with the lowest voltage rating effectively becomes the weakest link and limits the amount of voltage that can be applied to the parallel combination.

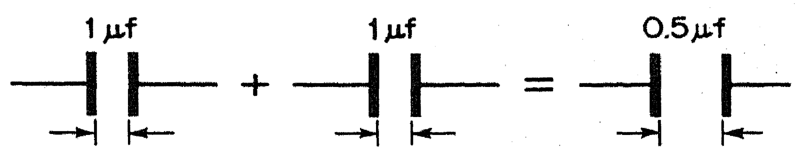
Connecting Two Capacitors in Parallel has the Effect of Increasing the Total Plate Area (capacitance increases)

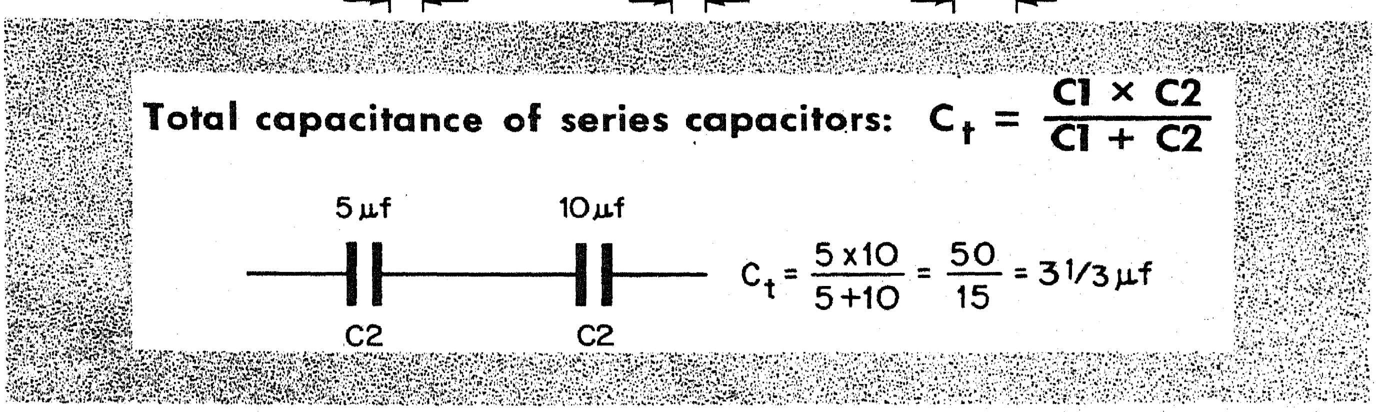


Connecting Capacitors in Series

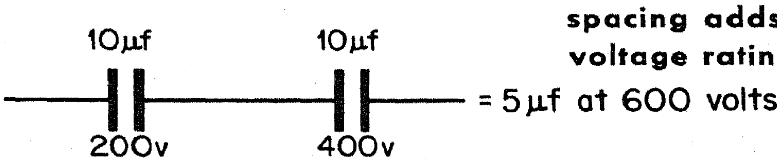
Connecting capacitors in series has the effect of reducing the total capacitance to a value <u>less than the lowest capacitance</u>. This is the equivalent of connecting resistors in parallel. The reason for the reduction in total capacitance when they are connected in series is that effectively, we have added together the spacing between the plates of all the capacitors. Since the capacitance of a capacitor varies inversely with the spacing between the capacitor plates, each series capacitor added to a series string reduces the total capacitance. From this, we get the formula for the total capacitance of a series circuit: $C_t = (C1 \times C2)/(C1 + C2)$. Thus, if a 10-µf and a 15-µf capacitor were connected in series, the total capacitance would be: $C_t = (10 \times 15)/(10 + 15) = 150/25$, or 6 µf. Where more than 2 capacitors are connected in series we can use the formula $C_t = 1/[(1/C1) + (1/C2) + (1/C3) + \ldots]$.

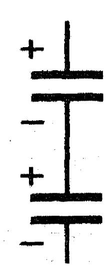
Connecting Two Capacitors in Series has the Effect of Increasing the Distance between Capacitor Plates (capacitance decreases)





Voltage rating of series capacitors equal to sum of individual voltage ratings. Each dielectric spacing adds to voltage rating





When connecting electrolytics in series, connect plus to minus, as when connecting cells in series

When capacitors are connected in series, the breakdown voltage of each unit is added to provide a breakdown voltage equal to the sum of each of the capacitor breakdown voltages. Two capacitors having a 600-volt and 1000-volt d-c working voltage provide a total safe working voltage of 1600 volts d-c when connected in series. Thus, series connection provides a <u>reduction</u> in capacitance and an <u>increase</u> in working voltage rating.

Capacitive Reactance (X_C)

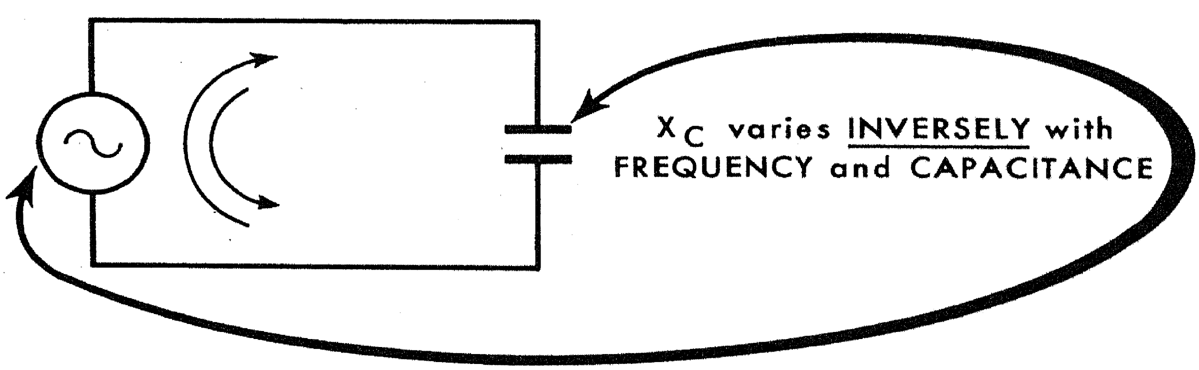
When an alternating voltage is applied to the plates of a capacitor, a certain amount of current will flow in the circuit. We recall that when a charge builds up in a capacitor, the voltage across the capacitor acts in opposition to the applied voltage. The amount of opposition that a capacitor offers to the flow of current in an a-c circuit depends upon the capacitance of the capacitor and upon the frequency of the a-c voltage source. The greater the size of the capacitor, the greater the amount of energy it can store, and the more the current that must flow to charge it. In addition, since it takes time to charge a capacitor, the lower the frequency of the a-c charging voltage, the slower will be the buildup of charge in a capacitor. The net effect of all this is to produce a certain opposition to the flow of current. This opposition in a capacitive circuit is called capacitive reactance and is measured in ohms.

capacitive reactance (X_C)

$$X_C = \frac{1}{2\pi fC}$$

in ohms — in cycles — in farads

CAPACITIVE REACTANCE is the OPPOSITION TO CURRENT FLOW offered by a Capacitor in an A-C Circuit

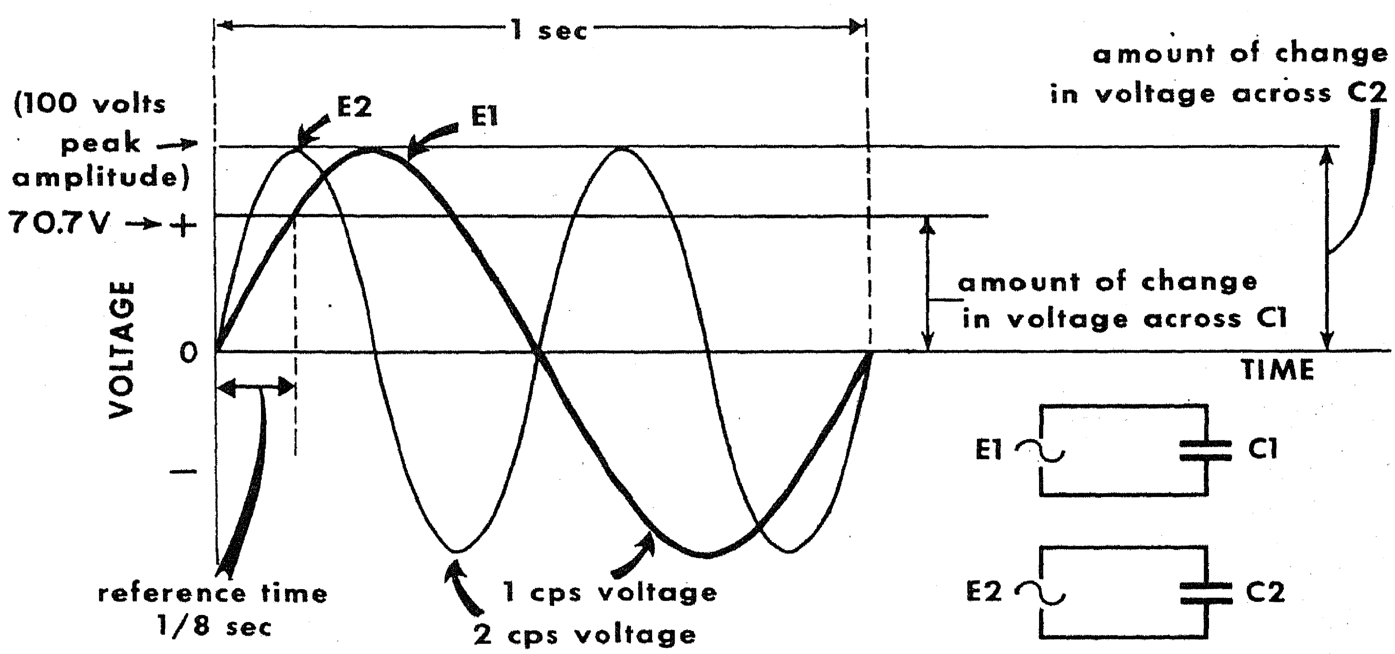


While different principles and effects are involved, the net effect of capacitive reactance on the flow of current in an a-c circuit is the same as that of inductive reactance. Whereas inductive reactance is expressed as X_L, capacitive reactance is expressed as X_C; both are measured in ohms. In addition, inductive reactance varies with inductance and frequency; capacitive reactance varies with capacitance and frequency — only inversely. The formula for computing the reactance of a capacitor is: $X_C = 1/2\pi fC$, where 2π equals 6.28, f is the frequency of the a-c source in cycles, and C is the capacitance of the capacitor in farads. From the formula, we can see an inverse relationship between capacitive reactance and frequency and capacitance. As f and C increase, X_C decreases; as f and C decrease, X_C increases.

Capacitive Reactance – Effect of Change in Frequency

To illustrate the change in capacitive reactance as frequency changes, imagine two sine waveform voltages, E1 (1 cycle) and E2 (2 cycles), each of 100 volts peak amplitude. These two frequencies are applied to two identical capacitors, C1 and C2. Voltage E1 rises from zero to peak amplitude (90°) in 1/4 second whereas E2 rises from zero to peak amplitude in 1/8 second (45°). These time intervals are arrived at by dividing the time duration of a single cycle by 4 since there are four quarter-cycles in a complete cycle. The time duration for E1 is 1 second; for E2 it is 1/2 second.

AS FREQUENCY INCREASES, CAPACITIVE REACTANCE DECREASES



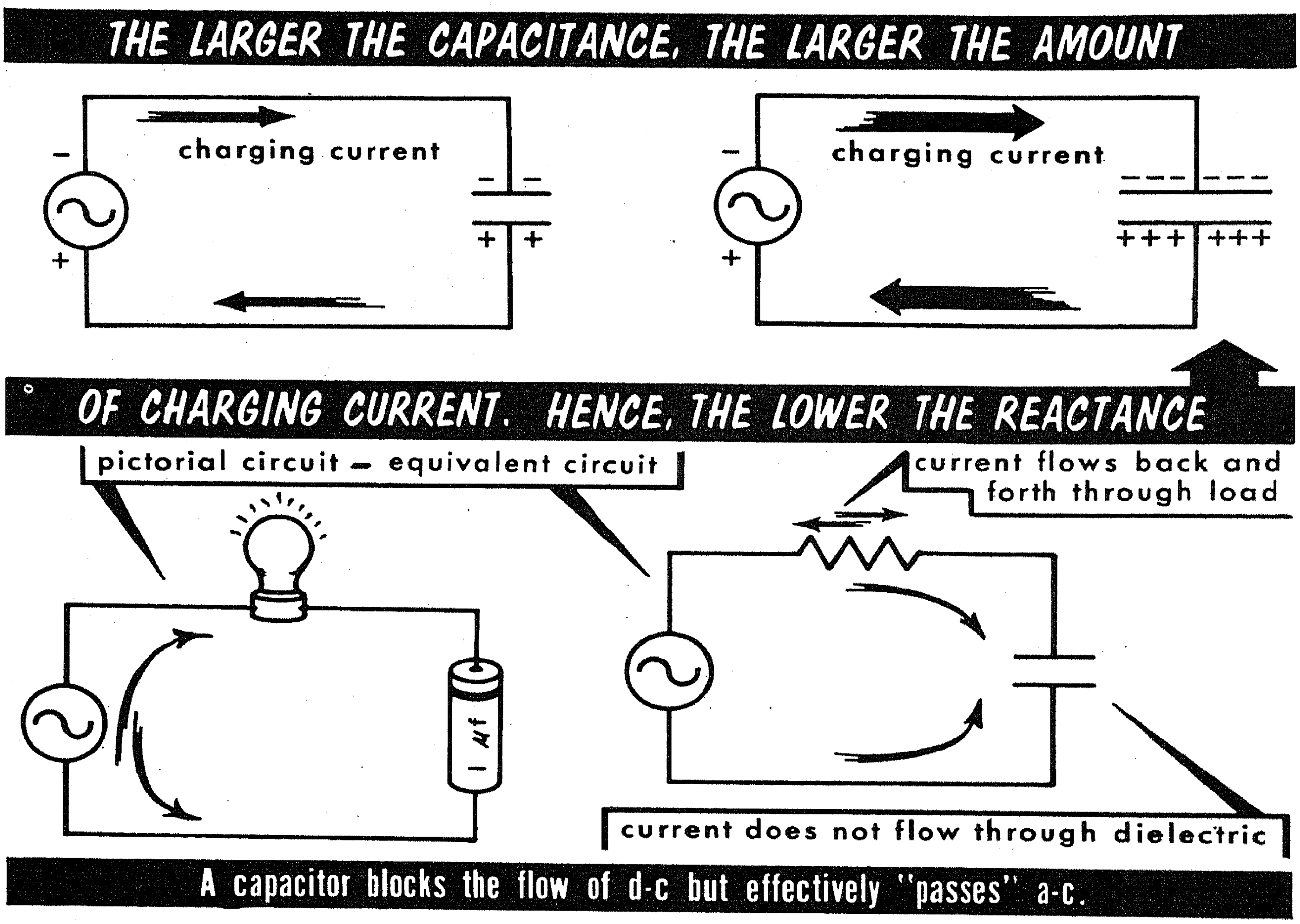
1/8 second = 90° for voltage E2 = peak × 1 or 100 volts
1/8 second = 45° for voltage E1 = peak × 0.707 or 70.7 volts

Now if we use 1/8 second as the reference time interval for both voltages, E2, being the higher-frequency voltage, changes in value more rapidly than E1. Voltage E2 rises to the peak value of 100 volts in 1/8 second, whereas E1 reaches only the 70.7-volt level in the same amount of time. Therefore, capacitor C2 receives maximum charge, whereas C1 receives less charge. For C2 to receive more charge than C1 in the same time interval, it is necessary that more current flow into C2 than into C1. Hence, the capacitive reactance of C2 for E2 is less than that of C1 for E1.

We can translate the above action by saying that the faster the rate of change of the charging voltage – or the higher the frequency of the charging voltage – the lower the capacitive reactance of any given capacitor. Of course, the opposite is true: the lower the frequency, the higher the capacitive reactance.

Capacitive Reactance — Effect of Change in Capacitance

Capacitive reactance decreases with an increase in capacitance and increases with a decrease in capacitance. Given two capacitors, C1 (1 μf) and C2 (5 μf), both subjected to a voltage of 100 volts peak amplitude, the larger capacitance can accept more charge. For this to be true when the two capacitors are charged by the same amount of voltage in the same amount of time, the current flowing into the larger amount of capacitance must be greater than the current flowing into the smaller amount of capacitance. Hence, the capacitive reactance of C2 must be less than that of C1. From this, we see that for a given frequency, the larger the capacitance, the lower the capacitive reactance; the smaller the capacitance, the higher the capacitive reactance.

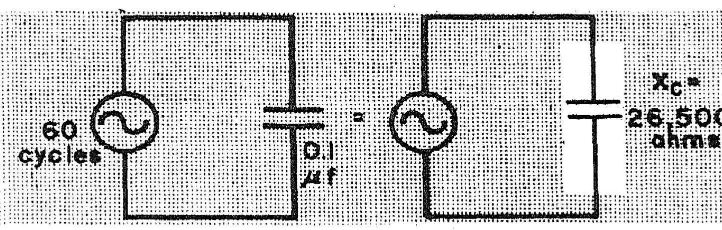


We have seen that during the charge and discharge of a capacitor, electrons flow back and forth in the circuit, first making one plate negative with respect to the other, and then making the other plate negative with respect to the first. It would seem that there is a complete closed circuit in which the current is alternating. Actually, of course, the plates of a capacitor are separated by an insulator (dielectric). No current flows through the dielectric, but since current flows back and forth from plate to plate, the current in a capacitive circuit takes on all the appearances of a closed-circuit arrangement. From this, we often use the expression that "current flows through a capacitor." Of course, this is not so — it only appears that way.

Using the Equation for Capacitive Reactance

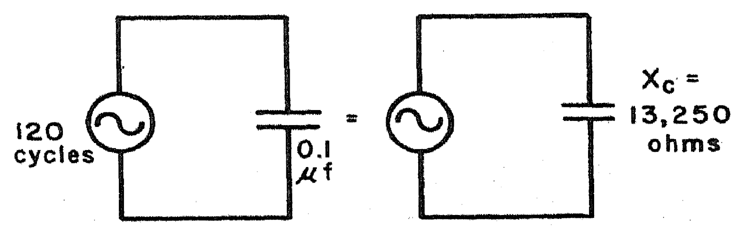
Referring again to the equation for capacitive reactance, let us solve several typical examples. While doing this, you must bear in mind that we are determining the opposition to current due only to the presence of the capacitance. The equation does not involve the resistance of the connecting wires or the actual value of the voltage applied. The illustration shows what happens when a 0.1-μf capacitor is used in a 60-cycle circuit.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 60 \times 0.0000001} = 26{,}500 \text{ ohms}$$



Leaving the capacitance fixed and doubling the frequency

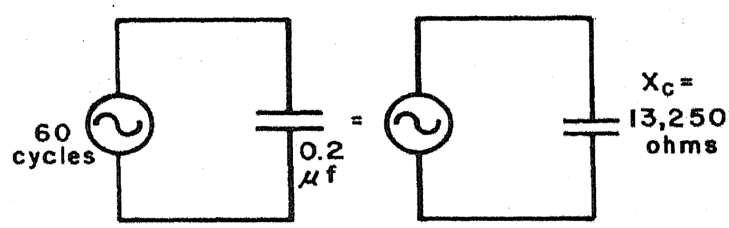
$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 120 \times 0.0000001} = 13{,}250 \text{ ohms}$$



DOUBLING THE FREQUENCY HALVES THE CAPACITIVE REACTANCE

Leaving the frequency fixed and doubling the capacitance

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 60 \times 0.0000002} = 13{,}250 \text{ ohms}$$



DOUBLING THE CAPACITANCE HALVES THE CAPACITIVE REACTANCE

Given high values of capacitance and high-frequency voltages, capacitive reactance can fall to extremely low values, even to the point where the capacitor behaves as a virtual short-circuit to the voltage. For example, a 1-μf capacitor subjected to a voltage of 5 megacycles (5,000,000 cycles) has a capacitive reactance of

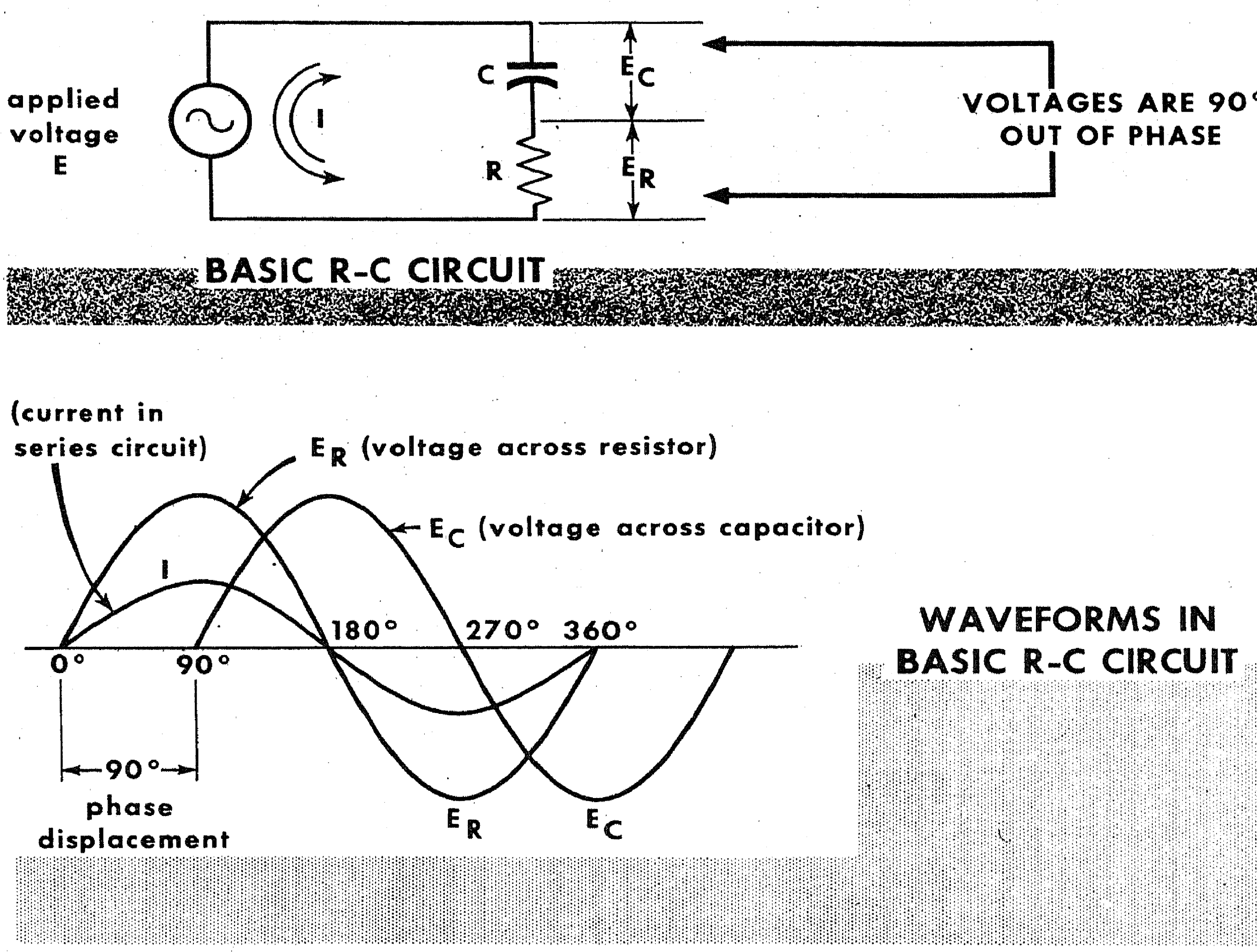
$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 0.000001 \times 5{,}000{,}000} = 0.0318 \text{ ohm}$$

On the other hand, if the capacitance value is very low, and the frequency of the applied voltage also is very low, the capacitive reactance can become so high as to behave like a virtual open circuit to the voltage. For instance, if the capacitance is 0.001 μf, and the frequency of the voltage is 5 cycles, the capacitive reactance rises to 31,847,000 ohms.

Current and Voltage in a Series R-C Circuit

In any circuit containing both capacitance and resistance, there is a 90° phase shift of current and voltage across the capacitance, and no phase shift across the resistance. As discussed in inductance, current in a series circuit is the same throughout and is, therefore, taken as the line of reference for both the capacitance and the resistance. Since the voltage across the resistance is in phase with the current through it, and the voltage across the capacitance is 90° out of phase with this same current, we can see that these two voltages are 90° out of phase with each other.

Current and Voltages in a Series R-C Circuit

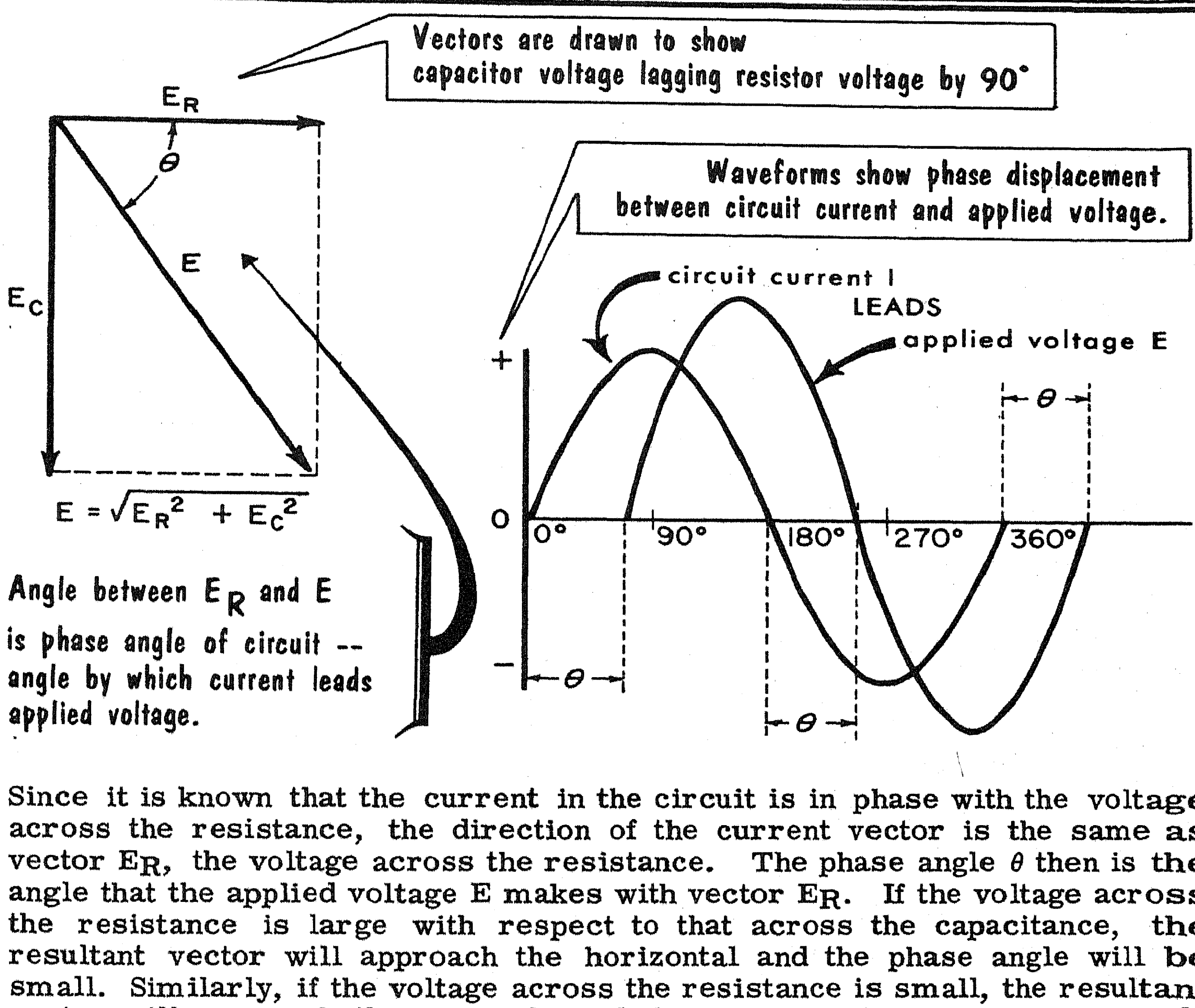


In the illustration, there is a basic series R-C circuit and the curves showing the relationship between the current and voltages across both components. The resultant voltage from the two voltage drops which are 90° out of phase is the voltage drop for the whole circuit and is, by Kirchhoff's law, equal to the applied voltage. The phase shift of the current in the circuit, measured with respect to the applied voltage, is called the phase angle of the circuit.

Current and Voltage in a Series R-C Circuit (Cont'd)

The relationship between the applied voltage, the voltage drops, and the phase angle of any series R-C circuit may be determined by means of vectors, as shown. The voltage across the resistance is plotted on the horizontal vector, and the voltage across the capacitance on the vertical vector. Since these two voltages are 90° out of phase, the angle between them is a right angle. By drawing in a parallelogram based on the two sides, the resultant vector E becomes the hypotenuse of a right triangle. By using the theorem that the square of the hypotenuse is equal to the sum of the squares of the other two sides, we get $E^2 = E_R{}^2 + E_C{}^2$, or $E = \sqrt{E_R{}^2 + E_C{}^2}$.

VECTORS AND WAVEFORMS OF CURRENT AND APPLIED VOLTAGE IN A SERIES R-C CIRCUIT

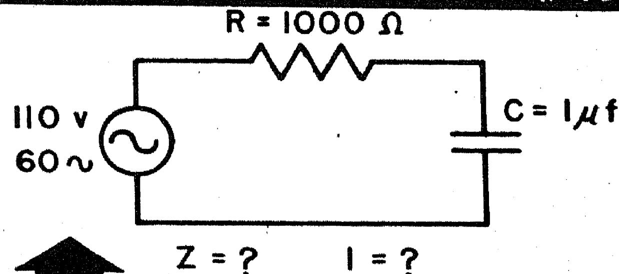


Since it is known that the current in the circuit is in phase with the voltage across the resistance, the direction of the current vector is the same as vector E_R, the voltage across the resistance. The phase angle θ then is the angle that the applied voltage E makes with vector E_R. If the voltage across the resistance is large with respect to that across the capacitance, the resultant vector will approach the horizontal and the phase angle will be small. Similarly, if the voltage across the resistance is small, the resultant vector will approach the vertical, and the phase angle will approach 90°. Hence, the presence of resistance in a capacitive circuit causes the current to lead the applied voltage by some angle less than 90°. The waveforms show the relative positions of current, voltage, and the phase angle θ.

Impedance of a Series R-C Circuit

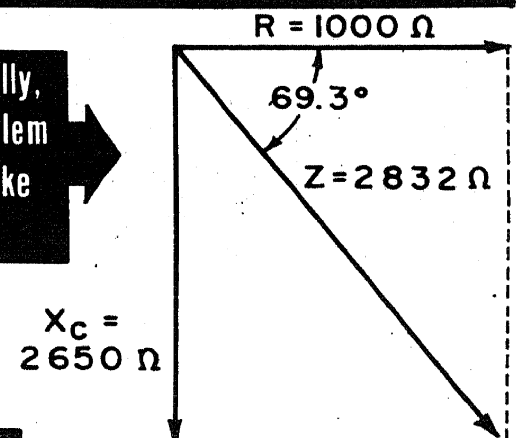
As stated in our discussion of R-L circuits, the total opposition offered by a circuit containing both a reactive element and a resistance is not the simple arithmetical sum of the reactance and the resistance. The capacitive reactance is added to the resistance in such a manner as to take into account the 90° phase difference between the two voltages in the circuit. To find the impedance of an R-C circuit, we use the same basic formula that we used in the R-L circuit; that is: $Z = \sqrt{R^2 + X_C^2}$. In short, the impedance of an R-C circuit is equal to the square root of the sum of the squares of the resistance and the capacitive reactance.

HOW TO FIND THE IMPEDANCE (Z) OF A SERIES R-C CIRCUIT



PROBLEM

Vectorially, the problem looks like this



SOLUTION

1st find the X_C of C

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 60 \times 0.000001}$$

$$= \frac{1}{0.000377}$$

$$= 2650\,\Omega$$

$$\text{Tangent } \theta = \frac{X_C}{R}$$

$$= \frac{2650}{1000}$$

$$= 2.65$$

$$\theta = 69.3^\circ$$

look up on tangent table

2nd find Z

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{1000^2 + 2650^2}$$

$$= \sqrt{1{,}000{,}000 + 7{,}023{,}000}$$

$$= \sqrt{8{,}023{,}000}$$

$$= 2832\,\Omega$$

$$I = \frac{E}{Z}$$

$$= \frac{110}{2832}$$

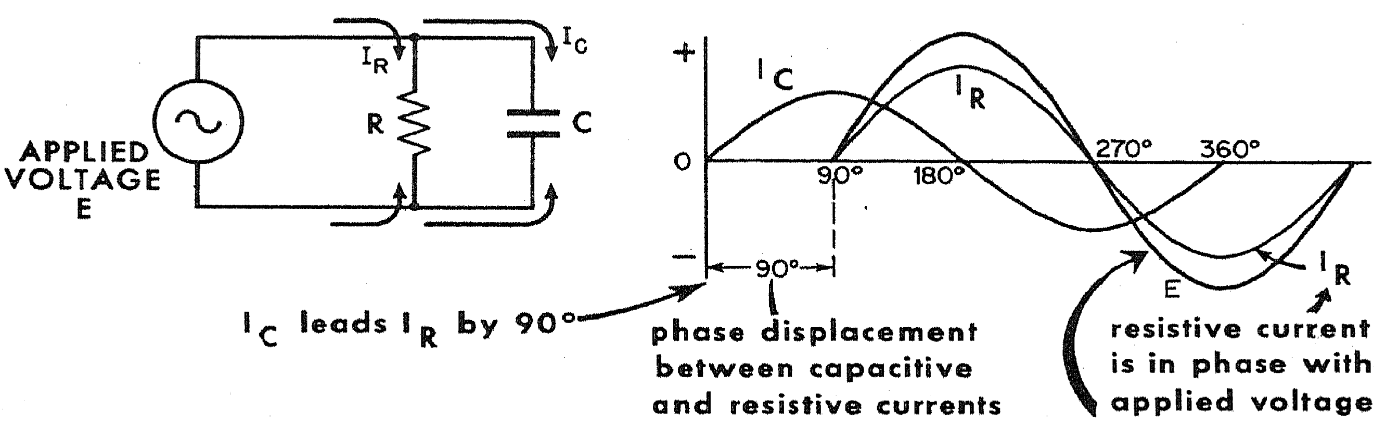
= 0.0389 Ampere, or 38.9 Milliamperes

The same result can be obtained by the use of vectors. Since the same current flows in C and R, the vectors can be made proportional to the resistance and the capacitive reactance. Note that the angle θ is the phase angle because the direction of the impedance vector is the same as that of the applied voltage vector. This angle may be determined by its tangent, X_C/R. The total current in an R-C series circuit can then be determined by the formula, $I = E/Z$.

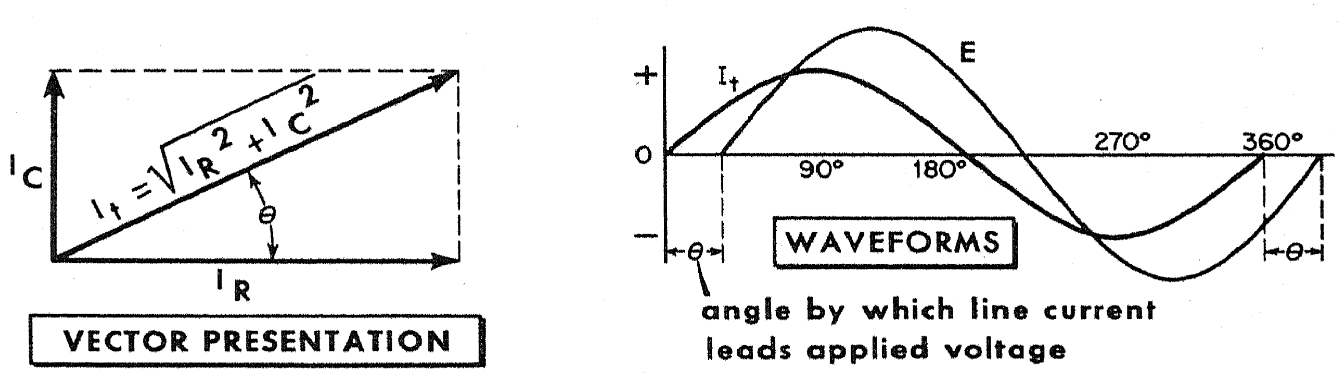
Parallel R-C Circuits

We show here capacitance C and resistance R connected in parallel across an a-c source. Since this is a parallel circuit, voltage is the same everywhere; thus, all voltages are in phase with each other. However, the current through the capacitor leads the applied voltage by 90°, and the current through the resistance is in phase with the applied voltage, as shown in the waveforms. Thus, the capacitive current leads the resistance current by 90°, and the resultant current, or total line current, is the vectorial sum of these two currents.

VOLTAGE AND CURRENTS IN PARALLEL R-C CIRCUIT



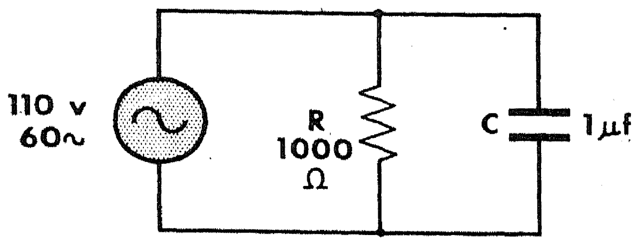
Total Current and Applied Voltage in Parallel R-C Circuit



In making vectors for this situation, the current through the resistance I_R is laid off on the horizontal vector, and the current through the capacitance I_C on the vertical vector. Because the capacitive current leads the resistive current, the I_C vector is laid off in the positive direction. The resistive current is taken as the reference vector, since it is in phase with the applied voltage and represents the direction of the applied voltage. The resultant vector I_t represents the total current in the circuit, and the angle this vector makes with the horizontal is the phase angle θ. The line current, then, is said to lead the applied voltage by the angle θ. The tangent of this angle is equal to R/X_C. The total current of this circuit is equal to: $I_t = I_R^2 + I_C^2$. Thus it can be seen that total current in a parallel R-C circuit, as in d-c circuits, is always greater than the current in either branch.

Impedance of a Parallel R-C Circuit

HOW TO FIND THE IMPEDANCE OF A PARALLEL R-C CIRCUIT



Z = ? I = ?

PROBLEM

$$I_t = \frac{E}{Z} = \frac{110}{937} = \begin{matrix}0.117 \text{ ampere}\\ \text{or}\\ 117 \text{ milliamperes}\end{matrix}$$

The line current leads the applied voltage by the angle θ

$$\text{Tangent } \theta = \frac{R}{X_C} = \frac{1000}{2650} = .377$$

Found in the Tangent Table → = .377

$$\theta = 20.7^\circ$$

SOLUTION

1st Find X_C of C

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 60 \times 0.000001}$$

$$= \frac{1}{0.000377}$$

$$= 2650 \text{ ohms}$$

2nd Find Z

$$Z = \frac{R X_C}{\sqrt{R^2 + X_C{}^2}}$$

Vectorial Addition of R + X_C

$$= \frac{1000 \times 2650}{\sqrt{1{,}000{,}000 + 7{,}023{,}000}}$$

$$= \frac{2{,}650{,}000}{2840}$$

$$= 937 \text{ ohms} \left\{ \begin{matrix}\text{Less than}\\ \text{R or } X_C\end{matrix} \right.$$

The impedance of a parallel R-C circuit may be calculated by using the same general formula as for finding the total resistance of resistors in parallel. Because we are dealing with vector quantities when we discuss R and X_C, we cannot add them arithmetically – they must be added vectorially. Thus we get the formula for R and C in parallel:

$$Z = \frac{R \times X_C}{\sqrt{R^2 + X_C{}^2}}$$

From this, we can see that the total impedance of a parallel R-C circuit always is somewhat less than either the resistance or the reactance.

Indirectly, the impedance also can be found by finding the total current and then using the formula $Z = E/I$. To find the total current, we must first find the individual currents that flow in R and in C. This is done by Ohm's law, $I = E/R$ and $I = E/X_C$. Knowing these currents, the total current then can be found by adding them vectorially. For instance, if 3 amperes flow through R and 4 amperes flow through C, the vectorial sum of 3 and 4 is 5 amperes, which represents the total current ($I_t = \sqrt{I_R{}^2 + I_C{}^2}$, or $5 = \sqrt{3^2 + 4^2}$).

A capacitor is any two conductors separated by an insulating material (dielectric).

Capacitance is a property of a circuit whereby energy may be stored in the form of an electric field between two conductors (plates) separated by a dielectric.

Capacitance in a circuit opposes any change in voltage.

The action of storing electricity in a capacitor is called charging.

The action of recovering the energy stored in a charged capacitor is known as discharging the capacitor.

In a capacitive circuit, current leads the applied voltage by 90°.

The ratio of the charge (Q) to the voltage (E) is the measure of capacitive action, and is called capacitance. $C = Q/E$.

A farad is the unit of capacitance. A capacitor has a capacitance of 1 farad if a 1-volt difference in potential results in the storage of 1 coulomb of charge.

A microfarad is one-millionth of a farad; a micromicrofarad is one-millionth of a microfarad, or one million millionth of a farad.

The capacitance of a capacitor varies directly with the plate surface area and inversely with the distance between the plate surfaces.

Dielectric constant (K) is the ability of a material, or medium, to permit the establishment of electric lines of force between oppositely charged plates.

Fixed capacitors are distinguished, according to the dielectric material used, as paper, oil, mica, ceramic, and electrolytic capacitors.

Electrolytic capacitors show polarity and are used principally in high-power, low-frequency filter circuits up to 600 volts.

Capacitors connected in parallel add like resistances in series; capacitors connected in series divide according to the parallel-resistance formula.

The opposition a capacitor presents to a-c is called capacitive reactance (X_C); the formula for capacitive reactance is $X_C = 1/2\pi fC$.

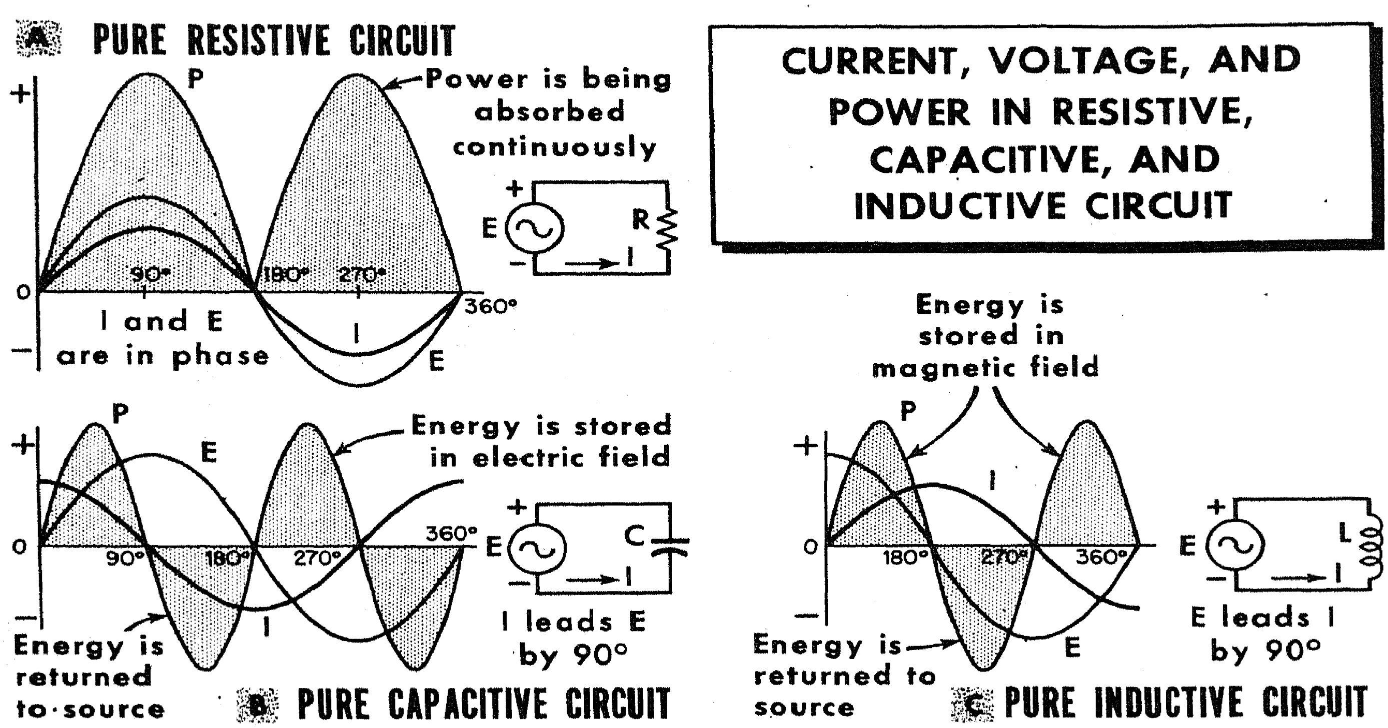
If either, or both, frequency or capacitance increases, X_C decreases, and vice versa. Impedance of a series R-C circuit is calculated by $Z = \sqrt{R^2 + X_C^2}$; impedance of a parallel R-C circuit is calculated by $Z = R \times X_C/\sqrt{R^2 + X_C^2}$.

REVIEW QUESTIONS

1. What is a capacitor? What is capacitance?
2. Describe the action of current flow in a capacitor when a d-c voltage is applied.
3. What factors determine the amount of current flow in a capacitor?
4. What is a dielectric constant?
5. Define capacitive reactance (X_C). Give the formula for it.
6. How does capacitance vary with respect to the area of the plates, the distance between them, and the dielectric constant?
7. Define the farad, the microfarad, and the micromicrofarad.
8. When does a capacitor discharge?
9. What two functions are characteristic of a capacitor?
10. Give the formulas for calculating capacitors in series and in parallel.
11. Give the formulas for calculating the impedances of series R-C circuits and parallel R-C circuits.

Resistive, Inductive, and Capacitive Circuits

The power absorbed by a resistance in a d-c circuit is expressed by $P = E \times I$, or by I^2R and or by E^2R. All lead to the same answer. The power absorbed by resistance in an a-c circuit is expressed by exactly the same equations. When an a-c voltage is applied to a pure resistance only, power is absorbed each instant, regardless of the direction of the current. The power consumed during a complete cycle is equal to the effective value of current multiplied by the effective value of voltage, shown simply as $P = E \times I$. The power curve for current flowing through a resistance is positive (power is being absorbed) for each half-cycle of the current; there are two positive power loops for the complete cycle of 360°.



If an a-c voltage is applied to a pure capacitance, current flows into the capacitor during the charging half-cycle. During this interval, the capacitor absorbs energy from the voltage source and stores it in the form of an electric field. Then, during the discharge half-cycle, the capacitor returns all of the energy it has stored to the voltage source. Thus, over a complete cycle, the power absorbed by the pure capacitance is zero. When illustrated by a power curve, the power absorption intervals are shown by positive loops, whereas the power return intervals are shown by negative loops. There are two power loops (one positive and one negative) for each half-cycle of current, or four power loops for a complete cycle.

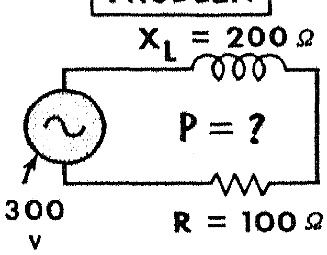
A similar situation prevails when an a-c voltage is applied to a pure inductance. Energy is absorbed during the time the current is building the magnetic field around the inductor, and power is returned to the source when the magnetic field collapses back into the inductor. As much electrical energy is returned as is absorbed; hence the net power consumed by a pure inductance during a complete cycle is zero.

Power Factor

The power consumed in a resistive a-c circuit is calculated in exactly the same manner as in d-c circuits ($P = E \times I$). To get an equivalent answer between d-c and a-c, we use the effective (rms) values of a-c voltages and currents. However, in inductive and capacitive circuits, the simple power formula $P = E \times I$ requires further consideration. We saw that, in purely inductive and capacitive circuits, all the energy stored in the form of magnetic and electric fields was returned to the source on the 2nd quarter-cycle, and that the net power dissipated was zero. The practical inductive or capacitive circuit always contains some resistance, however little. This resistance makes the phase angle between voltage and current somewhat less than 90°, and some power will be consumed – none will be returned to the source. If we were to measure the current and voltage in an inductive circuit that contained resistance, we would not get the true power consumed by multiplying $E \times I$, because we would be ignoring the partial power of the inductor, which is returned to the source.

When we measure E and I and then find their product, we get the <u>apparent power</u> consumed in the circuit. In a purely resistive circuit, the <u>apparent</u> power is the same as the <u>true power</u>. However, in an inductive or capacitive circuit, we must take into consideration the phase angle between E and I, using the formula $P = E \times I \times \cos\theta$. The use of "cosine θ" adds a <u>power factor</u> to our calculations. The cosine of 0° is 1. Thus, in a purely resistive circuit where the phase angle between current and voltage is 0°, power is simply $E \times I$. The cosine of 90° is 0; therefore, in a purely inductive or capacitive circuit, the power is $E \times I \times 0$, or zero.

PROBLEM



Power in an A-C Circuit is measured by

$$P = E \times I \times \text{cosine}\,\theta$$

SOLUTION

$P = E \times I \times \text{cosine}\,\theta$
$= 1.34 \times 300 \times \text{cosine}\ 63.4°$
$= 1.34 \times 300 \times 0.447$
$= \underline{179 \text{ watts}}$

A $Z = \sqrt{200^2 + 100^2} = \underline{224 \text{ ohms}}$

B $I = \dfrac{300}{224} = \underline{1.34 \text{ amperes}}$

C $\text{tangent}\,\theta = \dfrac{X_L}{R} = \dfrac{200}{100} = 2 \text{ or } \underline{63.4°}$

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

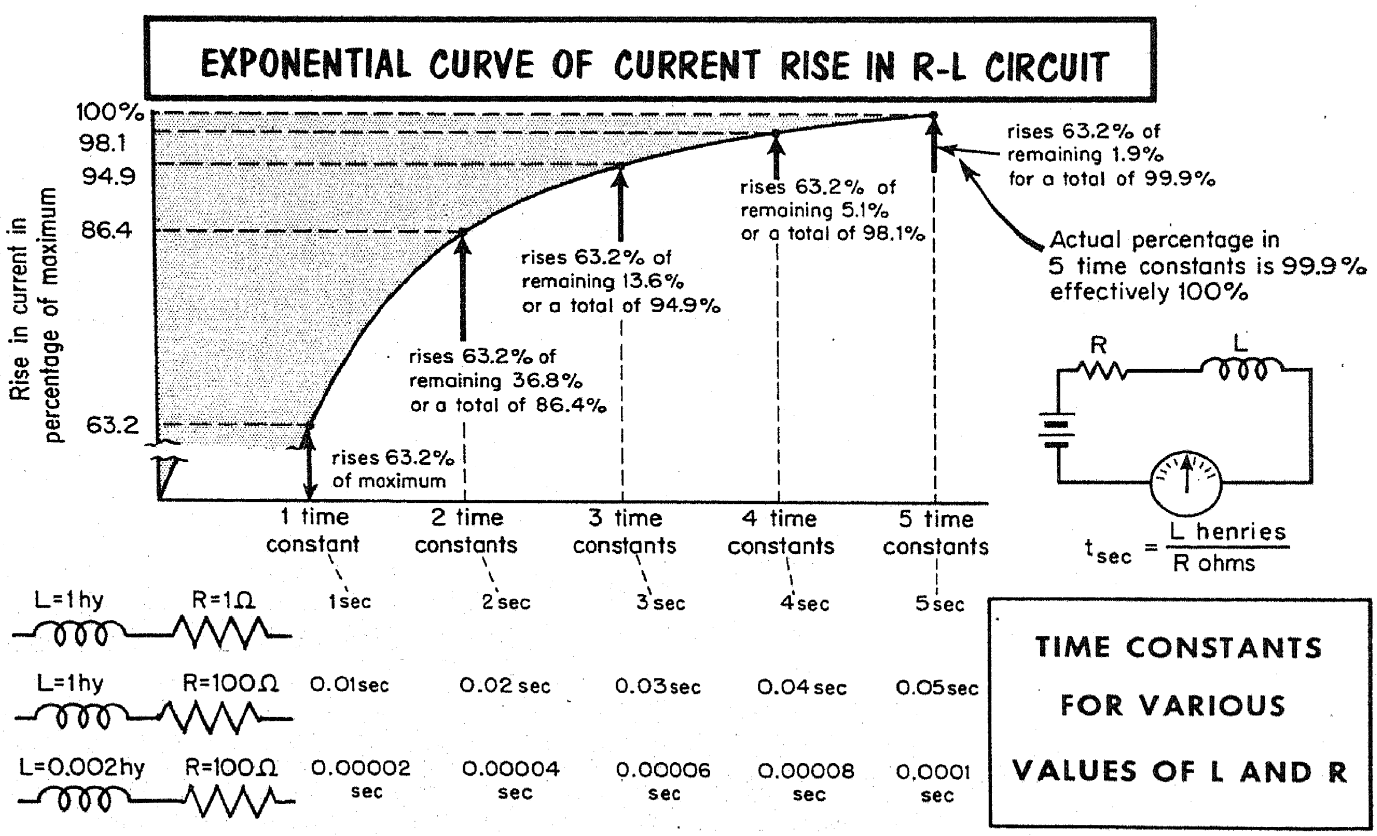
$$= \frac{179}{401}\ (1.34 \times 300) = \underline{0.447}$$

In all practical circuits containing C and R, or L and R, the cosine of the phase angle (θ) between E and I enables us to determine the true power consumed by the circuit. By dividing true power by apparent power, we get the power factor of the circuit – something between 0 and 1.

R-L Circuit Time Constant

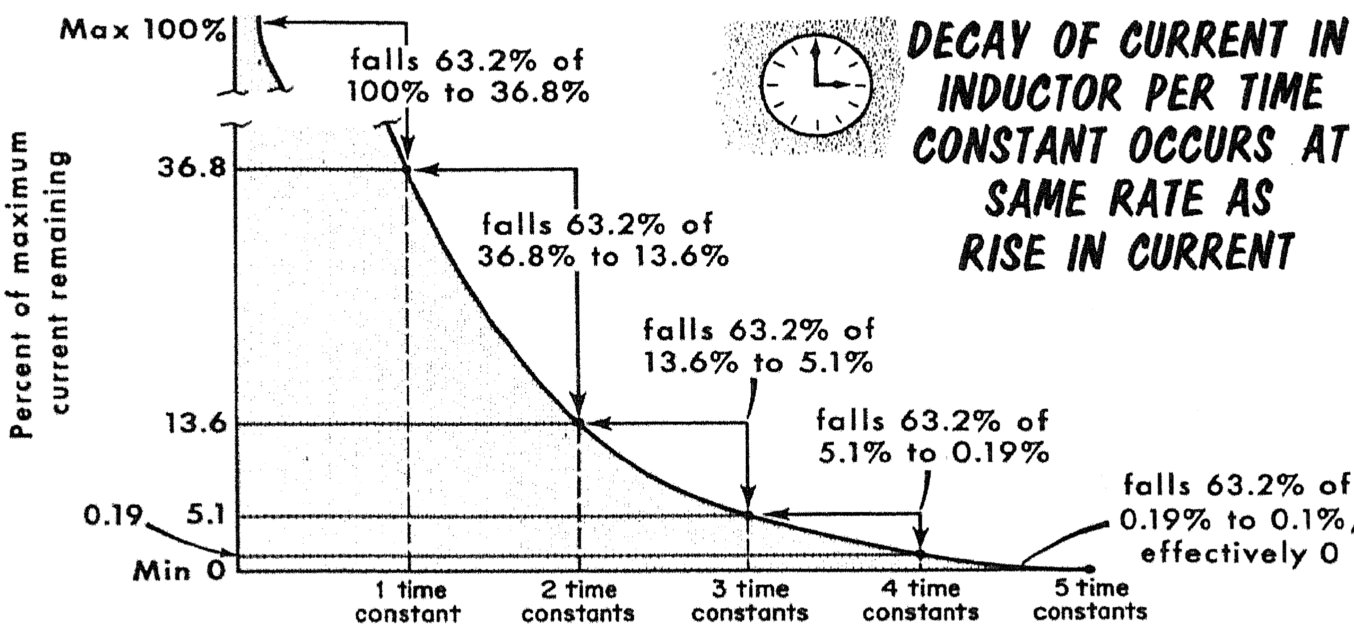
When a constant (d-c) voltage is applied to a resistance, the rise in current to a maximum value is instantaneous. If an inductance is connected in series with the resistance, time elapses while the current builds up to maximum. If we could observe the behavior of the current, we would see it rise rapidly from zero and then its rate of increase would progressively diminish. After a lapse of time, the current would reach a value which for all intents and purposes is maximum, equal to $I = E/R$. The relationship between the rise in the current to a given value and the time lapse while it is happening is determined by a term known as the time constant. Time constant is a means of comparing how rapidly the current in one R-L circuit rises to a given value relative to the current in another R-L circuit. Time constant is expressed in seconds, and is equal to the inductance L (in henries) divided by the resistance R (in ohms). The equation is: $t = L_{henries}/R_{ohms}$. If, for the moment, we assume L = 1 henry and R = 10 ohms, the time constant L/R = 0.1 second. The time constant varies in direct proportion to L and inversely with R.

The pattern of the increase in current in an R-L circuit is the same in all R-L circuits regardless of the specific values of L and R. When shown graphically, it is a singularly shaped curve known as an exponential curve. The characteristic of this pattern of change is that in a time equal to 1 time constant (1t), the current rises to 63.2% of its maximum value (regardless of what the maximum value may be). On this basis, in the numerical example given above, the current would rise to 63.2% of the maximum current in 0.1 second. The lapse of time corresponding to additional time constants permits current to rise to specific percentages of maximum (as illustrated).



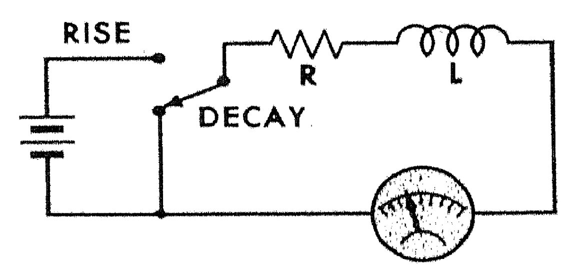
R-L Circuit Time Constant (Cont'd)

When a constant voltage is removed from a resistance, the current falls to zero instantly. When such a voltage is removed from an R-L combination, a time lapse occurs before the current decays to zero. As in the case of the rise in current, the decay in current follows an exponential curve, except that now the curve is an inverted version of the one which showed the rise in current. In a time equal to 1t, the current decreases 63.2% from the maximum; i.e., it falls to 36.8% of the maximum. In time 2t, it decays 86.4% from the maximum to a value equal to 13.6% of the maximum. As shown in the illustration, it decreases to 0.1% of the maximum; i.e., it falls 99.9% from the maximum or, effectively, to zero in 5t.



Units of Time Constant

When R is	When L is	Time constant is in
ohms	henries	seconds
megohms	henries	microseconds
ohms	millihenries	milliseconds
ohms	microhenries	microseconds



How do we calculate the momentary current? It is simple for whole-number time constants; for values in between, the chart is most convenient, as discussed later. Suppose L = 1 henry and R = 10 ohms. The applied voltage is 10 volts; t is 0.1 second. The maximum current is I = E/R = 10/10 = 1 ampere. If in constant 1t (0.1 second), the current rises 63.2% of maximum, it rises to 63.2% of 1 ampere or to 1 × 0.632 = 0.632 ampere. In 2t (0.2 second), it rises 86.4% of maximum, or to 1 × 0.864 = 0.864 ampere, etc., as shown on the previous page until in 5t (0.5 second), it reaches 0.999 ampere, effectively 1.0 ampere maximum. As to the decay of the current when the voltage is removed, in 1t (0.1 second) the current falls 63.2% of maximum, or decreases 63.2% of 1 ampere to 36.8% of the maximum, which amounts to 0.368 ampere.

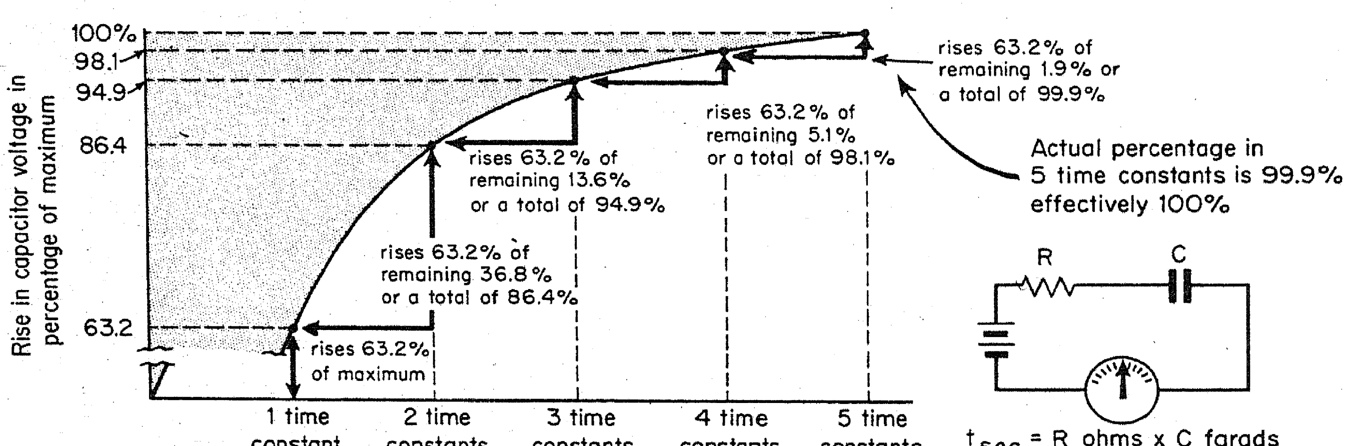
R-C Circuit Time Constant (Charging)

When a constant voltage is applied to a capacitance, the voltage built up in the capacitor by the charging current reaches the value of the charging voltage almost instantly. If a resistance (R) is connected in series with the capacitor (C), it tends to limit the amount of charging current and, in so doing, causes time to lapse while the capacitor acquires a charge. The factor that determines the rise in voltage in the capacitor of an R-C circuit relative to time is the time constant of the circuit. The R-C circuit time constant is also expressed in seconds, and equals the resistance (in ohms) multiplied by the capacitance (in farads). Expressed as an equation, it is:

$$t = R \times C$$

Imagine a C of 1 µf (0.000001 farad) in series with an R of 1000 ohms. The time constant t = 1000 × 0.000001 = 0.001 second. The applied voltage is 100 volts. The buildup of voltage in the capacitor is shown by exactly the same shaped curve as illustrated in the current rise of the R-L circuit. In 1t, the current flowing into the capacitor builds the capacitor voltage to 63.2% of the maximum (the applied voltage), or to 63.2 volts. In 2t, the capacitor voltage rises to 84.6%; in 3t, to 94.9%; in 4t, to 98.1%; and in 5t, it reaches 99.9% (effectively 100%). In voltage values, these percentages are 84.6, 94.9, 98.1, and 99.9 (or 100) volts. In 5t, the capacitor is, for all practical purposes, fully charged. Given the same applied voltage, but an R-C circuit with a t of 0.005 second, exactly the same percentages of maximum voltage would appear in the capacitor per time constant as before, but now it would require 0.005 second to build to 63.2%, and a correspondingly increased time to build to the higher voltage values.

EXPONENTIAL CURVE OF VOLTAGE ACROSS C IN AN R-C CIRCUIT



t_{sec} = R ohms x C farads

R=1000 Ω	C=1µf	0.001sec	0.002sec	0.003sec	0.004sec	0.005sec
R=10K	C=10µf	0.1 sec	0.2 sec	0.3 sec	0.4 sec	0.5 sec
R=2Meg	C=0.0025µf	0.005sec	0.01sec	0.015sec	0.02sec	0.025sec

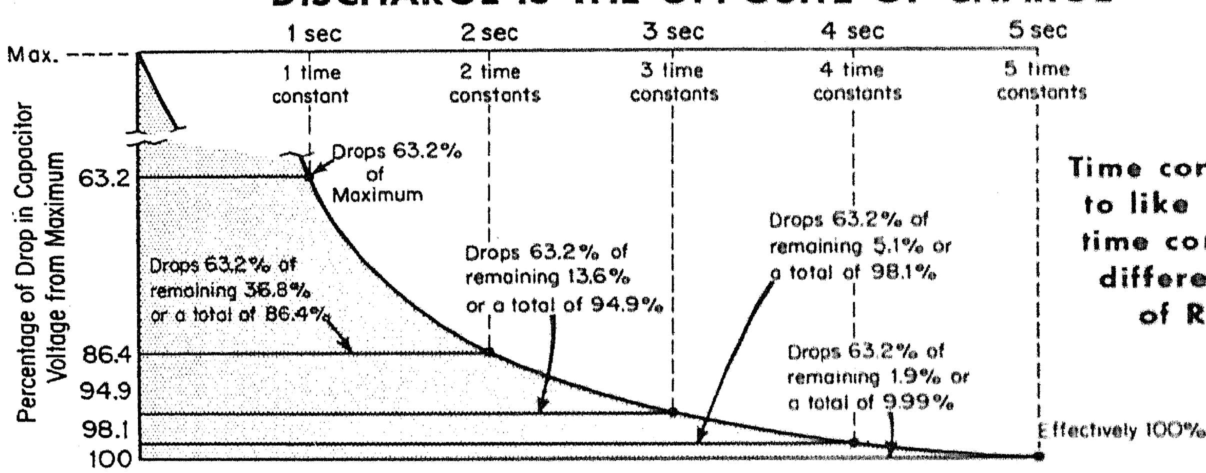
Time Constants for Various Values of R and C

R-C Circuit Time Constant (Discharging)

The action during discharge of the capacitor in the R-C circuit is the opposite of capacitor charging. The time required for complete discharge and for the capacitor voltage to fall effectively to zero is extended over that when no resistance is present in the circuit. The curve which shows the decrease of the capacitor voltage is exactly the same as the one which shows the decrease in inductor current. It is the inverted version of the curve which shows the rise in voltage across the capacitor during charging (see preceding page).

The percentage fall of the capacitor voltage from its maximum value relative to time is a function of the time constant of the circuit. You have learned that the capacitor acquires 63.2% of its maximum charge during the first period amounting to 1t. During discharge the capacitor loses .632 of its full charge in the first time interval equal to 1t. Thereafter, during the time interval equal to each succeeding time constant, it loses .632 of the charge still remaining in it.

IN THE R-C CIRCUIT, THE ACTION DURING DISCHARGE IS THE OPPOSITE OF CHARGE



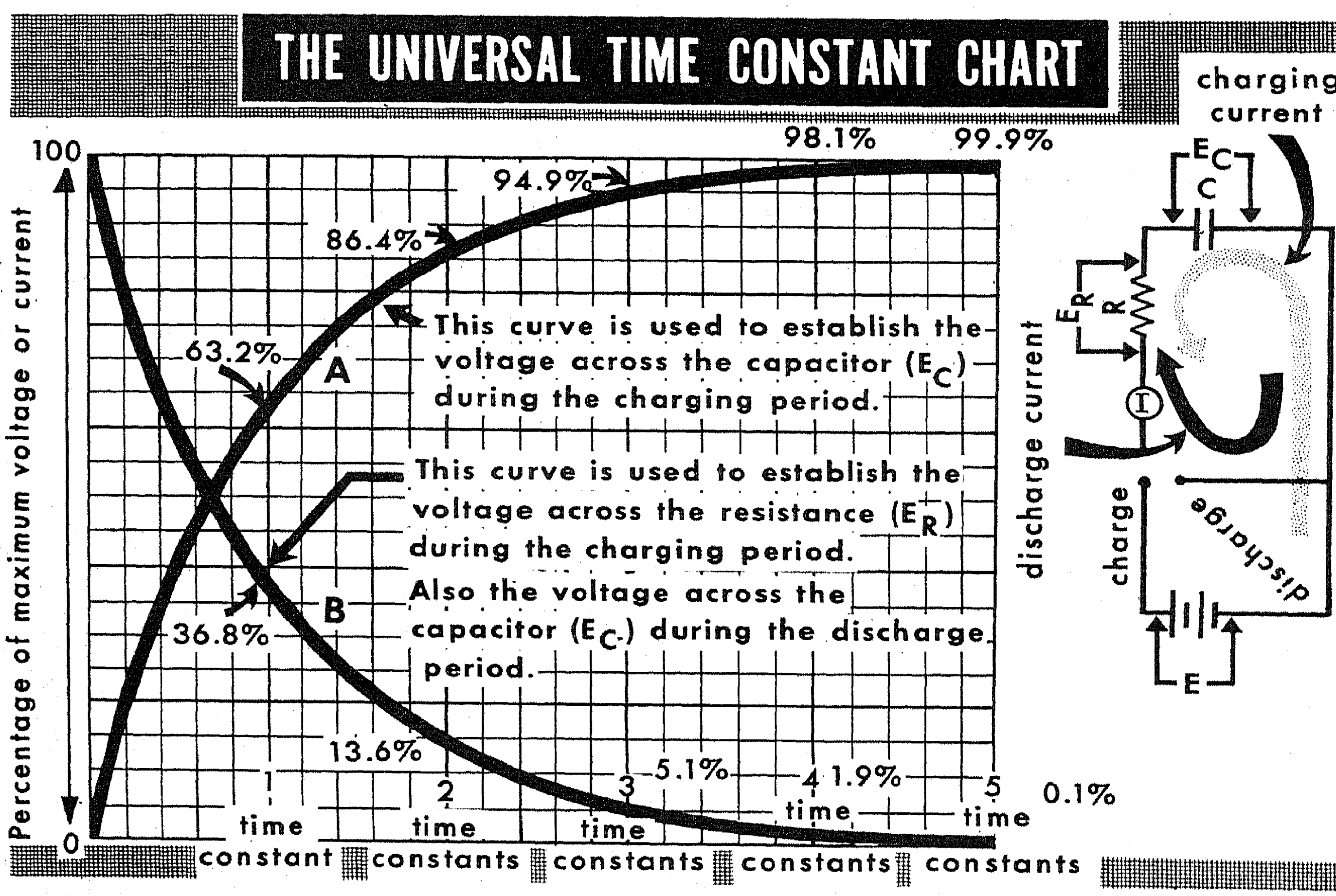
Time corresponding to like number of time constants for different values of R and C

R=1000, C=1μf	0.001 sec	0.002 sec	0.003 sec	0.004 sec	0.005 sec
R=10K, C=10μf	0.1 sec	0.2 sec	0.3 sec	0.4 sec	0.5 sec
R=2 Meg, C=0.0025μf	0.005 sec	0.01 sec	0.015 sec	0.02 sec	0.025 sec

Consider the first time interval equal to 1t. Assume that the capacitor is charged to a maximum of 100 volts; it loses 63.2% of its maximum charge. If 100 volts = 100%, the capacitor loses 63.2 volts of its charge; hence, there remains 100 − 63.2, or 36.8 volts of charge in the capacitor. In 2t, the capacitor loses 63.2% of its charge, or 63.2% of the 36.8 volts that still remain in it. Thus, in time equal to 2t, the capacitor loses a total of 86.4% of the original maximum charge, leaving 13.6% or 13.6 volts in the capacitor, and so on as illustrated. At the end of 5t, the capacitor has lost 99.9% of its charge, or 99.9 of the original 100 volts. Effectively, this is considered as leaving zero voltage in the capacitor.

Applying the Universal Time Constant Chart

We have said that the pattern of current rise in an R-L circuit is the same for all values of R and L. The curve that shows the decay of current in the R-L circuit similarly suits all values of R and L. The same two curves apply to the R-C circuit. The curve that shows the rise in capacitor voltage while charging is the same as the one that shows the rise in current in the R-L circuit. The curve that shows the fall in capacitor voltage during discharge is identical with the curve that shows the decay of current in the R-L circuit. Because of these similarities, the two curves are known as <u>Universal Time Constant Curves.</u> They are shown on a single chart. The horizontal axis is calibrated in units totalling 5 time constants, whereas the vertical axis is divided uniformly in percentages of rise and fall of current in the R-L circuit, or rise and fall of capacitor voltage in the R-C circuit.



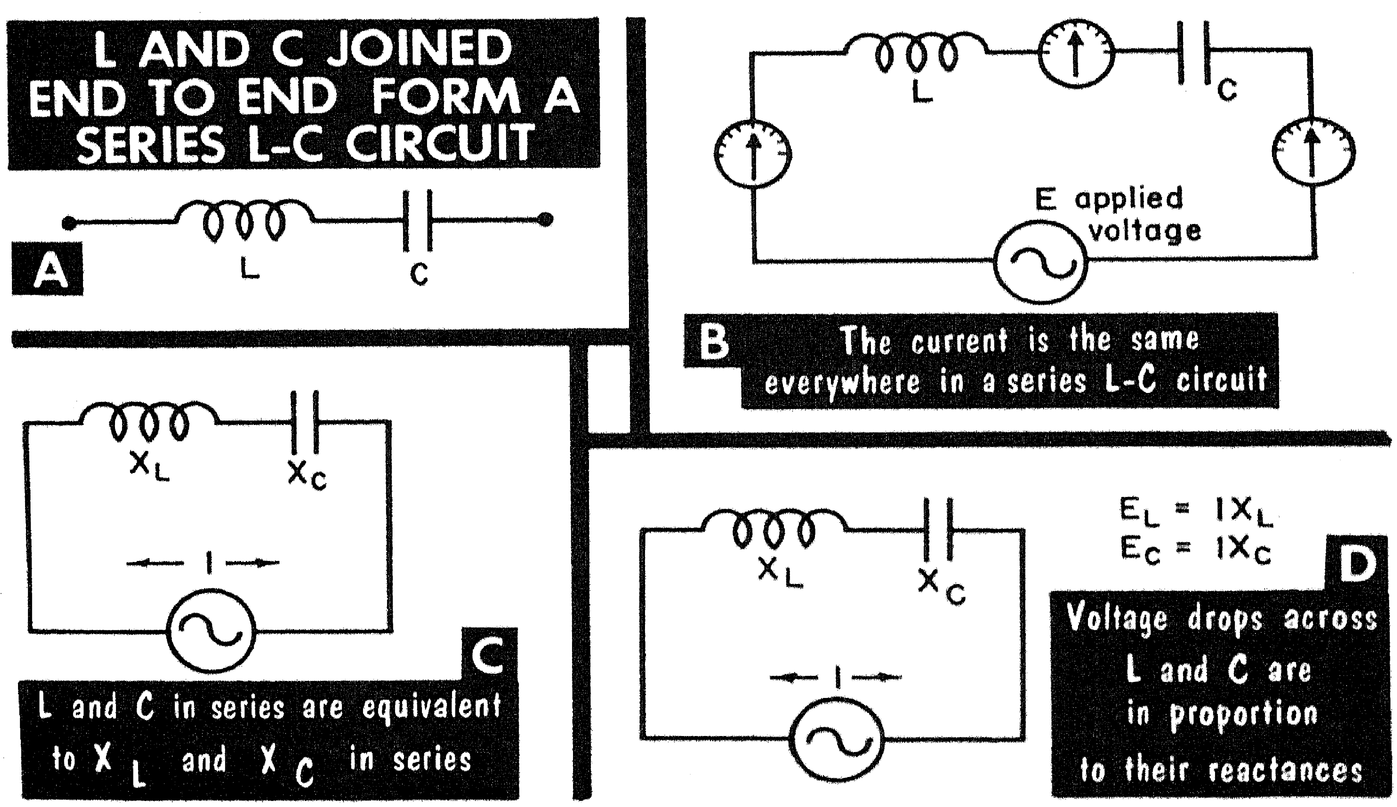
Assume that 100 volts is applied to an R-C circuit. t = 0.1 second. In 1t, the capacitor voltage will rise to 63.2 volts. What will be the voltage in 1.5t? Projecting 1.5t from the horizontal axis to curve A and its point of intersection with the curve to the vertical axis, we see that the capacitor voltage builds up to approximately 77.5% or 77.5 volts. At 0.5t, the voltage in the capacitor is just under 40% of maximum, or just under 40 volts. How low does the capacitor voltage fall in 0.4t? Using curve B, it is seen to fall 32.5% of the full charge to 67.5 volts. How low does the voltage fall in 2.6t? Using curve B, the answer is approximately 93% of maximum, leaving 7% in the capacitor. Hence, the capacitor voltage is 100 × 0.07 = 7 volts.

Relationship of L and C in Series (General)

When an inductance L and a capacitance C are joined end to end, a series L-C circuit is formed, as shown in A. Let us assume L to be a pure inductance and C to be a pure capacitance. In other words, the circuit has no d-c resistance. In practice, this electrical situation is never realized. The assumption is valid, however, because in most practical cases, the d-c resistance is so little as to be unimportant in the analysis of the circuit. Later, we shall discuss the effects of resistance.

If an a-c E is applied to a series L-C circuit, a current I will flow. Since a series circuit offers only one path for the current, the same current flows in the inductance and the capacitance – i.e., through the inductive reactance of L and the capacitive reactance of C (as in B). The characteristic that limits the current in an a-c circuit is the impedance of the circuit. In the series L-C circuit assumed to be free of d-c resistance, the circuit impedance consists of reactance only – inductive reactance X_L in series with capacitive reactance X_C (as in C).

Inductive and capacitive reactances are, with reference to their opposition to current flow, comparable to resistance, although they are different phenomena. The factor common to them is that current flow through a reactance or a resistance results in the appearance of a voltage drop across each. Current through a resistance results in voltage drop $E_R = IX_R$; current through an inductive reactance results in voltage drop $E_L = IX_L$; current through a capacitive reactance results in voltage drop $E_C = IX_C$ (see D). These voltages differ in one major respect – voltage across resistance is in phase with current; voltage across an inductance leads current by 90°; voltage across capacitance lags current by 90°.



L and C in Series (Impedance)

Imagine a circuit in which inductance L is 1 henry (negligible d-c resistance), in series with a capacitance C of 10 μf. Applied voltage E is 120 volts, and frequency f is 60 cycles. Solving for the reactances:

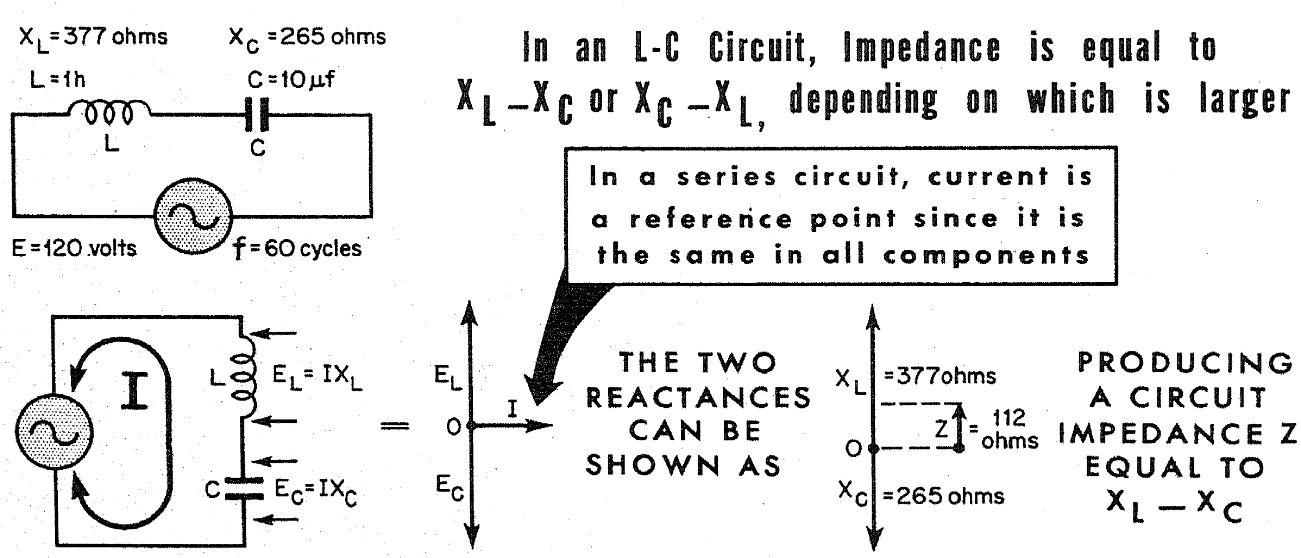
$$X_L = 2\pi f L = 6.28 \times 60 \times 1 = 376.8 \text{ or } 377 \text{ ohms}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{6.28 \times 60 \times 0.00001} = 265.3 \text{ or } 265 \text{ ohms}$$

Now how do we establish the circuit impedance? We find the answer in a rule, "The impedance of a series L-C circuit having no resistance is equal to the difference between the ohmic values of the inductive and capacitive reactances." Therefore, for our circuit:

$$\begin{aligned} \text{Impedance } Z &= X_L - X_C \\ &= 377 - 265 \\ &= 112 \text{ ohms (inductive)} \end{aligned}$$

Reactance X_C was subtracted from reactance X_L because X_L was the greater quantity. If the situation were reversed, X_L would be subtracted from X_C.



We illustrate this rule vectorially by arranging the X_L vector pointing up from a reference point, and the X_C vector pointing down from the <u>same</u> point. The vectors point in opposite directions. If we make each vector length proportional to its ohmic value, using the same scale for both, the shorter vector can be subtracted from the longer. The remainder is the difference between the two and is the <u>net reactance</u>, which we call <u>impedance</u> (Z). Since X_L in our circuit is greater than X_C, the impedance consists of inductive reactance; hence, the Z vector has the same direction as the X_L vector. This accounts for the reference, "inductive," in connection with the impedance value. More about this later.

Impedance, and Calculating Impedance Problems

The references to inductive and capacitive impedances (Z) require clarification. Where X_L exceeds X_C, Z equals $X_L - X_C$. With X_C greater than X_L, Z equals $X_C - X_L$. In either case, the remainder (called impedance) is net reactance: in one case, a certain amount of X_L; in the other, a certain amount of X_C.

For a given frequency, changes in the values of L and C result in different circuit impedances. The same is true if L and C are held constant and the frequency of E is varied. For example, assume L = 10 h (ten times that of the previous example) while C = 10 µf (as before). Frequency remains at 60 cycles. Impedance is as shown in A.

Now reduce capacitance by making C = 1 µf (a tenfold decrease), and restore L to 1 h. Frequency stays at 60 cycles. Impedance now is as shown in B.

Compare this amount of impedance with the example on page 2-98 (112 ohms inductive). Reducing C increases X_C so that it exceeds X_L. The circuit impedance therefore becomes capacitive.

Now change E from 60 to 160 cycles. Everything else stays as in the example immediately above. Impedance is as shown in C.

Changing frequency causes the two reactances to approach equality. Each individual reactance remains relatively high, but impedance has fallen to 10 ohms. In certain special cases (resonance), this net reactance can fall to zero. Lower frequency to 40 cycles, leaving everything else as above (D).

A study of these examples shows that the impedance can be low, medium, or high, and inductive or capacitive, depending on the values of L, C, and f.

CALCULATIONS OF IMPEDANCE IN SERIES L-C CIRCUITS

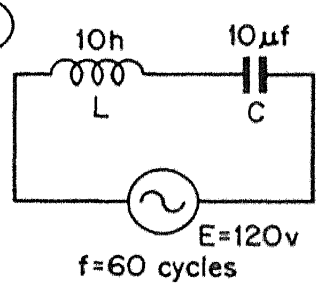
A

$X_L = 2\pi fL = 3768$ ohms

$X_C = \frac{1}{2\pi fC} = 265$ ohms

then $Z = X_L - X_C =$

$\boxed{3768 - 265 = 3503 \text{ ohms, inductive}}$



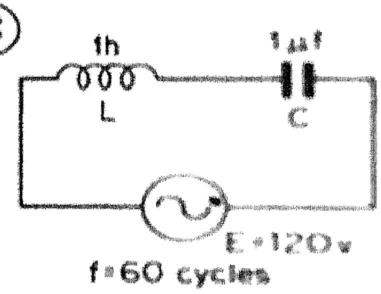
B

$X_L = 2\pi fL = 377$ ohms (actually 376.8Ω)

$X_C = \frac{1}{2\pi fC} = 2653$ ohms

then $Z = X_C - X_L =$

$\boxed{2653 - 377 = 2276 \text{ ohms, capacitive}}$



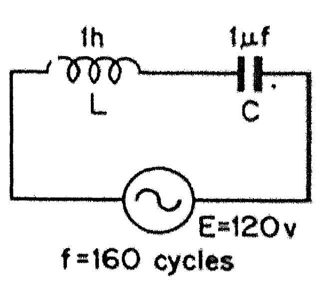
C

$X_L = 2\pi fL = 1005$ ohms (actually 1004.8Ω)

$X_C = \frac{1}{2\pi fC} = 995$ ohms (actually 995.2 Ω)

then $Z = X_L - X_C =$

$\boxed{1005 - 995 = 10 \text{ ohms, inductive}}$



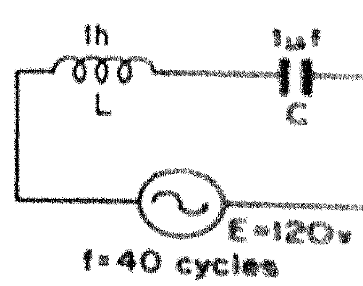
D

$X_L = 2\pi fL = 251$ ohms (actually 251.2Ω)

$X_C = \frac{1}{2\pi fC} = 3981$ ohms (actually 3980.8Ω)

then $Z = X_C - X_L =$

$\boxed{3981 - 251 = 3730 \text{ ohms, capacitive}}$



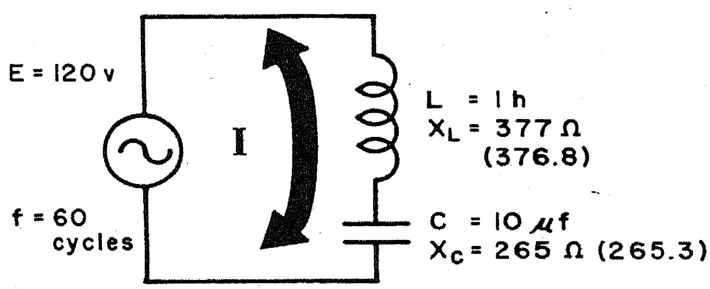
Calculating Current Problems

Given any value of circuit impedance for L and C in series, and knowing the applied voltage, the circuit current can be computed by applying Ohm's law for a-c. The equation is: $I = E/Z$, where E is the applied voltage, and Z is the circuit impedance. If you examine the equation, you can see that current varies in direct proportion to the applied voltage, and in inverse proportion to changes in Z. The latter relationship is shown in the different examples on this page.

The values of L and C, E, and f, as well as the current used in the examples, are not typical of series L-C circuits found in radio receivers. L and C usually are much smaller, as is the current. The values used here were selected because it was felt they helped to clarify the discussion.

CALCULATING CURRENT

A



$Z = X_L - X_C$

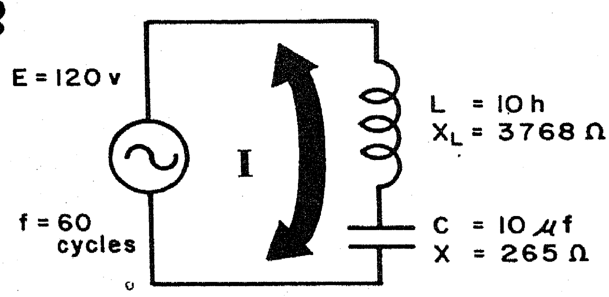
$= 112\ \Omega$, inductive

Then $I = \frac{E}{Z} = \frac{120}{112}$

$= 1.07$ Amperes in round numbers

B

HIGH IMPEDANCE means LOW CURRENT



$Z = X_L - X_C$

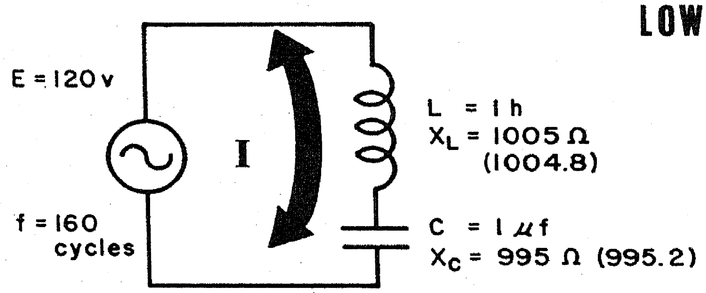
$= 3503\ \Omega$, Inductive

Then $I = \frac{E}{Z} = \frac{120}{3503}$

$= 0.034$ Ampere, or 34 milliamperes in round numbers

C

LOW IMPEDANCE means HIGH CURRENT



$Z = X_L - X_C$

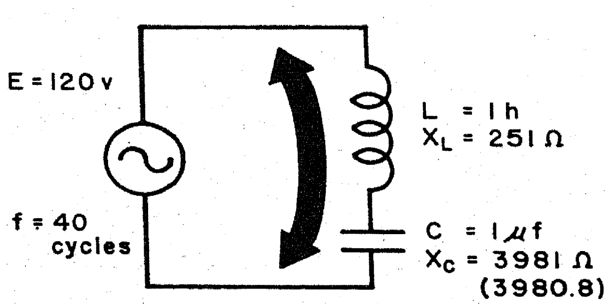
$= 10\ \Omega$, Inductive

Then $I = \frac{E}{Z} = \frac{120}{10}$

$= 12$ Amperes in round numbers

D

HIGH IMPEDANCE means LOW CURRENT



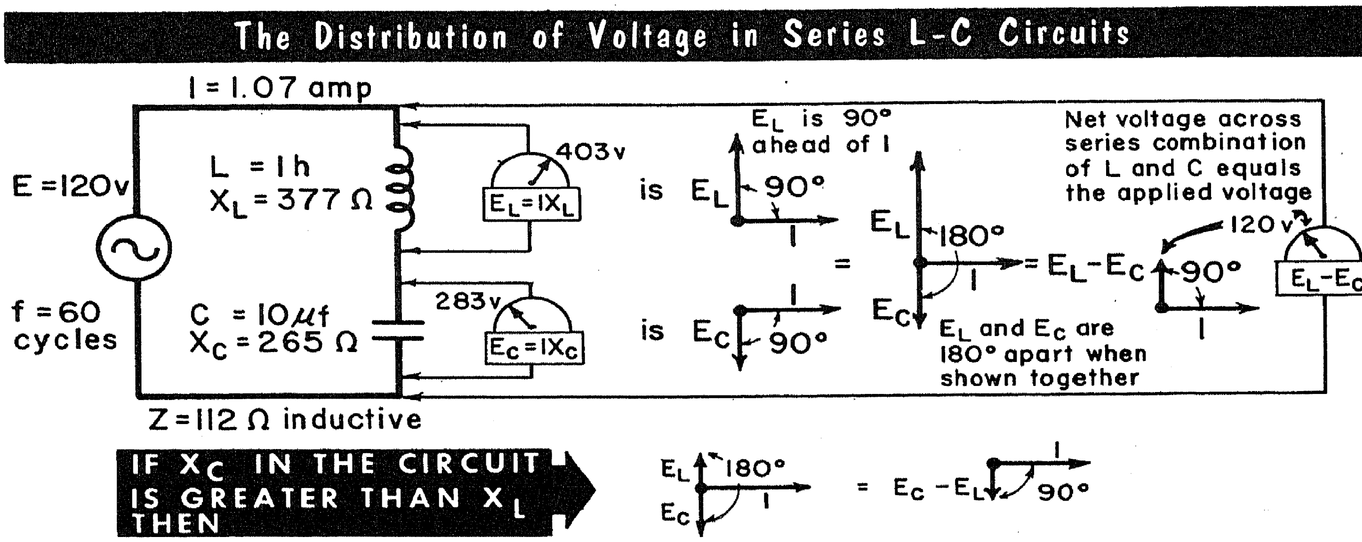
$Z = X_C - X_L$

$= 3730\ \Omega$, Capacitive

Then $I = \frac{E}{Z} = \frac{120}{3730}$

$= 0.0322$ Ampere, or 32 milliamperes in round numbers

Analyzing the Distribution of Voltage



In the circuit under discussion, E_L = 403 volts and E_C = 283 volts. Note that both of these are higher than the applied voltage, which is 120 volts. How can this be? It does happen in series L-C circuits; in fact, one or the other of these two voltages is always higher than the applied voltage, and, as shown in this example, both can be higher. The reason is that for any given circuit current, the individual voltage drops are determined by the individual reactances. The higher the reactance for a given current, the greater the individual voltage developed across it by the current. It is a characteristic of series L-C circuits that very high voltages can develop across L and C.

Let us analyze the voltages in the circuit discussed on page 2-98. As the result of current flow through L and C, a voltage $E = IX_L$ appears across L, and a voltage $E_C = IX_C$ appears across C. Substituting the circuit values in the equations for E_L and E_C, we get:

$$E_L = IX_L = 1.07 \times 377 = 403 \text{ volts}$$

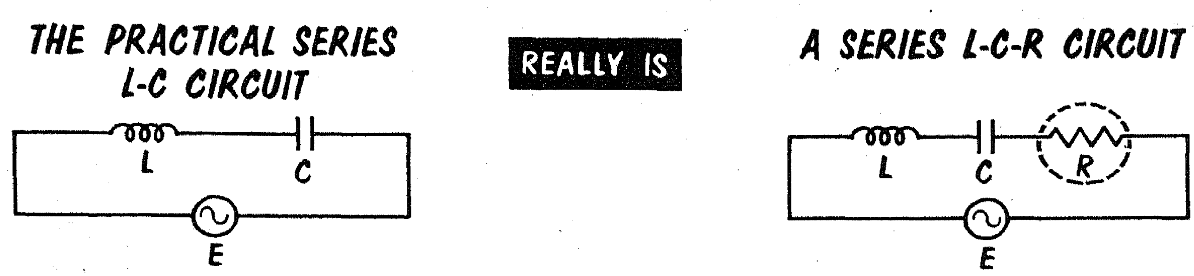
$$E_C = IX_C = 1.07 \times 265 = 283 \text{ volts}$$

Several significant details are associated with these two circuit voltages: each is independent of the other; each is measurable with a suitable voltmeter. Also, voltage E_L leads circuit current I by 90°, while voltage E_C lags I by 90°. If we show these two phase relationships in a single vector presentation using I as the reference vector, it is seen that E_L and E_C are 180° apart. They thus tend to offset each other in their effects on the circuit. This leads to the conclusion that the voltage present across the series L-C circuit as a whole (i.e., across the series combination of L and C), is the difference between the two voltages. In this instance, it is $E_L - E_C$, or 403 - 283 = 120 volts (the applied voltage). Since the current flowing through impedance Z develops the same voltage as the voltage difference between E_L and E_C, we can conclude that the voltage across the circuit impedance always equals the applied voltage.

Calculating the Impedance

The practical L-C circuit really is a series L-C-R circuit. The R is the inherent d-c resistance of the connecting wires and of the coil itself. The ohmic value of R equals the total d-c resistance of the circuit.

When R is a substantial quantity, the manner of determining impedance is different from when R is negligible. The net reactance of X_L and X_C must be determined, to which the circuit resistance is added. But this cannot be done by simple addition because the voltage and current relationships in the reactance differ in phase by 90°, while they are in phase in the resistance. To calculate the circuit impedance, the net reactance and the resistance must be added vectorially. We can do this using the right triangle relationship expressed by the equation $Z = \sqrt{R^2 + X^2}$, where X is the net reactance and R is the d-c resistance. Because the net reactance is the difference between X_L and X_C, the equation is changed to read $Z = \sqrt{R^2 + (X_L - X_C)^2}$, when X_L is greater than X_C, and to $Z = \sqrt{R^2 + (X_C - X_L)^2}$ when X_C is greater than X_L.

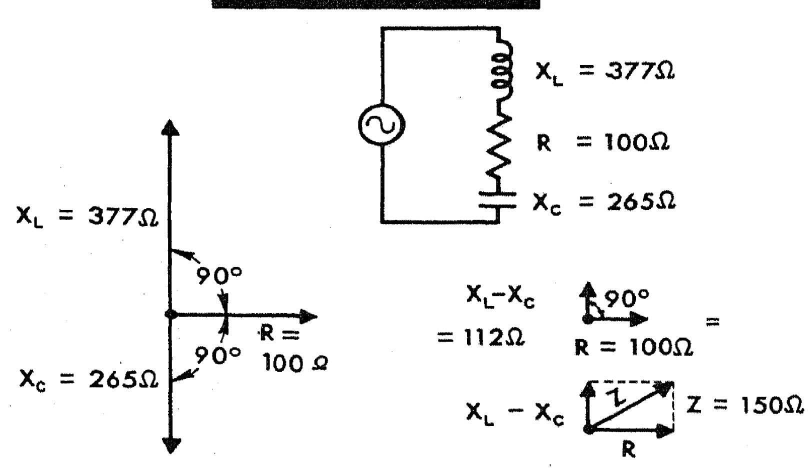


The Solution of the Impedance Z of Series L-C-R

by the Pythagorean Equation Method

$$
\begin{aligned}
Z &= \sqrt{R^2 + (X_L - X_C)^2} \\
&= \sqrt{100^2 + (377 - 265)^2} \\
&= \sqrt{100^2 + 112^2} \\
&= \sqrt{10{,}000 + 12{,}544} \\
&= \sqrt{22{,}544} \\
&= 150 \text{ ohms}
\end{aligned}
$$

by Vector Presentation



For the problem to be solved, we refer back to a previous example: L = 1 henry; C = 10 μf; and f = 60 cycles. Here X_L was calculated to be 377 ohms and X_C 265 ohms. With R = 100 ohms, and the net reactance $X_L - X_C$ equal to 112 ohms (inductive), the circuit impedance Z equals 150 ohms. It constitutes an impedance (equivalent to an L-R circuit) in which the voltage is neither 90° ahead of the current, nor in phase with the current. The angle of lead of the voltage relative to the current is expressed by $\tan \theta = (X_L - X_C)/R$. This equals $112/100 = 1.12$, or $\theta = 48.3°$.

L-C-R in Series (Impedance)

The relative values of R and the net reactance of the series L-C-R circuit create a variety of impedance and circuit conditions. Imagine several circuits in which L and C are of such value as to present the indicated capacitive and inductive reactances. The resistance R of each circuit is as indicated.

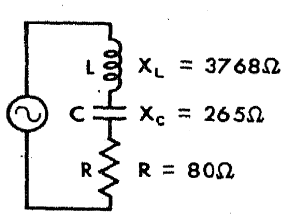
When the circuit resistance is small, as in circuit A, in comparison to the net reactance, it contributes little to the circuit impedance. As a whole, the circuit behaves as if it were an inductance of such value as to present a reactance of 3504 ohms.

When the circuit resistance is high in comparison to the net reactance (circuit B), the impedance, for all intents and purposes, is made up of the resistance. As a whole, circuit B behaves like a resistance of 200 ohms. The greater the ratio between the net reactance and the circuit resistance, the greater the contribution of the resistance to the circuit impedance.

In circuit C too, the circuit resistance is very small in comparison to the net reactance (capacitive). Therefore, the resistance contributes very little to the circuit impedance. Circuit C behaves as though it were a capacitance with a reactance of 3730 ohms.

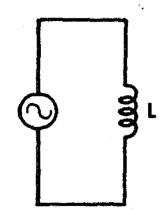
SERIES L-C-R CIRCUIT . . . . HAS Z OF . . . . AND BEHAVES AS

A



$$Z = \sqrt{80^2 + (3768 - 265)^2} = \sqrt{80^2 + 3503^2} = \sqrt{6400 + 12{,}271{,}009} = 3504 \text{ ohms}$$

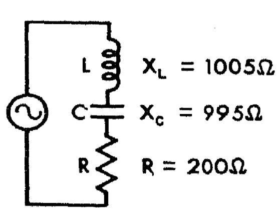
=



with X_L = 3504Ω at 60 cycles in which voltage leads current by $\tan\theta = \frac{X_L - X_C}{R}$ =43.7 or 88.7°,

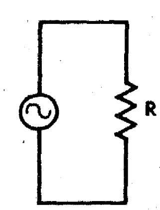
effectively by 90°

B



$$Z = \sqrt{200^2 + (1005 - 995)^2} = \sqrt{200^2 + 10^2} = \sqrt{40{,}000 + 100} = 200 \text{ ohms}$$

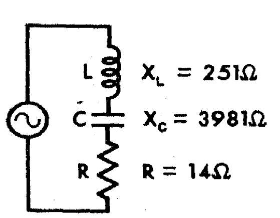
=



R = 200 ohms (actually 200.28 ohms) in which voltage leads current by $\tan\theta = \frac{X_L - X_C}{R}$ $= \frac{10}{200}$ = 0.05 or 2.9°,

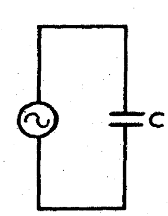
effectively 0°

C



$$Z = \sqrt{14^2 + (3981 - 251)^2} = \sqrt{14^2 + 3730^2} = \sqrt{196 + 13{,}912{,}900} = 3730 \text{ ohms}$$

=



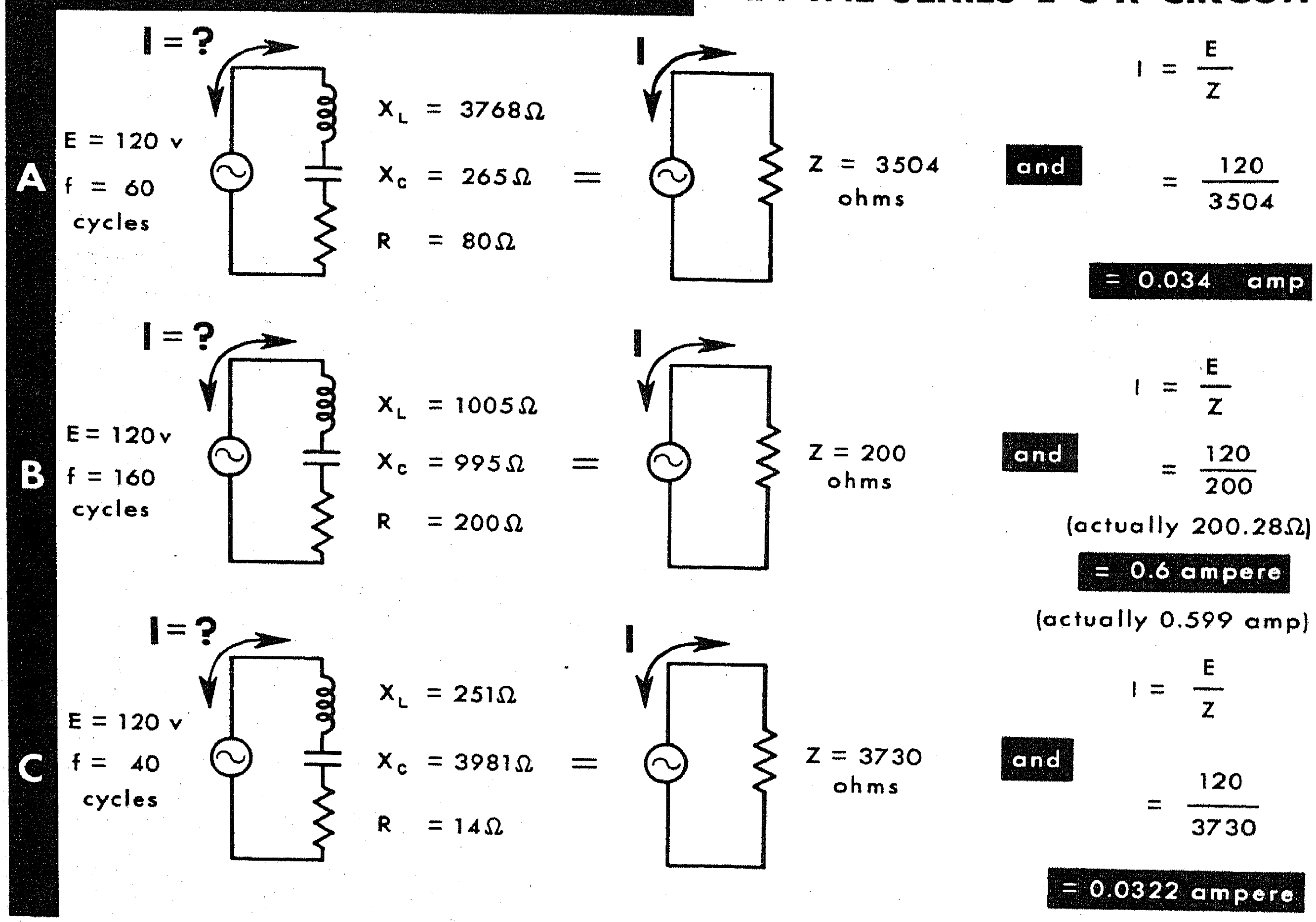
X_C = 3730 ohms voltage **lags** current by $\tan\theta = \frac{X_C - X_L}{R}$ $= \frac{3730}{14}$ = 266 or 89.7°,

effectively by 90°

L-C-R in Series

Given an applied voltage E and circuit impedance Z, the circuit current I equals E/Z. The nature of Z – whether it is inductive, capacitive, or resistive – is unimportant in the calculation. It is only after the current is known, as with the voltage across the impedance, that the nature of the impedance becomes of interest. The same equation applies to all cases for Z. As you can see, the equation for current I is Ohm's law for a-c and is identical for the theoretically ideal (resistanceless) L-C circuit and for the practical L-C-R circuit. Let us apply the equation to the three examples of circuit impedance shown on the preceding page. Assume that applied voltage E = 120 volts.

THE SOLUTION OF CURRENT IN THE SERIES L-C-R CIRCUIT



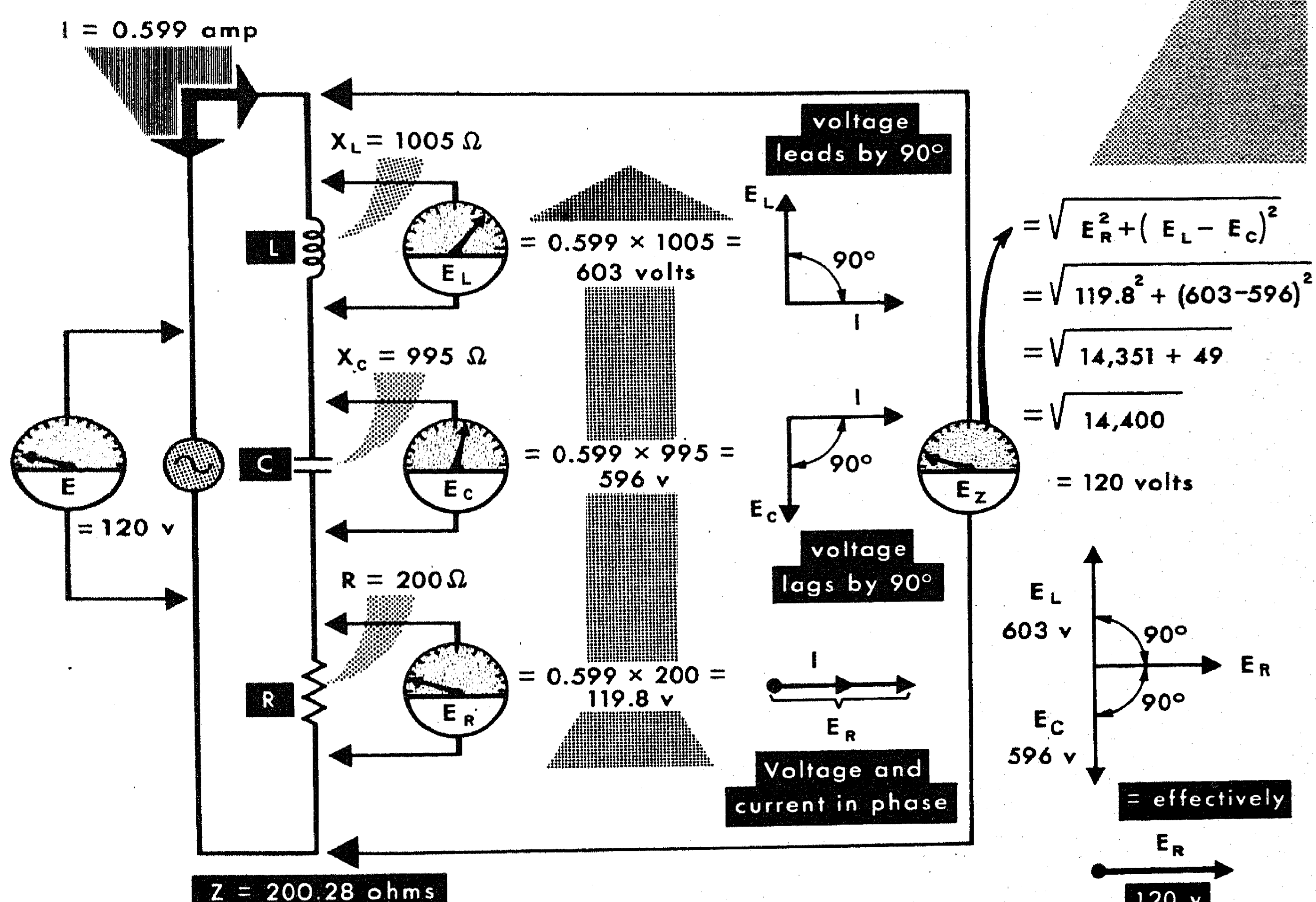
If you compare the current flowing in examples A on this page with example B on page 2-100, you will note no change in current despite the presence of a resistance of 80 ohms. The reason is that the prime control of the current is X_L; the additional current limitation imposed by the 80 ohms of resistance causes only a trifling reduction in current, which we do not show. On the other hand, if you compare example B above with example C on page 2-100, you will note that the presence of 200 ohms resistance reduces the current from 12 amperes to 0.6 ampere – a tremendous change.

L-C-R in Series (Voltages)

The voltages present across the components of the series L-C-R circuit are calculated in the same way as for the theoretically ideal (resistanceless) L-C circuit. The individual voltage drops (E_X) across the reactances are $E_X = IX$, where X stands for reactances; the voltage drop across the resistance is $E_R = IR$; and the voltage across the circuit as a whole (i. e., across the impedance Z) is $E_Z = IZ$. The illustration is of a typical case.

The addition of the two reactive voltages E_L and E_C to the resistive voltage E_R is done in the same way as the addition of reactances and resistance; that is, solving for Z. The same answer can be obtained by using vectors, but the equation method is much easier to use because many values are difficult to read from vector dimensions. When the three voltages are shown in the same vector presentation, the voltage E_R is the reference voltage. It is the voltage that is in phase with the series circuit current. The other two voltages, E_L and E_C, differ by 90° from E_R; one (E_L) leading E_R, and the other (E_C) lagging E_R by 90°. When all the voltages in the circuit are added, the voltage across the circuit as a whole is equal to the applied voltage.

THE DISTRIBUTION OF VOLTAGES IN THE SERIES L-C-R CIRCUIT



Resonant Frequency

Radio communications involves the transmission and reception of signals of selectable frequency. Such selection is possible because every combination of L and C responds better to voltages of one frequency than to voltages of other frequencies. The single frequency at which the circuit responds best is called the resonant frequency of the circuit.

Resonance occurs when the amount of inductance and the amount of capacitance in a circuit present equal amounts of reactance; i.e., $2\pi fL = 1/(2\pi fC)$, or $X_L = X_C$. Resonance in a series L-C-R circuit is, therefore, a particular condition in the circuit. Resistance R plays no part in determining the resonant frequency, although, as you will see, it does limit the amount of current at resonance and affects the behavior of the circuit off resonance. The equation used for calculating the resonant frequency of a circuit is:

$$\text{frequency of resonance, } f = \frac{1}{2\pi\sqrt{LC}}$$

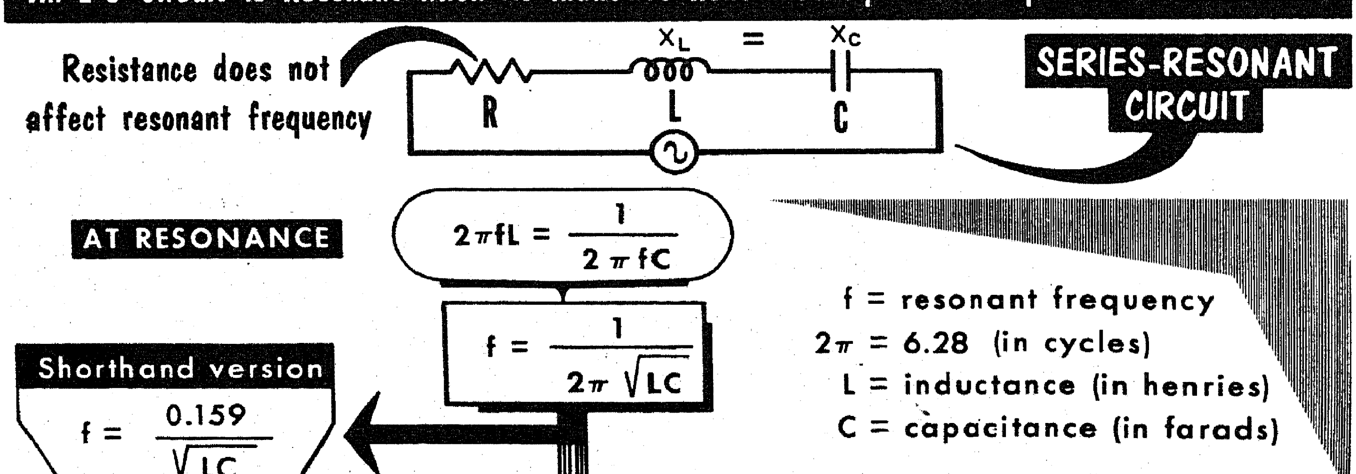
where π is a constant 3.1416, L is the inductance in henries, and C is the capacitance in farads. To illustrate the application of the equation, assume that L = 2 mh (0.002 henry) and C = 80 μμf (0.00000000008 farad). What is the resonant frequency of the circuit?

$$f = \frac{1}{6.28\sqrt{0.002 \times 0.00000000008}} = \frac{1}{6.28\sqrt{0.00000000000016}}$$

$$= \frac{1}{6.28 \times 0.0000004} = \frac{1}{0.000002512} = 398,089 \text{ cycles or } 398 \text{ kc (round numbers)}$$

A change in either L or C, or both, results in a change in resonant frequency, except when the product of L and C remains the same. For example, 0.002 × 80 μμf = 0.160. If L were 0.001 h (instead of 0.002), and C were 160 μμf (instead of 80), the product of 0.001 × 160 would be 0.160 — the same as above. Hence the two circuits would resonant at the same frequency — 398,089 cycles.

An L-C Circuit is Resonant when its Inductive Reactance equals its Capacitive Reactance



Determining the Impedance

Resonance is a particular condition in an L-C circuit at which $X_L = X_C$. Since the net reactance of a series L-C-R circuit is the difference between X_L and X_C, when these two reactances are equal (at resonance), the net reactance is zero. This leaves only the resistance as the current-limiting factor in the circuit. Thus, at the resonant frequency, the impedance of the series L-C-R circuit must be the lowest possible for the circuit, for at this frequency, the total circuit impedance is the circuit resistance.

Let us examine the impedance conditions at resonance. We shall use the constants mentioned on the preceding page. L = 2 mh; C = 80 μμf; and the resonant frequency is 398,089 cycles. Now we include the circuit resistance of 100 ohms. Then,

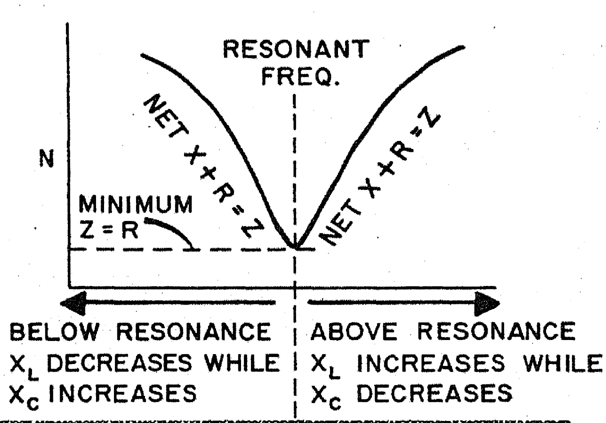
$$X_L = 2\pi fL = 6.28 \times 398{,}089 \times 0.002 = 5000 \text{ ohms (in round numbers)}$$

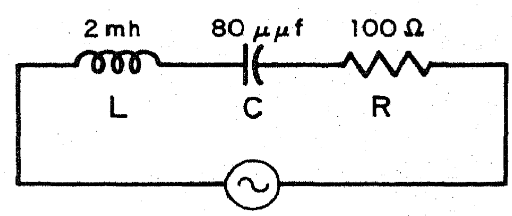
$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 398{,}089 \times 0.00000000008} = 5000 \text{ ohms}$$

Therefore,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{100^2 + (5000 - 5000)^2} = \sqrt{100^2 + 0} = 100 \text{ ohms}$$

At all other frequencies the net reactance is not zero; it has a finite value which increases (as does Z) as the operating frequency moves farther above or below the resonant frequency. We show this in the table for a range of frequencies from 100 to 600 kc. Examine the values in the Z column. The references to Z relative to frequency can be translated into a graph known as the impedance curve of the series resonant L-C-R circuit. Of course, the curve is for this particular set of circuit values.





Impedance Calculations At, Above, and Below Resonance

Frequency kc/sec	X_L ohms	X_C ohms	X ohms	R ohms	Z ohms
100	1,256	19,890	18,634	100	18,634
200	2,512	9,945	7,433	100	7,433
300	3,768	6,630	2,862	100	2,862
*398	5,000	5,000	0	100	100
450	5,620	4,410	1,240	100	1,240
500	6,280	3,978	2,302	100	2,302
600	7,536	3,315	4,221	100	4,221

*RESONANT FREQUENCY

Variations in Current

We have established that the circuit impedance of the series L-C-R circuit is minimum at the resonant frequency. For any given value of applied voltage, this means that the current is maximum. In fact, the occurrence of minimum impedance and maximum current at resonance is the identifying feature of the series resonant L-C-R circuit. Two equations can be used to compute the current: I = E/Z, for general application during the off resonant condition, and I = E/R, at the resonant frequency. Assume that E equals 30 volts at all frequencies and R equals 100 ohms. The rest of the circuit has the constants shown on the preceding page. Therefore, current at the resonant frequency of 398 kc (actually 398,089 cycles) is:

$$I = \frac{E}{R} = \frac{30}{100} = 0.3 \text{ ampere} = 300 \text{ milliamperes}$$

The table shows the frequency, impedance, and current for the circuit under discussion, over a frequency range of from 100 kc to 600 kc.

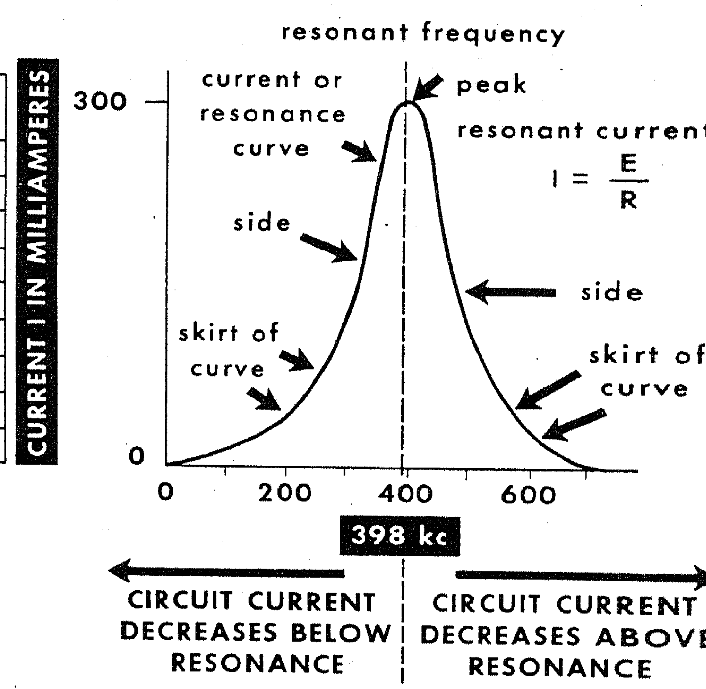
MAXIMUM CURRENT OCCURS AT ONE FREQUENCY -- THE RESONANT FREQUENCY

CHART OF VALUES

frequency (kc)	Z ohms	E(volts)	I(milliamps)**
100	18,634	30	1.61
200	7,433	30	4.0
300	2,862	30	10.48
350	1,270	30	23.6
398*	100	30	300
400	122	30	246
450	1,240	30	24.2
500	2,302	30	13.0
600	4,221	30	7.1

* 398 = resonant frequency

** When computing the current I, the answer will be in amperes, which must be converted to milliamperes.

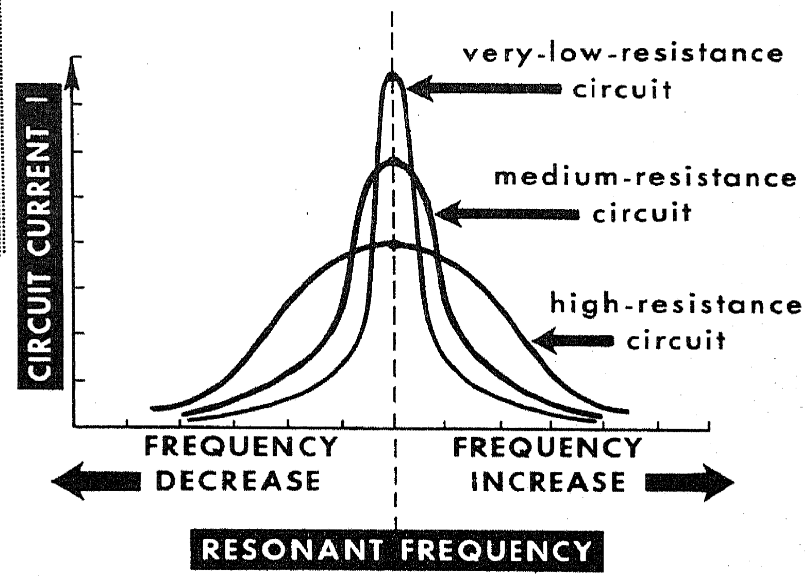


The variation in circuit current with the change in frequency can be shown graphically. It is known as the current curve, or resonance curve, of the circuit. Frequency is shown along the horizontal axis, and the current amplitude is scaled along the vertical axis. Note that while maximum current flows at the resonant frequency, some current flows at frequencies off resonance. The high current at one single frequency (resonance) and reduced currents at other frequencies can be interpreted as discrimination by the circuit against frequencies other than the resonant frequency. This action of the circuit is the basis of its use.

The Effect of Resistance

We have explained the action of resistance in the series L-C-R circuit as being the factor which contributes to the control of the current. When we think of a circuit as a frequency-resonant system, we must take note of other effects of circuit resistance – that is, its effect on the circuit current or resonance curve. The lower the circuit resistance when $X_L = X_C$, the higher is the circuit current. If the current is shown graphically as a resonance curve, the peak of the current curve will be higher with lower circuit resistance. For instance, if the resistance of the circuit discussed on the preceding page were 10 ohms instead of 100 ohms, the resonant frequency current would have been 3 amperes instead of 300 milliamperes.

The lower the resistance of the series-resonant L-C circuit...



...the greater the change in current per unit change in frequency -- hence the SHARPER the circuit tuning.

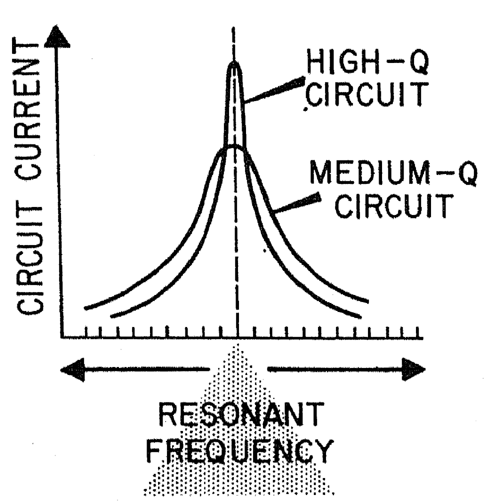
Equally important is another effect – this time on the shape of the resonance curve. The lower the circuit resistance, the greater the relative control of the current by the net reactance of the circuit as the frequency changes from resonance to off resonance. The result is that the circuit current falls rapidly as the frequency changes, and the sides of the resonance curve are steep. Conversely, the higher the circuit resistance, the less will be the relative control of the circuit current by the net reactance as the frequency is changed. The result is a broadening of the resonance curve. The steeper the sides of the resonance curve, the sharper will be the tuning of the circuit. This means that the circuit discriminates more against frequencies that are off resonance. In other words, the lower the resistance of a circuit, the sharper its performance as a frequency selector. Note the difference in peak amplitudes at the resonant frequency for the three conditions of resistance in the circuit. Note also the limited change in current for resonance and off resonance conditions with high resistance, compared with the large change in current for resonance conditions when there is very low resistance.

Effective Resistance and Q

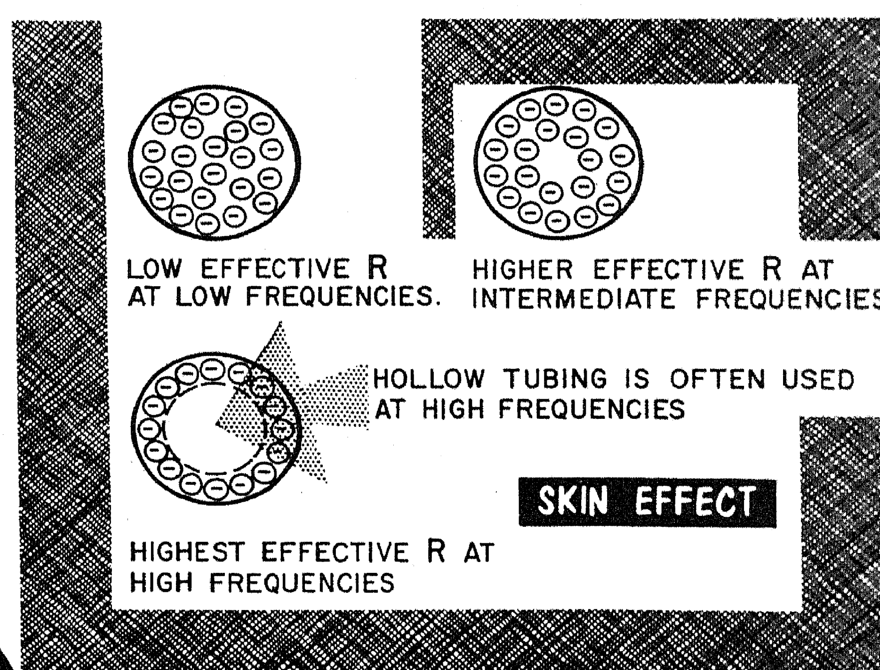
All inductive windings, i.e., all coils which demonstrate the property of inductance, can be rated in terms of a figure of merit called Q. It is expressed as a number – for instance, a Q of 100. The higher the number, the more effectively the coil acts as an electrical device. When stated as an equation: $Q = X_L/R$, or $2\pi fL/R$.

In this equation, R has a somewhat different meaning than d-c resistance. Here, it stands for effective resistance, a term used to describe all forms of resistance (not reactance) which tend to retard the flow of current in a circuit. At d-c and low frequencies, the only opposition to the flow of current is the ohmic resistance of the wire. But at high frequencies, another kind of resistance – a-c resistance – appears and adds to the effective resistance of the coil, lowering its Q. This resistance is produced by skin effect. When skin effect is present, the electron flow is redistributed over the conductor cross section so as to make most of the electrons flow where they are encircled by the least number of magnetic flux lines. Because a greater number of flux linkages exists in the center of a conductor, the inductance at the center is greater than near the surface. Thus, at high frequencies, the reactance is great enough to affect the flow of current, most of which flows along the surface of the conductor. Therefore, the effective resistance is increased, since, in effect, the useful cross section of the conductor is greatly reduced.

Skin effect can be minimized by forming the conductor from a large number of small enameled wires connected in parallel at their ends, but insulated from each other throughout the rest of their length and interwoven. Each conductor will then link with the same number of flux lines as every other one, and the current will divide evenly among the strands, thus greatly increasing the useful cross section of the wire. A stranded cable like this is called a Litz conductor.



The higher the Q, the sharper the resonance curve



SKIN EFFECT

Time constant in an R-L circuit is a means of comparing how rapidly the current in one R-L circuit rises to a given value relative to the current in another R-L circuit. It is expressed in seconds and is calculated by the equation t (seconds) = L (henries)/R (ohms).

Time constant in an R-C circuit is calculated by the equation: t (seconds) = R (ohms) × C (farads).

In an R-C circuit, voltage rises to 63.2% of its maximum value and falls 63.2% from its maximum value in the first unit of time.

If X_L is greater than X_C, the circuit acts inductively and the current lags the applied voltage (E) by the phase angle θ. Tan $\theta = X_L - X_C / R$.

If X_C is greater than X_L, the circuit acts capacitively and the current leads the applied voltage (E) by the phase angle θ. Tan $\theta = X_C - X_L / R$.

If X_L is equal to X_C, the circuit is resistive and the current is in phase with the applied voltage (E).

The resonant frequency of the circuit is the particular frequency at which the circuit responds best.

The frequency at which an L-C-R circuit resonates is found by the formula $f = 1/2\pi\sqrt{LC}$.

At resonance, the reactances cancel, current is a maximum, impedance is a minimum, and the phase angle is 0°.

At resonance, the voltages across the reactances are maximum, and circuit current is maximum.

In a series L-C-R circuit, the voltages across the reactive elements are 180° out of phase and may be subtracted directly: $E = \sqrt{E_R^2 + (E_L - E_C)^2}$; $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Q is a measure of the selectivity of a circuit, and varies inversely with the resistance.

The Q of a series resonant circuit is the ratio of the inductive reactance to the effective resistance, and is equal to X_L/R.

In considering the Q of a coil, it is important to consider skin effect – an a-c resistance which at high frequencies adds to the effective resistance of a coil, causes losses, and lowers the Q.

REVIEW QUESTIONS

1. Give the formula for finding the impedance of an R-L circuit and of an R-C circuit?
2. What is the formula for finding the impedance of a series L-C-R circuit?
3. What determines the voltage drop across any single element in an a-c circuit?
4. In an L-C-R circuit, what is the phase relationship between current and voltage when X_L is equal to X_C.
5. What is the phase relationship between the voltages across the reactive elements in a series L-C-R circuit?
6. Give the formula for calculating the resonant frequency of a circuit.
7. Name the conditions present in a series resonant L-C circuit.
8. What is the relationship between current and impedance at resonance?
9. What is the Q of a resonant circuit? How is it determined?
10. What effect does the resistance in a circuit have upon the frequency of resonance?
11. In a series resonant circuit, what is the relationship between the voltage across either reactance and the applied voltage?

Branch Currents in the Basic Parallel L-C Circuit

A capacitor and an inductor connected in parallel across a voltage source make up a basic parallel L-C circuit. Since they are in parallel, the applied voltage appears across both L and C. We refer to the voltage across L as E_L and across C as E_C.

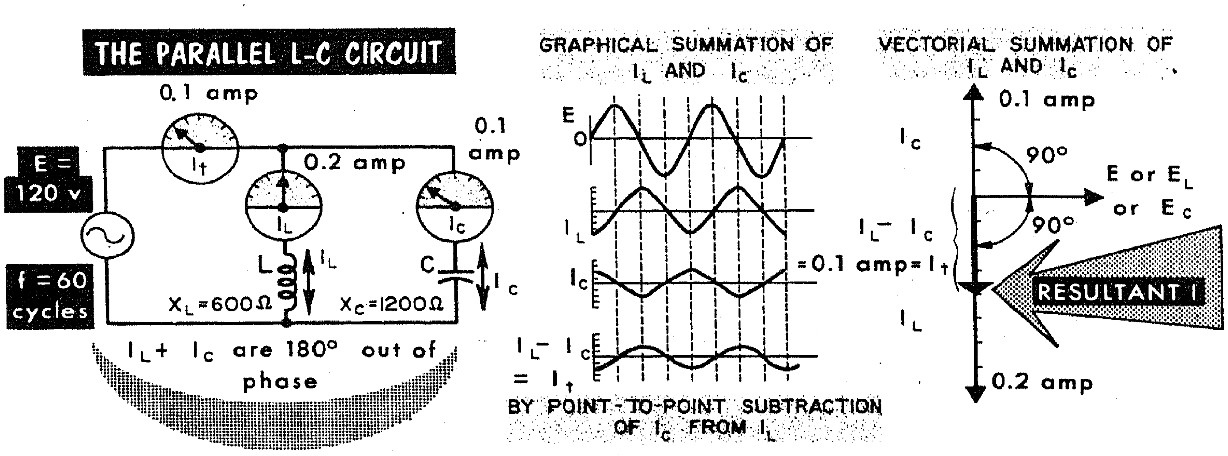
Assume that a parallel L-C circuit is made up of pure inductance and capacitance. The inductance is of such an amount as to present an X_L of 600 ohms; the capacitance is such as to present an X_C of 1200 ohms. The applied voltage is 120 volts at 60 cycles. Thus, the two branch currents are:

$$I_L = \frac{E}{X_L} = \frac{120}{600} = 0.2 \text{ ampere or 200 milliamperes}$$

$$I_C = \frac{E}{X_C} = \frac{120}{1200} = 0.1 \text{ ampere or 100 milliamperes}$$

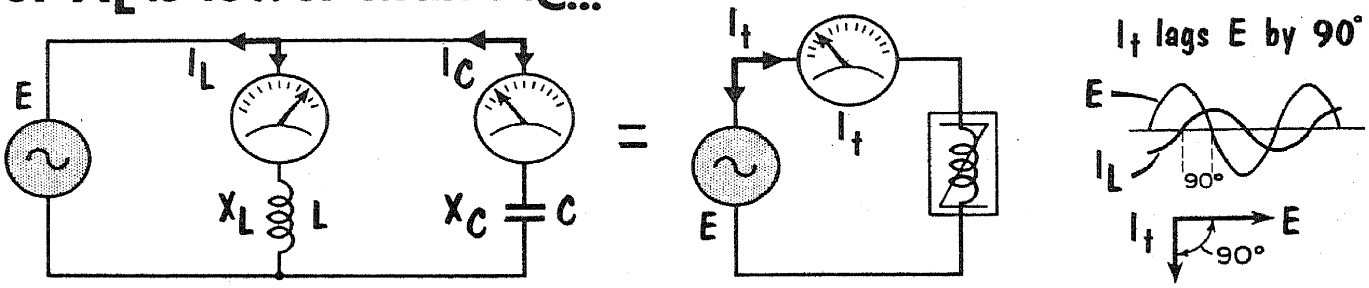
Note that the inductive branch presents the lesser amount of reactance; hence, it passes the greater amount of current.

Because the branch currents differ in phase by 180°, the line current in the parallel L-C circuit is determined by vectorial addition of the branch currents. When arranging the vectors, the applied voltage (identical to E_L and E_C) has the same phase across each branch; hence, it is suitable for use as the reference vector. With the inductive current I_L lagging E_L by 90°, the I_L vector is positioned 90° behind the voltage vector. The length of the I_L vector is determined by using any desired scale compatible with the value. The capacitive current I_C leads E_C by 90°; hence, the I_C vector leads the reference voltage vector by 90°. The scale used for vector I_C must be the same as for vector I_L. The current-voltage relationships shown establish the two currents as being 180° out of phase. The resultant of two vectors 180° out of phase is the difference between their magnitudes. So we subtract the smaller vector I_C from the larger vector I_L. The resultant is the total line current ($I_t = I_L - I_C$). The line current I_t equals $0.2 - 0.1 = 0.1$ ampere. Since the inductive current I_L is the predominant current in the parallel network, the resultant current I_t has the same phase as the original I_L; that is, it lags the applied voltage by 90°.

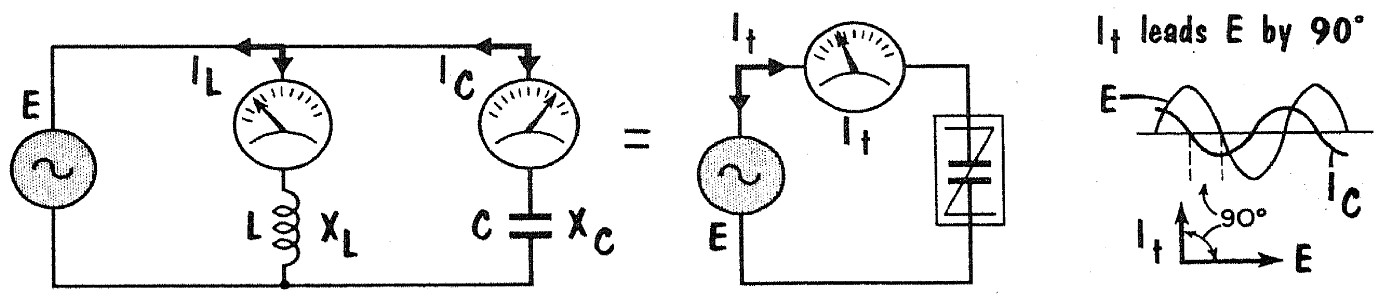


Branch Currents and Circuit Impedance

When I_L is greater than I_C, or X_L is lower than X_C... ...the circuit behaves as an inductance. Z is inductive.



When I_C is greater than I_L, or X_C is lower than X_L... ...the circuit behaves as a capacitance. Z is capacitive



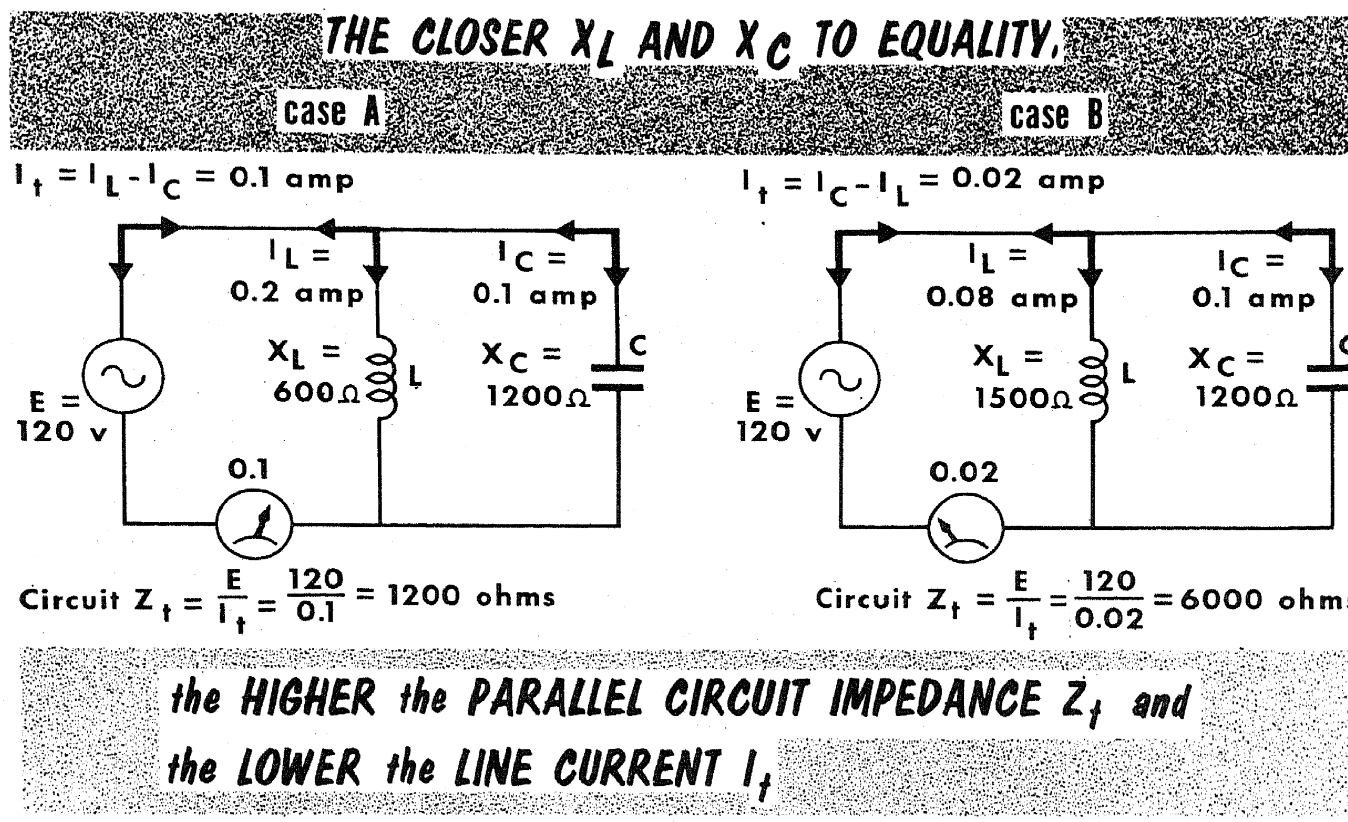
There are several interesting aspects of the parallel L-C circuit which we shall discuss, but first we must establish the total circuit impedance Z_t. Since we know that line current I_t equals 0.1 ampere and applied voltage equals 120 volts, the circuit impedance Z_t equals E/I_t. Substituting the appropriate numbers in the equation:

$$Z_t = \frac{120}{0.1} = 1200 \text{ ohms (inductive)}$$

The voltage source "looking" into the parallel circuit "sees" an impedance of 1200 ohms. The reference to inductive for the impedance has a meaning similar to that in the series L-C circuit; namely, the behavior of the impedance as an inductance or as a capacitance. The impedance of the parallel L-C circuit can be inductive or capacitive; this is determined by which form of opposition to the current flow is most prominent in the parallel network (assuming that X_L does not equal X_C). The predominant branch reactance is the lesser one, since it permits the greater amount of branch current to flow. Therefore, it is most prominent in the resultant line current. In this way, the lesser reactance determines the overall behavior of the circuit, as well as the phase relationship between line current and applied voltage. The line current can have two relationships relative to the applied voltage — leading or lagging. (Later on, you will learn of a third possible identity, this being the in-phase condition when the L-C circuit is resonant and behaves like a resistance.)

Line Current and Circuit Impedance

The branch and line currents have been established in circuit A and are: I_L = 0.2 ampere; I_C = 0.1 ampere; I_t = 0.1 ampere; Z_t = 1200 ohms. Line current I_t is seen to be less than one of the branch currents. This is not unusual in an a-c circuit when the parallel network consists of L and C. The currents flowing through the branches are 180° out of phase with each other; therefore, they tend to cancel in the path which carries the two currents. If the two branch currents differ greatly, as in case A, the line current I_t is less than the higher of the two branch currents; if the two branch currents do not differ by too much, the current can be less than either of the two branch currents. The example which follows illustrates this point.

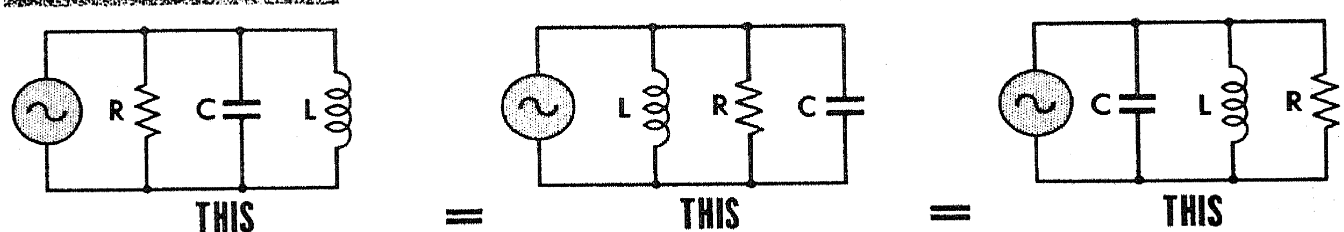


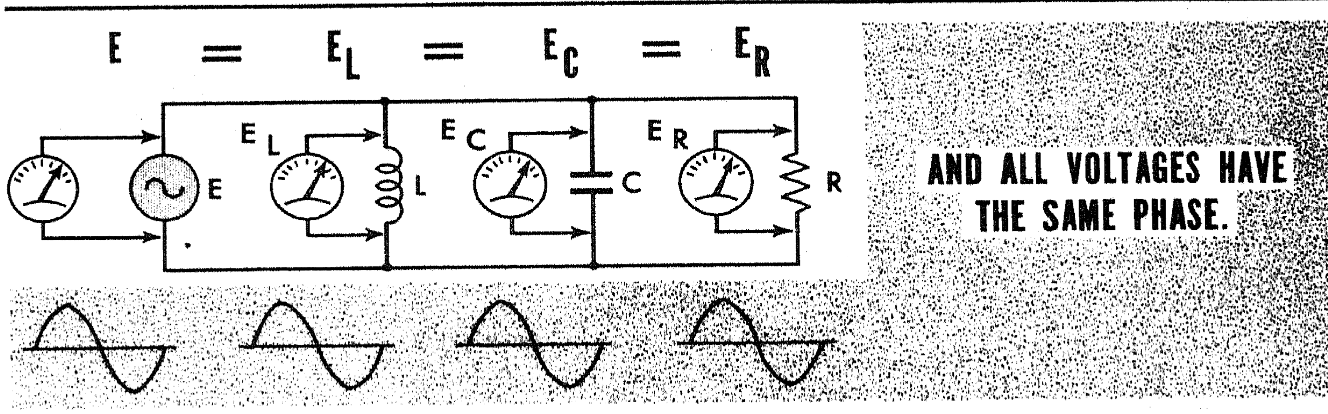
Assume a second parallel L-C circuit (B) in which E = 120 volts at 60 cycles, X_L = 1500 ohms, and X_C = 1200 ohms. Applying Ohm's law for current: $I_L = E/X_L$ = 120/1500 = 0.08 ampere and $I_C = E/X_C$ = 120/1200 = 0.1 ampere, from which $I_t = I_C - I_L$ = 0.1 - 0.08 = 0.02 ampere, an amount less than either branch current. Then the circuit impedance $Z_t = E/I_t$ = 120/0.02 = 6000 ohms. If now we study closely the constants of circuits A and B, two extremely important situations are brought to light. By comparing the two values of line current and the respective reactances in the circuits, it is seen that the closer to equality X_L and X_C are, the less the line current; the more one reactance differs from the other, the greater the line current. Since line current I_t is the denominator in the equation for the circuit impedance, the closer to equality X_L and X_C are for any given voltage, the higher the circuit impedance; the greater the difference between X_L and X_C, the lower the circuit impedance. These two electrical conditions are important to remember.

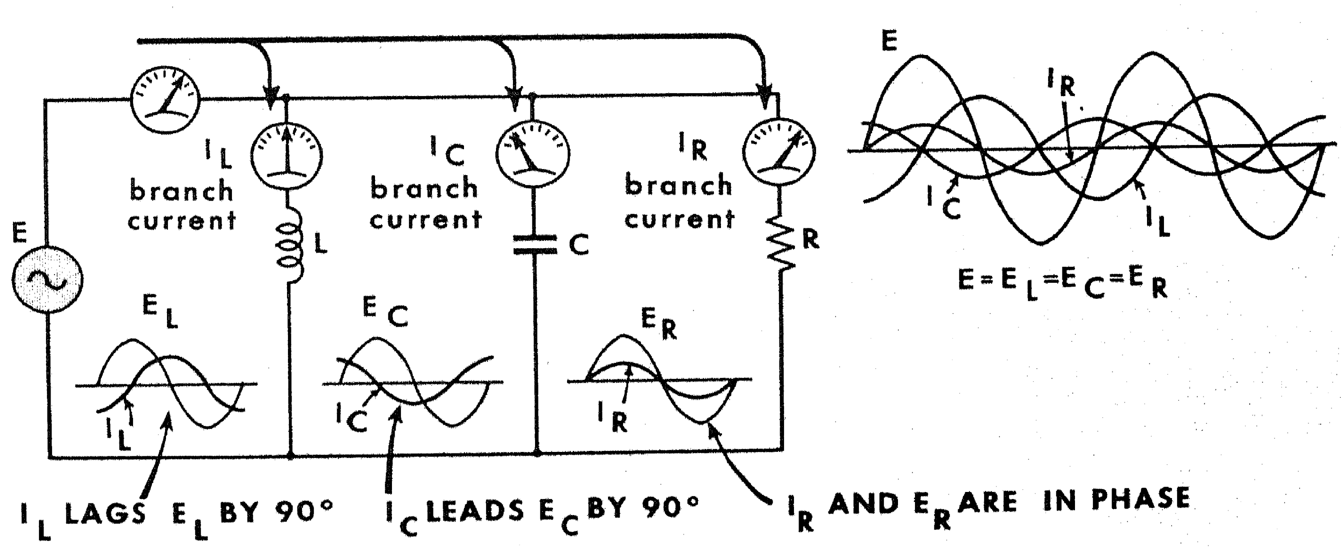
The Basic Parallel L-C-R Circuit (General)

Let us now consider an L-C circuit with parallel resistance R added. Elements L and C remain "pure" inductance and capacitance. The addition of the parallel resistance does not change the individual actions of L and C. Now $E = E_L = E_C = E_R$; therefore, all voltages are in phase. R is simply another branch across which the applied voltage appears as E_R, and through which a current I_R equal to E_R/R flows. Branch current I_R is not influenced by the presence of I_L or I_C. There is, however, a difference in the phase relationship between the voltage and current associated with R. Voltage E_R and current I_R are in phase, while voltage E_L leads current I_L by 90°, and voltage E_C lags current I_C by 90°. Thus, the resistive current is 90° out of phase with the inductive and capacitive currents, which are 180° out of phase with each other.

The order of appearance of the circuit elements in a schematic is unimportant







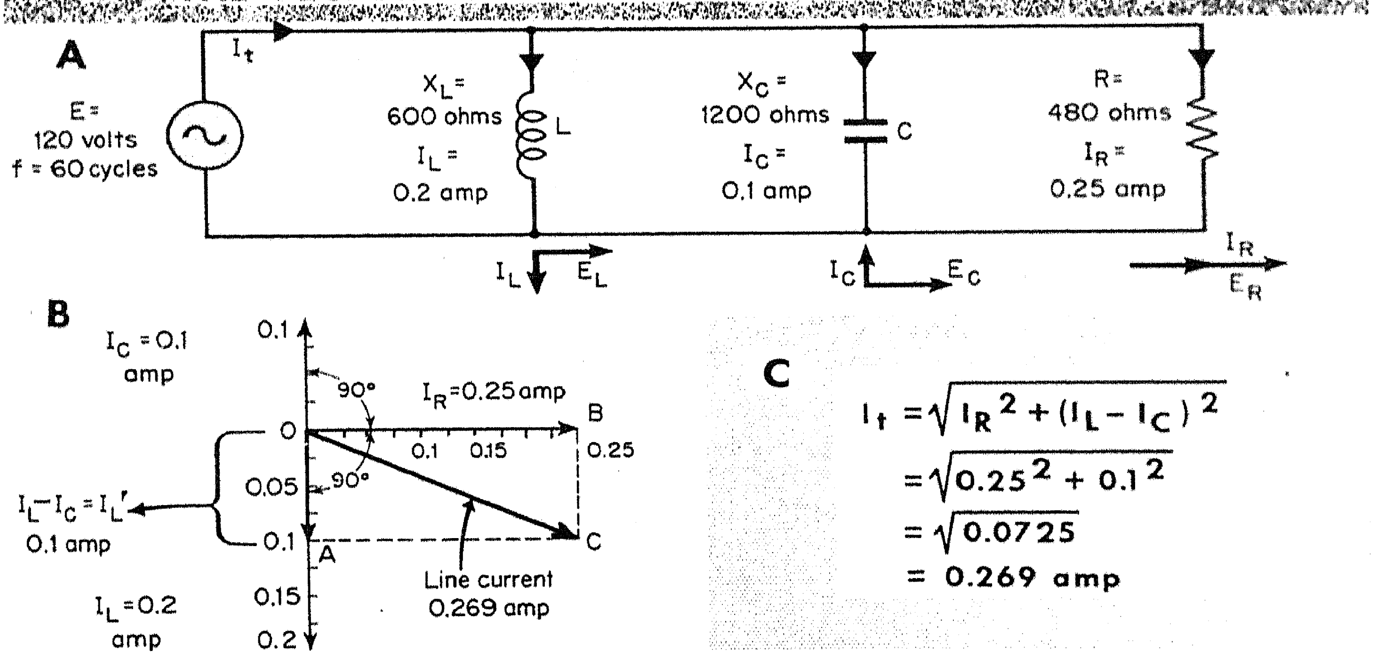
Branch Currents and Line Current

Let us say that a parallel L-C-R circuit (A) contains an inductive branch in which X_L equals 600 ohms, a capacitive branch with X_C equal to 1200 ohms, and a resistive branch with R equal to 480 ohms. The applied voltage is 120 volts at 60 cycles; hence, E_L equals E_C equals E_R equals 120 volts. $I_L = E_L/X_L$ = 0.2 ampere; $I_C = E_C/X_C$ = 0.1 ampere; and $I_R = E_R/R$ = 0.25 ampere.

When the three branch currents are known, the line current is determined by vectorial addition (B). Because the current and voltage across the resistance are in phase, I_R is used as the reference vector. The capacitive branch current leads its voltage (E_C) by 90°; hence, it is positioned 90° ahead of the I_R vector. The inductive current lags its voltage (E_L) by 90°; hence, it is positioned 90° behind I_R. The I_C and I_L vectors are 180° apart; therefore, their resultant is the difference between them, or $I_L - I_C$ = 0.1 ampere. Current I_L is greater than current I_C; therefore, the difference between them has the same direction as I_L. This difference current is 90° out of phase with I_R. The two currents can be added by the parallelogram method (B), or by the equation method (C). Completing the parallelogram and drawing the diagonal OC furnishes the answer. With all vectors similarly calibrated, the dimension of OC can be read directly as the amount of line current. The answer is 0.269 ampere. The same answer is arrived at by the equation method (C). With both inductive and resistive current present in the line current, the line current therefore lags the applied voltage by the angle θ. Using a protractor on the vector presentation shows the angle of lag to be 21.8°. Expressed mathematically, the lag of the line current is:

$$\tan\theta = \frac{I_L}{I_R} = \frac{0.1}{0.25} = 0.4 \text{ or } \theta = 21.8°$$

THE VECTOR ADDITION OF THE BRANCH CURRENTS



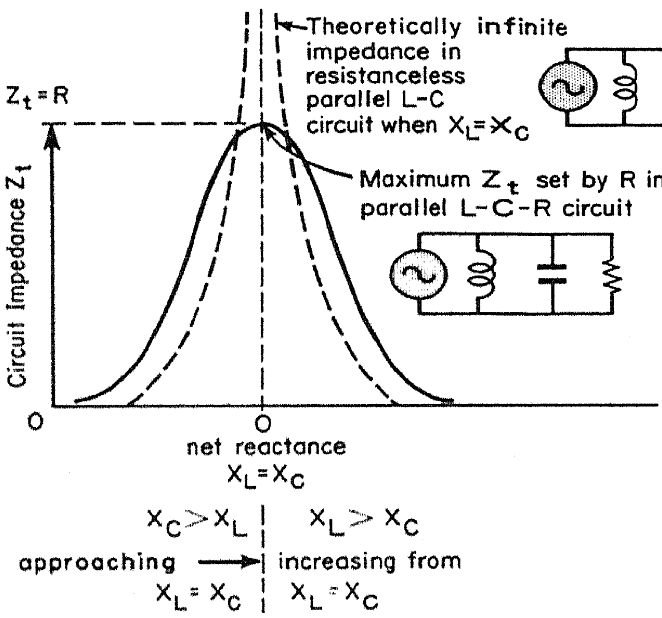
Line Current and Circuit Impedance

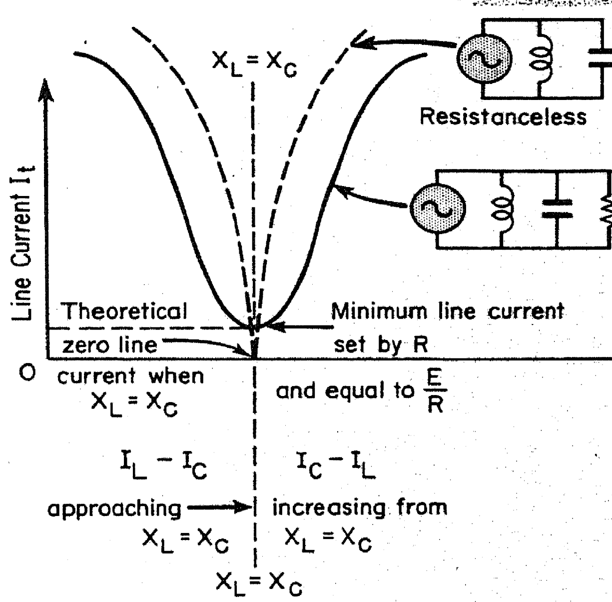
Since we know the line current (developed on the preceding page), we can determine the circuit impedance Z_t.

$$Z_t = \frac{E}{I_t} = \frac{120}{0.269} = 446 \text{ ohms}$$

Comparison of the line current with the individual branch currents shows that the line current is greater than the highest branch current. The impedance is less than the lowest ohmic value among the branches. The calculated circuit impedance $Z_t = 446$ ohms compared to $R = 480$ ohms shows this to be true. Although it is true in this instance, the circuit impedance of a parallel L-C-R circuit is not always less than the lowest ohmic value among the branches. For example, imagine I_L equal to I_C because X_L equals X_C at some frequency. Then the two reactive currents cancel each other as far as line current is concerned, and the only current appearing in the line current is the one flowing through the resistive branch. When this is true, circuit impedance Z_t equals R for all practical purposes.

THE PARALLEL R IN THE PARALLEL L-C-R CIRCUIT LIMITS THE MAXIMUM CIRCUIT IMPEDANCE AND SETS THE MINIMUM CURRENT ABOVE ZERO WHEN $X_L = X_C$





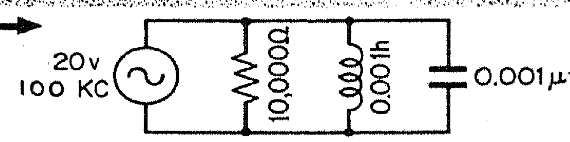
But a much more important point is that when R is in parallel with paralleled L-C, the circuit impedance can never rise higher than the ohmic value of R, even though the individual reactances X_L and X_C may be very much higher. On page 2-114, we found that when X_L and X_C approached equality, the circuit impedance of the resistanceless circuit increased greatly. With parallel R present, such an increase cannot take place. The parallel R prevents the parallel L-C circuit from presenting a very high impedance when $X_L = X_C$. This situation is important when working with parallel-resonant L-C circuits, as explained later.

Line Current and Circuit Impedance (Cont'd)

To emphasize the action of R in a parallel-connected L-C-R circuit, we examine two more sets of circuit constants.

The presence of the parallel-connected resistive branch (R = 10,000 ohms) appears to contribute very little to the operation of the circuit. When the difference between the inductive and capacitive branch currents is much greater than the resistive branch current, the R branch has very little effect on the circuit action. As the figures show with R in the circuit, the line current I_t is 0.018 ampere and is lagging the applied voltage by 83°. The circuit impedance Z_t is 1111 ohms. With R removed, the I_t would be 0.0185 ampere and lagging the applied voltage by 90° – very little difference. The circuit impedance would be 1081 ohms – again, very little difference. It would be good practice to solve for the circuit current and impedance with R out of the circuit.

R HAS LITTLE EFFECT ON THIS CIRCUIT ($I_L - I_C$ MUCH GREATER THAN I_R)



$X_L = 2\pi f L$
$= 6.28 \times 100{,}000 \times 0.001$
$= \boxed{628}$ ohms

$X_C = \frac{1}{2\pi f C}$
$= \frac{1}{6.28 \times 100{,}000 \times 0.000000001}$
$= \boxed{1594}$ ohms

$R = \boxed{10{,}000}$ ohms

$I_L = \frac{E_L}{X_L} = \frac{20}{628} = \boxed{0.031}$ ampere

$I_C = \frac{E_C}{X_C} = \frac{20}{1594} = \boxed{0.0125}$ ampere

$I_R = \frac{E_R}{R} = \frac{20}{10{,}000} = \boxed{0.002}$ ampere

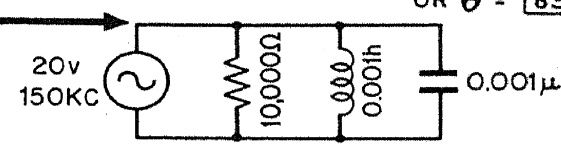
$I_t = \sqrt{I_R^2 + (I_L - I_C)^2}$
$= \sqrt{.002^2 + (.031 - 0.0125)^2}$
$= \boxed{.018}$ ampere

$Z_t = \frac{E}{I_t} = \frac{20}{0.018} = \boxed{1111}$ ohms

$\tan\theta = \frac{I_L - I_C}{I_R} = \frac{0.0185}{0.002} = 9.25$

OR $\theta = \boxed{83^\circ}$

R HAS CONSIDERABLE EFFECT ON THIS CIRCUIT ($I_L - I_C$ LESS THAN I_R)



$X_L = 2\pi f L = \boxed{942}$ ohms

$X_C = \frac{1}{2\pi f C} = \boxed{1063}$ ohms

$R = \boxed{10{,}000}$ ohms

$I_L = \boxed{0.020}$ ampere

$I_C = \boxed{0.0188}$ ampere

$I_R = \boxed{0.002}$ ampere

$I_t = \sqrt{I_R^2 + (I_L - I_C)^2}$
$= \boxed{0.00297}$ ampere

$Z_t = \boxed{6730}$ ohms

$\tan\theta = \frac{I_L - I_C}{I_R} = \frac{0.0022}{0.002} = 1.1$

OR $\theta = \boxed{47.7^\circ}$

Let us now change the operating conditions by raising the frequency of the applied voltage to 150 kc. It changes the values of X_L and X_C, making the difference between I_L and I_C nearly I_R. X_L almost equals X_C. Hence, the presence of the R branch displays a major effect. In the absence of R, the circuit impedance Z_t would be 9090 ohms, and the line current would lag the applied voltage by 90°. But with R present, Z_t = 6730 ohms and the line current lags the applied voltage by only 47.7°, a major change. It is evident, therefore, that the parallel R displays its greatest effect in the vicinity of, and at, resonance.

Calculating the Resonant Frequency

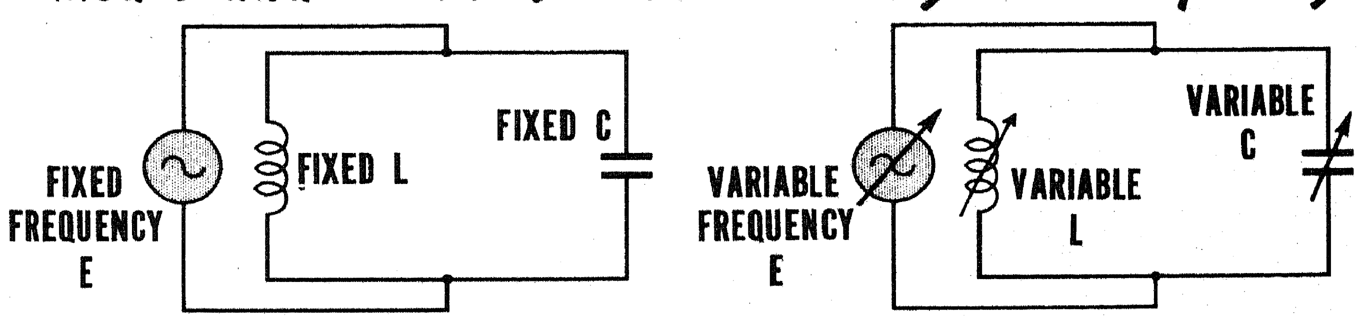
The parallel resonant L-C circuit differs from the ordinary parallel L-C circuit in one respect — resonance. Resonance occurs when the inductive reactance ($2\pi fL$) equals the capacitive reactance ($1/2\pi fC$), or $X_L = X_C$. For any given fixed amount of L and C, parallel resonance occurs at only one frequency (the same as in the series-resonant L-C circuit). The frequency of resonance is expressed by $f = 1/2\pi\sqrt{LC}$, or $1,000,000/2\pi\sqrt{LC}$. Both equations are exactly the same as used for the series-resonant circuit and both yield the same result. They differ only in the units which are used for L and C — the first is in henries and farads; the second is in microhenries and microfarads. To illustrate the application of the two equations, assume a parallel L-C circuit in which L = 100 microhenries (or 0.0001 henry), and C = 100 micromicrofarads (or 0.0001 microfarad, or 0.0000000001 farad). Using each equation for the solution, the resonant frequency (often indicated f_o) is:

$$\begin{aligned}
\text{f cycles} &= \frac{1}{6.28 \times \sqrt{0.0001 \times 0.0000000001}} \\
&= \frac{1}{6.28\sqrt{0.00000000000001}} \\
&= \frac{1}{6.28 \times 0.0000001} \\
&= \frac{1}{0.000000628} \\
&= 1{,}592{,}197 \text{ cycles} \\
&= 1.5922 \text{ mc}
\end{aligned}$$

or

$$\begin{aligned}
\text{f cycles} &= \frac{1{,}000{,}000}{6.28\sqrt{100 \times 0.0001}} \\
&= \frac{1{,}000{,}000}{6.28\sqrt{0.01}} \\
&= \frac{1{,}000{,}000}{6.28 \times 0.1} \\
&= \frac{1{,}000{,}000}{0.628} \\
&= 1{,}592{,}197 \text{ cycles} \\
&= 1.5922 \text{ mc}
\end{aligned}$$

Fixed L and C Can Resonate at Only One Frequency



Parallel L or C Can Be Tuned to Resonate at a Variety of Frequencies

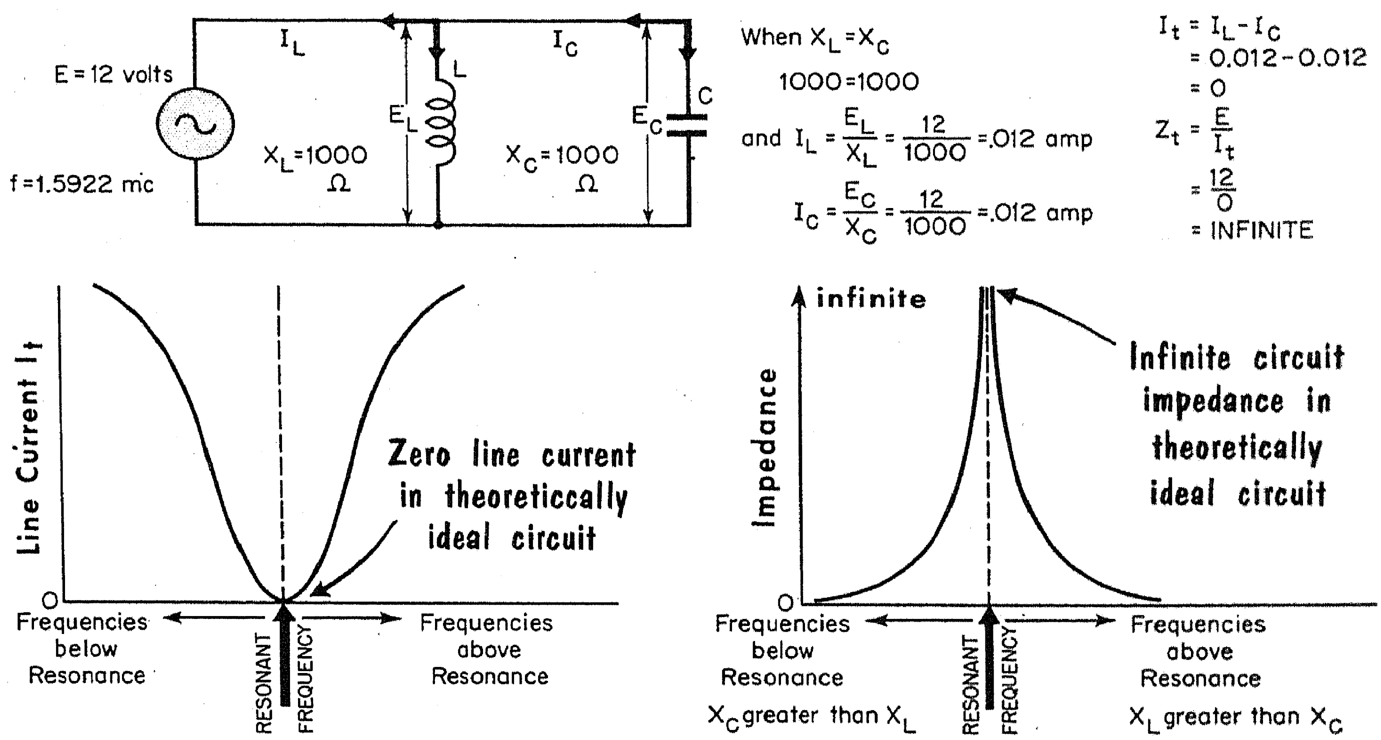
The values of L and C in a parallel-resonant circuit may be precalculated so as to be resonant to a single frequency. On the other hand, the requirement may be for resonance over a band of frequencies. This condition is provided for by making either L or C (sometimes both) variable between a minimum and a maximum amount. Each setting within the range of variation then affords a different resonant frequency. For instance, if in the example above, L were fixed at 100 μh while C were variable between a minimum of 20 μμf and a maximum of 100 μμf, the circuit could be resonated individually to all frequencies between a low of 1.5922 mc to a high of 3.56 mc.

Line Current and Impedance

You have learned that the series resonant L-C circuit offers minimum impedance at the resonant frequency. The behavior of the parallel resonant L-C circuit is the exact opposite — at resonance, the circuit impedance is maximum. When equality between X_L and X_C is reached, the respective branch currents I_L and I_C are equal. Since the line current I_t has been established as the difference between the branch currents, and the difference between two equal amounts is zero, the parallel resonant L-C circuit shows no line current. From the viewpoint of the voltage source, it is applying voltage across a circuit having infinite impedance.

Let us illustrate this situation. E = 12 volts at 1.5922 mc; L = 100 microhenries, for which X_L is 1000 ohms; C = 0.0001 microfarad, for which X_C = 1000 ohms. Then:

PARALLEL RESONANCE PRODUCES MINIMUM LINE CURRENT and MAXIMUM IMPEDANCE

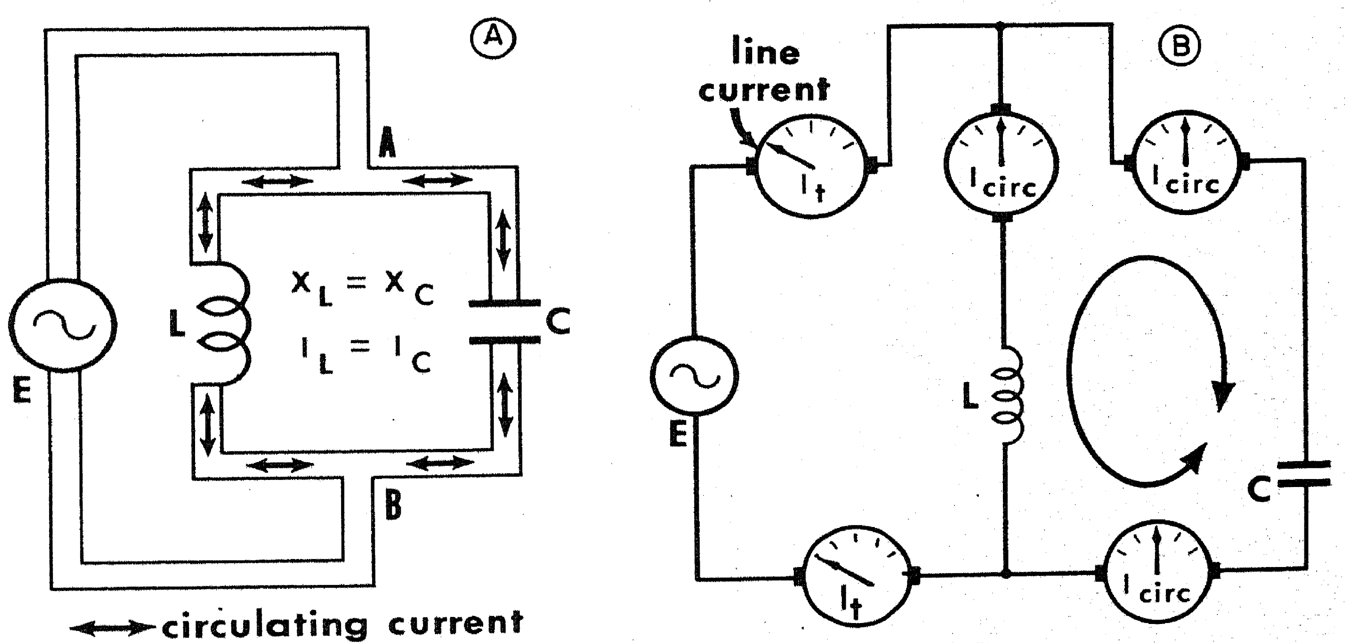


The ideal case of zero line current and infinite circuit impedance in the resonant parallel L-C circuit is not realizable in practice; nevertheless, we assume that they are attainable because they enable us to establish clearly the limiting conditions for later comparison with the practical circuit. There is similarity in behavior between the two. If we show a graphic picture of the change in line current with change in frequency for the ideal case, the line current is seen to be zero at resonance, and increases for frequencies on both sides of resonance. The graph of circuit impedance vs. frequency shows infinite impedance at resonance and reduced impedance off resonance. With equality between the branch reactances and branch currents, infinite impedance is interpreted as infinite resistance.

Circulating Current

The infinite impedance of the resistanceless parallel resonant L-C circuit should be understood as being the impedance "seen" by the voltage source as it "looks" into the parallel L-C network. The voltage source "sees" an open circuit. Also, the zero line current condition should not be mistaken for zero current conditions inside the parallel network. Interestingly enough, a significant current flow situation prevails inside the parallel L-C circuit. Let us examine the current conditions inside the parallel L-C network at resonance.

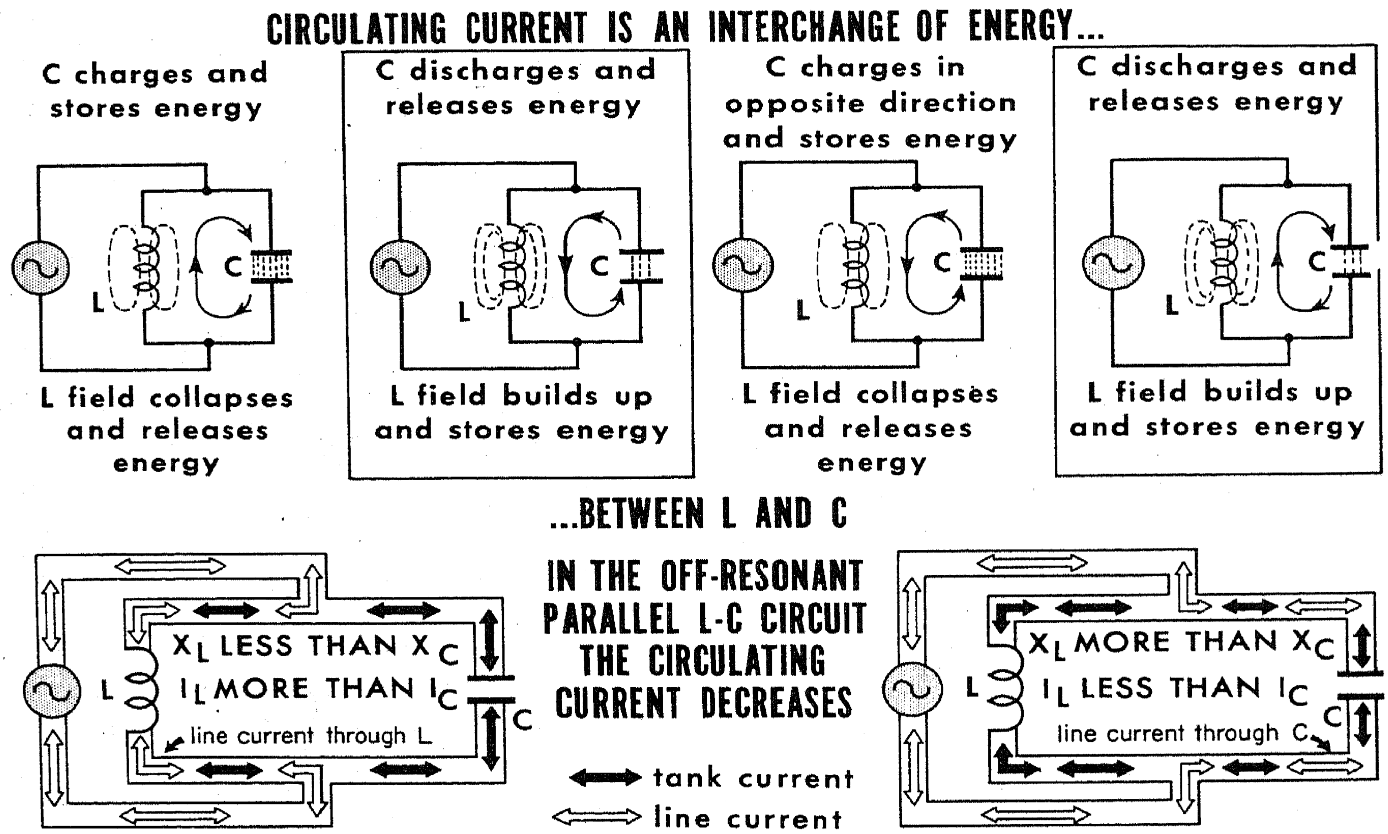
The circuit is resistanceless, $X_L = X_C$, and $I_L = I_C$. The two branch currents are 180° out of phase as the result of their 90° lag and lead relationships with their respective voltages, E_L and E_C. Examination of the flow of the two branch currents shows that they move in opposite directions through their respective branch elements. When the polarity of the applied voltage changes, the two branch currents reverse their directions of flow. Now, if we take points A and B as references, and examine the directions of the two branch currents, they are seen to have like directions. All the current which flows into A moves away from A; all the current which flows into B moves away from B. In other words, the two branch currents have become one and the same current, as far as to-and-fro circulation through L and C inside the parallel-connected circuit is concerned. In fact, as far as the circulating current is concerned, the parallel L-C network is really a series circuit, since there is only one path for the circulating current. The circulating current is equal to E_L/X_L or E_C/X_C.



The CIRCULATING CURRENT in the PARALLEL RESONANT CIRCUIT is EVERYWHERE THE SAME in the L CIRCUIT and in the C CIRCUIT

Circulating Current (Cont'd)

What is the importance of the circulating current? The parallel resonant L-C circuit is sometimes called a "tank" or storage circuit. The circulating current charges C, momentarily storing energy in it. When C discharges, the discharge current flows into L and builds a magnetic field, in which electrical energy is stored. When the magnetic field collapses, the current again flows into C, recharging it. Discharging again builds a magnetic field around the coil, thus effecting an interchange of electrical energy between C and L of the parallel resonant L-C circuit. This energy is maximum at resonance; hence, maximum energy is available for transfer to another circuit or to be kept within the circuit for a purpose.



What happens when the circuit is not resonant? Several actions occur. Assume X_L to be greater than X_C. Then I_C is greater than I_L. Suppose that I_C = 200 ma and I_L = 50 ma. We have learned that the line current I_t equals the difference between the two, or in this case 200 - 50 = 150 ma. Circulating current also flows in the nonresonant state of the circuit, but now it is equal to the lesser of the two branch currents – in this case, to I_L = 50 ma. An equal amount of the greater branch current (50 ma) becomes part of the circulating current. The remaining 150 ma of the capacitive branch current is the line current, and flows through the parallel L-C circuit via the capacitive branch. If the situation were reversed and I_L equalled 200 ma and I_C equalled 50 ma, the circulating current would be 50 ma while the line current of 150 ma would flow through the circuit via the inductance. The farther away from resonance, the less the circulating current and the greater the line current.

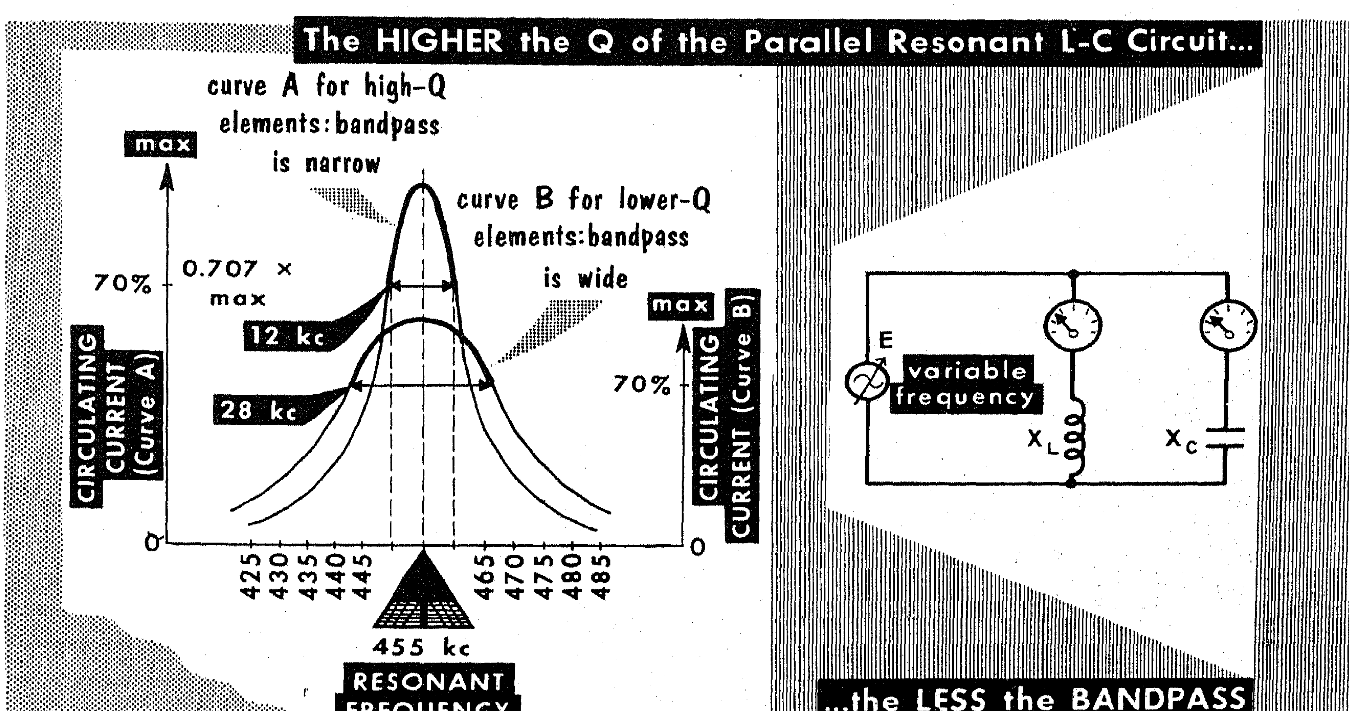
Resonance Curve

Like the series resonant L-C circuit, the performance of the parallel resonant L-C circuit can be portrayed by a resonance curve. A convenient way of preparing this curve is by plotting the circulating current vs. frequency. A current meter is used in each branch circuit.

Current at the resonant frequency is determined first. Both meters will indicate the same amount of current. As the frequency of the applied voltage is lowered, X_L decreases while X_C increases. The inductive branch current thus increases, whereas the capacitive branch current decreases. Since the circulating current has the value of the lesser branch current, the X_C branch meter is used as the current indicator. The lower the frequency relative to resonance, the lower will be the indication on the X_C branch meter. Since the inductive branch current exceeds the capacitive branch current, the parallel current as a whole behaves like an inductance.

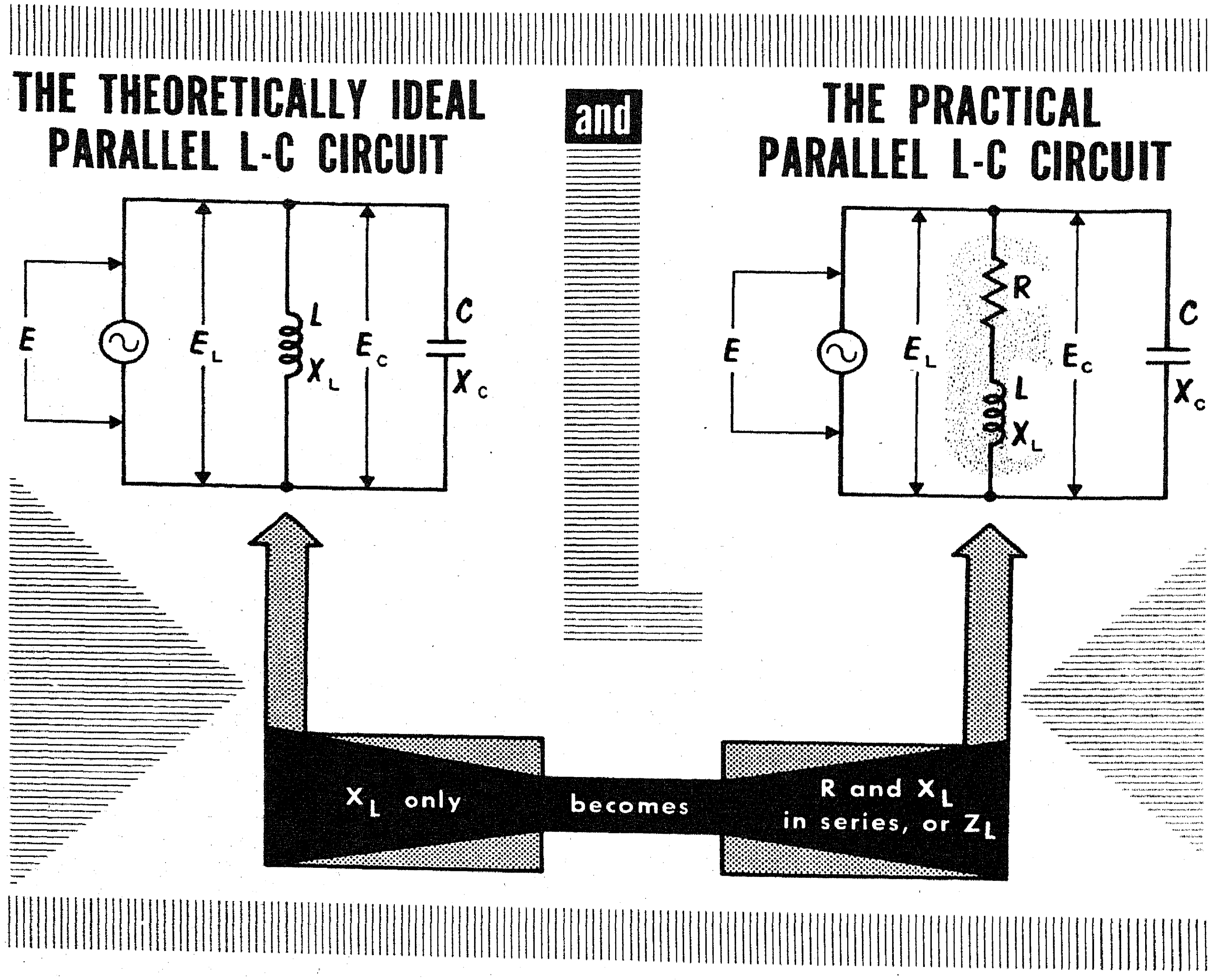
Then, the circulating current is plotted as the frequency is increased above resonance. As frequency increases, X_C decreases and X_L increases. The capacitive branch current now increases, whereas the inductive branch current decreases. The indication on the meter in the X_L branch is plotted for a range of frequencies above resonance. Since the capacitive branch current exceeds the inductive branch current, the parallel circuit behaves like a capacitance.

As in the series resonant circuit, the parallel resonant circuit also affords a selective frequency bandpass. It is the band of frequencies embraced by this curve at a level corresponding to a circulating current of 70% (70.7% exactly) of the maximum circulating current. The higher the circuit Q, the steeper the sides and the narrower the bandpass. The lower the Q, the broader the sides and the wider the bandpass.



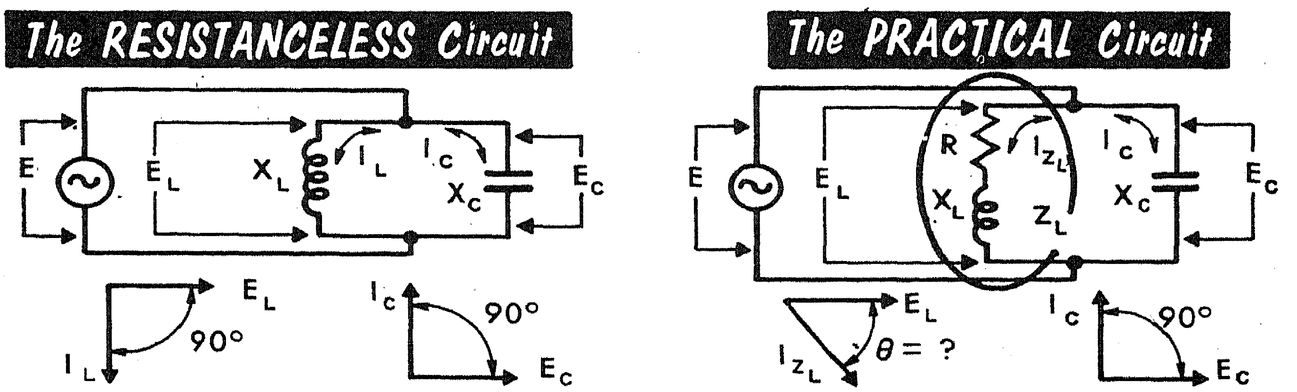
Comparison with Theoretically Ideal Parallel L-C Circuit

The practical parallel L-C circuit differs from the theoretically resistanceless version in one major respect: the presence of resistance. It exists in the inductance, in the capacitance, and in the interconnecting wires. Of these resistance sources, only the resistance contained in the inductance is important, so we disregard the others.



The resistance is treated as if it were a resistance in series with the coil L. The capacitive branch is assumed to consist of X_C only, whereas the inductive branch consists of R in series with X_L. In low-frequency circuits, L could have a great many turns; hence, R could be relatively high. In high-frequency circuits, L usually is small and therefore R is small; in fact, R is often so small that it is completely negligible. Yet we must recognize that R is present, especially (as you will see) when $X_L = X_C$ during resonant conditions. In the theoretically resistanceless circuit, when $X_L = X_C$, the line current is zero and the circuit impedance is infinite. In practical systems using parallel resonant L-C circuits, X_L can equal X_C, yet there is a finite amount of line current flowing due to the resistance in the circuit.

Branch Currents



A

In the theoretically resistanceless parallel resonant circuit, the inductive branch current I_L lags the inductive branch voltage by 90°, the capacitive branch current I_C leads its voltage E_C by 90°, and the line current I_t is zero. In the practical version of the circuit (with R in series with L), the capacitive branch current still leads its voltage E_C by 90°, but the inductive branch no longer is only reactance X_L. Now the branch is impedance Z_L made up of R and X_L in series.

In the study of the behavior of the series R-L circuit, we found that while the current is everywhere the same, the voltage across R is in phase with the current, but leads the current in X_L by 90°. When R is not negligible, the current in the series circuit lags the voltage across the series combination by some amount less than 90°. Applying these conditions to the practical parallel resonant L-C circuit shows that the two branch currents are not 180° out of phase. Resonance occurs, nevertheless because $X_L = X_C$. To determine the inductive branch current, we must first establish the impedance of the circuit. Assume that E = 12 volts, X_L = 775 ohms, R = 300 ohms (a completely unrealistic amount, but one which will illustrate the point), and X_C = 775 ohms. Then:

Inductive branch Z_L

$$Z_L = \sqrt{R^2 + X_L^2} = \sqrt{300^2 + 775^2} = \sqrt{90{,}000 + 600{,}625} = \sqrt{690{,}625} = \boxed{831 \text{ OHMS}}$$

Inductive branch current I_{Z_L}

$$I_{Z_L} = \frac{E_L}{Z_L} = \frac{12}{831} = 0.0144 \text{ ampere} = \boxed{14.4 \text{ MA}}$$

Capacitive branch current I_C

$$I_C = \frac{E_C}{X_C} = \frac{12}{775} = 0.0154 \text{ ampere} = \boxed{15.4 \text{ MA}}$$

B

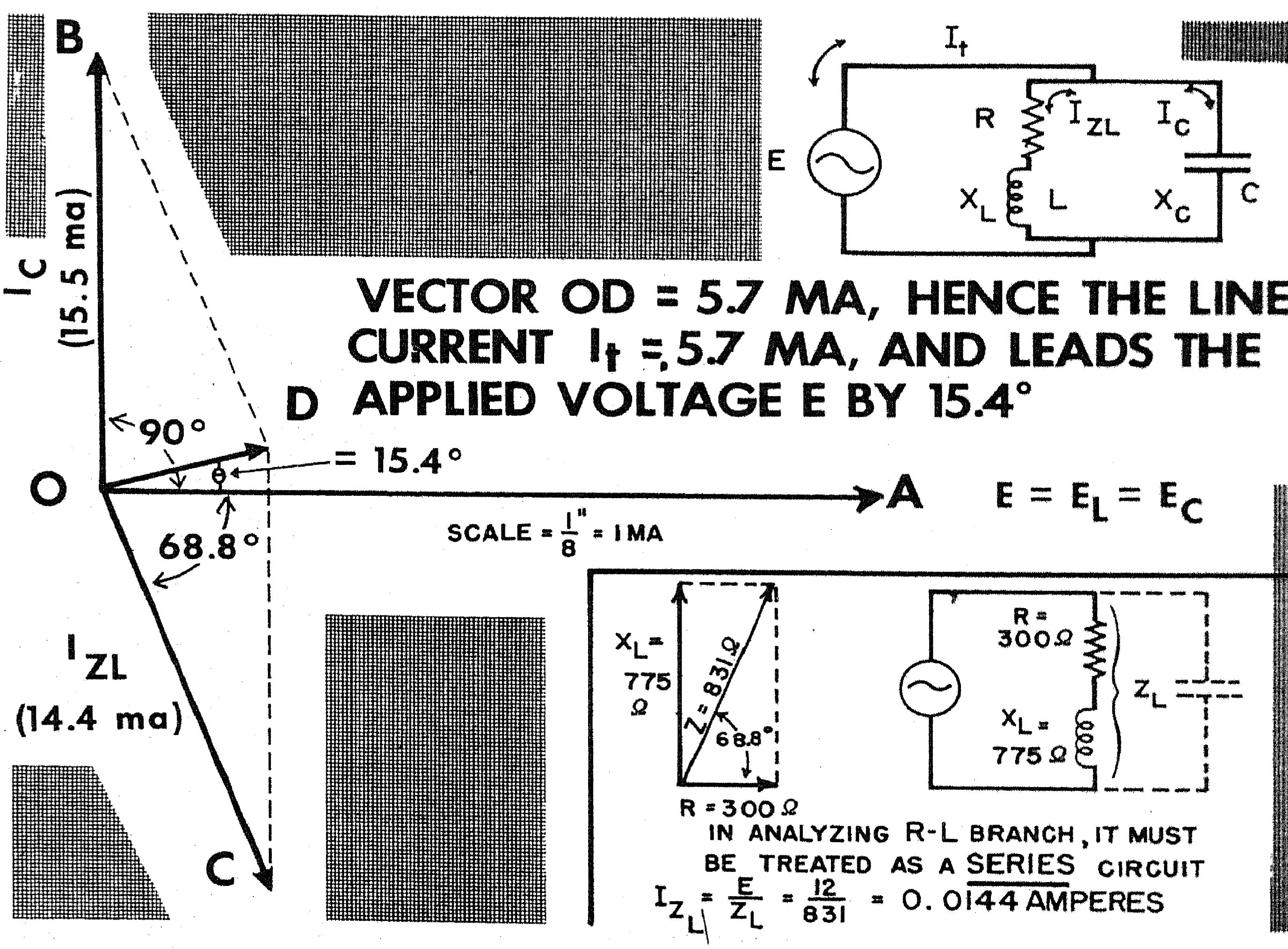
$$\tan\theta = \frac{X_L}{R} = \frac{775}{300} = 2.58$$

which from trigonometric table = **68.8° CURRENT LAGGING**

We now know that Z = 831 ohms; I_{ZL} = 0.0144 ampere (14.4 ma) while I_C = 0.0154 ampere (15.4 ma). The phase angle between the inductive branch current I_{ZL} and its voltage E_L is 68.8°.

Line Current

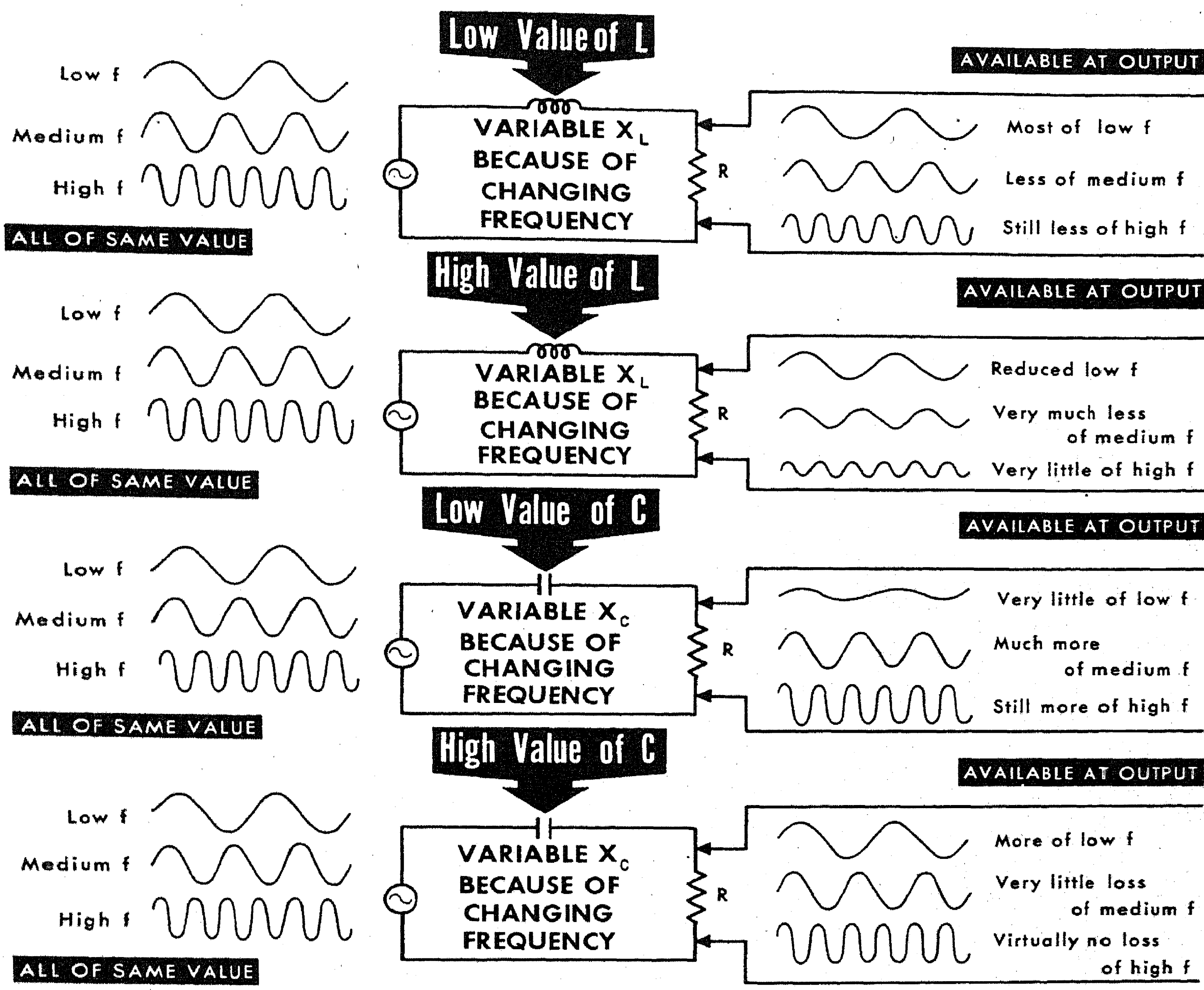
To determine the line current I_t in the practical parallel resonant circuit, we must resort to trigonometric means and use vectors. If we show the answers derived on the preceding page in a vector presentation, the voltages $E = E_L = E_C$ form the reference vector OA. Its length is arbitrary. Then we position the capacitive branch current vector; OB = 15.5 ma, 90° ahead of the reference voltage vector. The length of the capacitive current vector conforms with whatever scale is decided upon. In this case, it is 1/8 inch = 1 ma. The same scale is used for all currents, thereby permitting direct reading of current values from the dimensions of the lines. Using a protractor, we locate the inductive branch current I_{ZL} vector; OC = 14.4 ma, at 68.8° behind the voltage vector. This completes one portion of the branch. Because of the difference in the phase relationships between I_C and E_C, and I_{ZL} and E_L, we cannot subtract I_{ZL} from I_C and assume that the arithmetical difference is equal to the line current. To establish the line current, we must lay down two additional sides – side CD equal and parallel to vector OB, and side BD equal and parallel to vector OC. Now, the diagonal OD represents the line current. By measuring its length, vector OD equals 5.7 ma.



Vector OD is located above the reference voltage vector OA; therefore, I_t leads applied voltage E by angle AOD. Using a protractor, we find it to be 15.4°.

Effects of L and C at Different Frequencies

To aid in understanding the action of the filter circuits, it is best to review some of the effects of C and L at different frequencies. The diagrams show the effect of series-connected small and large values of inductance and capacitance on low-, medium-, and high-frequency voltages.



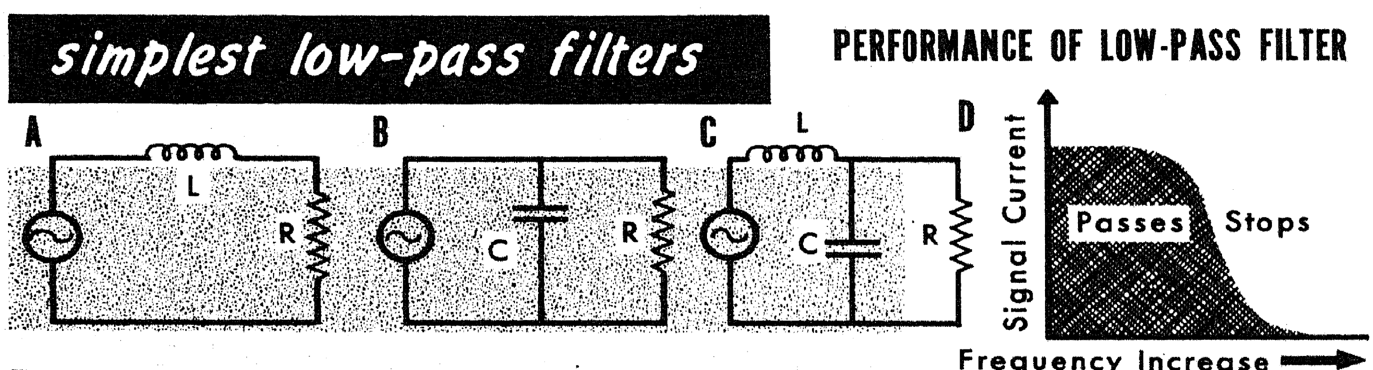
The higher the frequency of the applied voltage for any given value of L, the larger will be the voltage drop across L; hence, the less the signal voltage available at the output. The higher the frequency for any given value of C, the less will be the voltage drop across C and the higher the available signal voltage at the output.

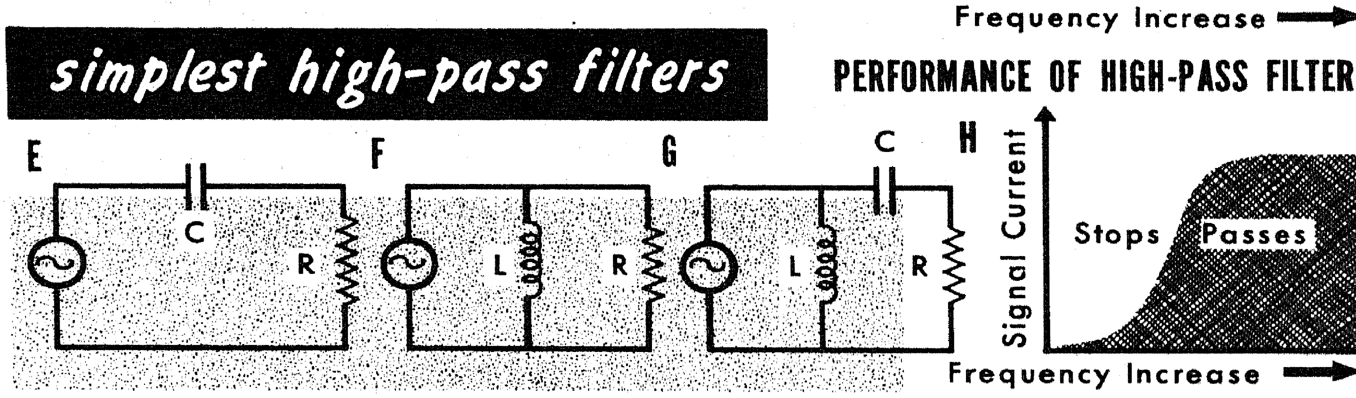
L and C are used in various ways in different kinds of filters. Sometimes they form resonant circuits; sometimes they form networks which will pass a wide band of frequencies and reject others, or reject a wide band of frequencies and pass others. Examples appear on the following pages, with more detailed discussions in subsequent volumes dealing with receivers and transmitters.

Low-Pass and High-Pass Filters

A filter circuit that passes all the low frequencies below a particular frequency, and rejects all higher frequencies, is called a low-pass filter. A filter circuit that passes all the high frequencies above a particular frequency and rejects the lower frequencies is called a high-pass filter.

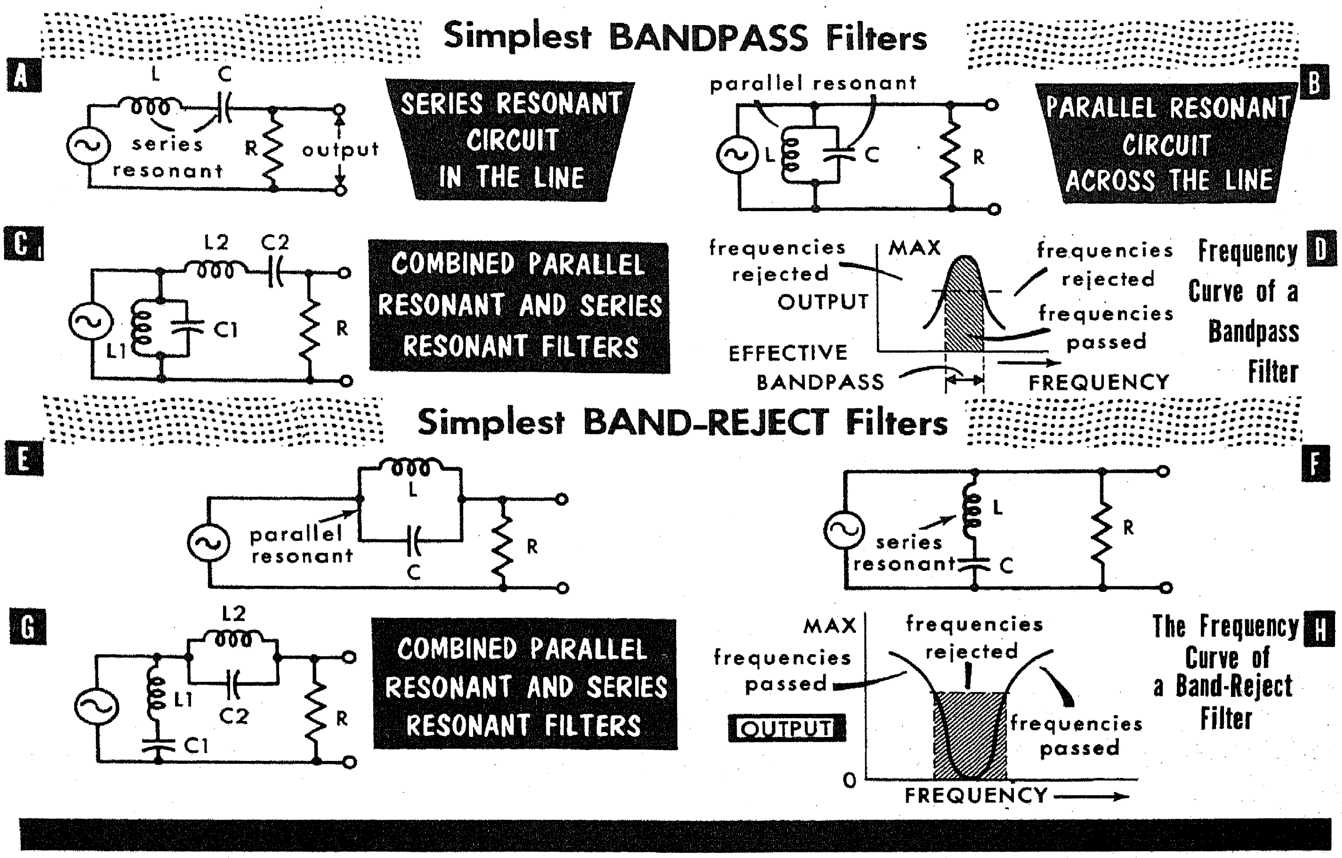
The simplest form of low-pass filter is an inductor in series with the line as in A, or a capacitor connected in parallel with the line as in B. The inductor presents a low X_L at the low frequencies but a high X_L at the high frequencies. The capacitor presents a high X_C at the low frequencies and progressively less X_C as the frequency increases. When L and C are combined, they form a low-pass filter as shown in C, with their performance curve shown in D. The low reactance of L at low frequencies provides an easy path for the signal. At the same time, the shunt capacitance presents a high impedance to the low-frequency signal currents; therefore, very little is lost across C. At high frequencies, the high reactance of L presents increasing opposition to the flow of signal currents. At the same time, the progressively decreasing reactance of C at high frequencies offers an easy bypass path for the currents.





The simplest form of high-pass filter would be either C in series with the line (as in E) or an inductor across the line (as in F). When combined (G), the high reactance of C at low frequencies offers high opposition to their path. The low reactance of L at low frequencies will effectively bypass them. At high frequencies, the reactance of C is low, and it readily passes these signal currents. On the other hand, the increasing reactance with increasing frequency of L minimizes signal-current bypass through the coil. The net result is to pass all the higher frequencies readily to the load, but to reject the lower frequencies (H).

Bandpass and Band-Reject Filters



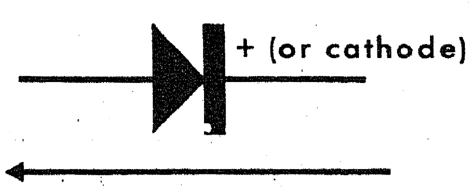
Bandpass filters permit passage of a narrow band of frequencies while rejecting all other undesired frequencies. The simplest form of bandpass filter would be a series resonant circuit in the line as shown in A. Another simple bandpass filter using a parallel resonant circuit across the line is shown in B. Combining the two as in C provides a more effective bandpass filter. The series resonant circuit offers low impedance and readily passes the desired frequencies, while offering high impedance and blocking undesired frequencies. The parallel resonant circuit offers high impedance to the desired band of frequencies, preventing any bypass action; the undesired frequencies find the parallel resonant circuit a low-impedance path and are effectively bypassed through it. The characteristic curve of a bandpass filter is shown in D.

Band-reject filters are used to block the passage of a narrow band of frequencies while passing all other frequencies. The simplest form of band-reject filter would be a parallel resonant circuit in the line as shown in E. A simple band-reject filter using a series resonant circuit across the line is shown in F. Combining the two as in G provides an effective band-reject filter. The parallel resonant circuit offers high impedance to the desired band of frequencies to be rejected, while offering a low-impedance path to all other frequencies. The series resonant circuit across the line offers a low impedance bypass path to the band of frequencies to be rejected, while offering a high-impedance path to all other frequencies. The characteristics curve of a band-reject filter is shown in H.

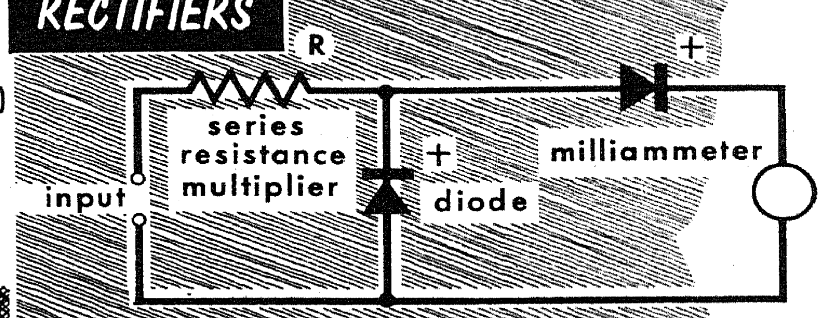
Characteristics and Operation

The D'Arsonval moving-coil meter used for measuring voltages and currents in Volume I is basically a d-c meter. If we apply a-c to it, one half-wave tries to make it read in the normal way; the other half-wave tries to make it read backward. As the meter pointer does not have time to move back and forth so rapidly, it either stands still or vibrates rapidly around zero. However, the D'Arsonval movement can be used to measure a-c if we first change the a-c to d-c. This can be done through the use of rectifiers or diodes. These are electrical devices that have a special characteristic — they permit current to flow through them in one direction (low resistance), but not in the other (high resistance). While many kinds of rectifiers can be used, a-c ammeters and voltmeters most often use the copper-oxide rectifier. Adding a rectifier circuit to the D'Arsonval movement gives us an a-c meter.

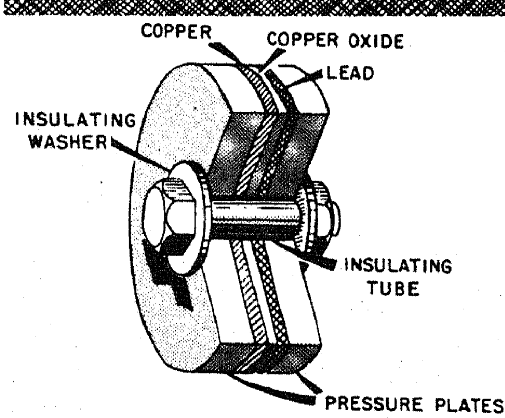
Symbol for Rectifier



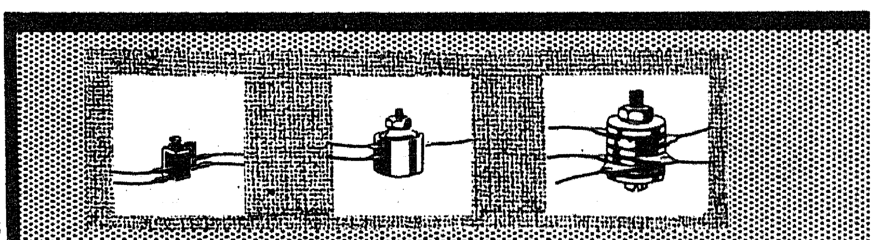
RECTIFIERS



Half-Wave — the Simplest Rectifier Meter



Copper-Oxide Type Dry-Plate Rectifier



Typical Copper-Oxide Instrument Rectifiers

Copper-oxide rectifiers generally provide good rectification for a-c up to about 20,000 cycles. They are constructed of a series of copper discs clamped together flat in a stack. On one side of each disc is a coating of copper oxide which forms a layer of this material between adjacent discs. Washers made of lead are clamped against each oxide surface to improve the contact, and the complete assembly is held together by an insulated bolt running through the center. Two types of rectifier circuits are used in meters to a great extent — the half-wave and full-wave types.

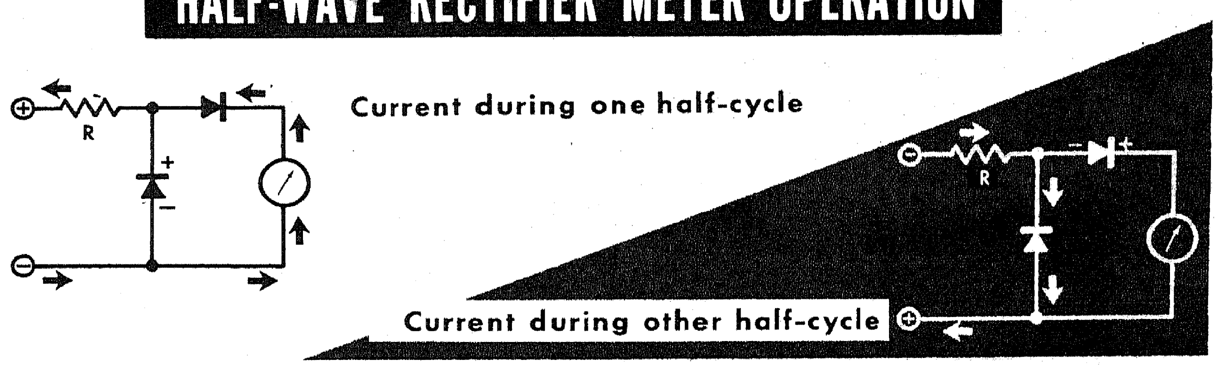
Characteristics and Operation (Cont'd)

In the half-wave type (which uses two rectifiers), one half-wave is bypassed through one rectifier, while the other half goes through the meter. The meter pointer will not have time to follow the fluctuations, so it will average out the current that flows through it.

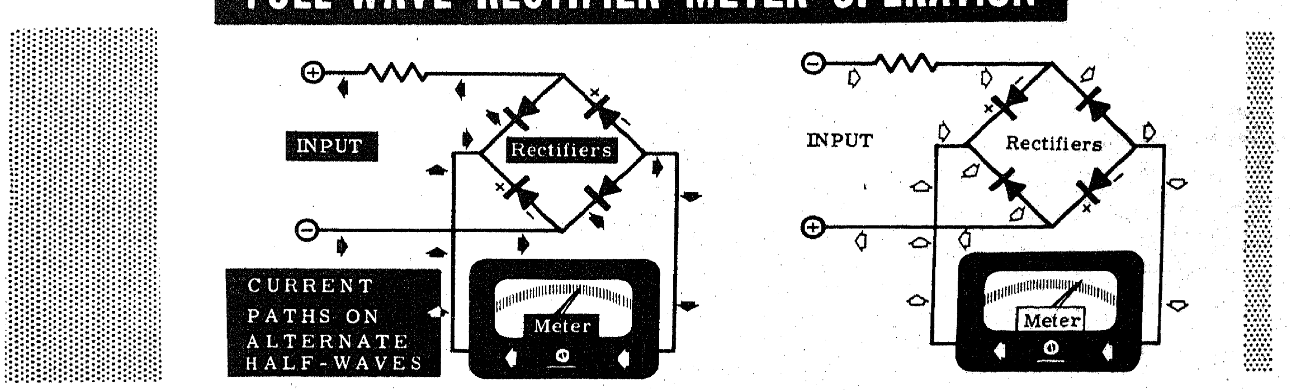
During one half-wave, no current flows through the meter, while during the other half-wave, it follows half a sine wave in form. The average of half a sine wave is 0.637 of peak value. However, during half the time, no current goes through the meter; therefore, the average over the whole time will be half of 0.637 or 0.3185. (If an ordinary d-c meter movement is used in this circuit to measure an alternating voltage, a 1-volt peak – 2 volts peak to peak – voltage will give a reading of only 0.3185 volt.) In the full-wave bridge-type rectifier, the meter gets both halves of the wave, and it will read 0.637 of the peak voltage if a regular d-c meter is used. To provide practical a-c voltage scales, multiplier resistors must be used in series with the meter movement and the rectifier, with appropriate shunts connected across the meter for current measurements.

The electrodynamometer-type wattmeter discussed in Volume I is equally useful for the measurement of a-c power. Making use of the voltage across a circuit as well as the current flow through it, this type wattmeter is ideal for measuring the actual or true power in an a-c circuit. Since the torque on the moving coil is proportional to the applied power, the "cosine θ," or phase angle between the current and voltage, is automatically taken into consideration.

HALF-WAVE RECTIFIER METER OPERATION



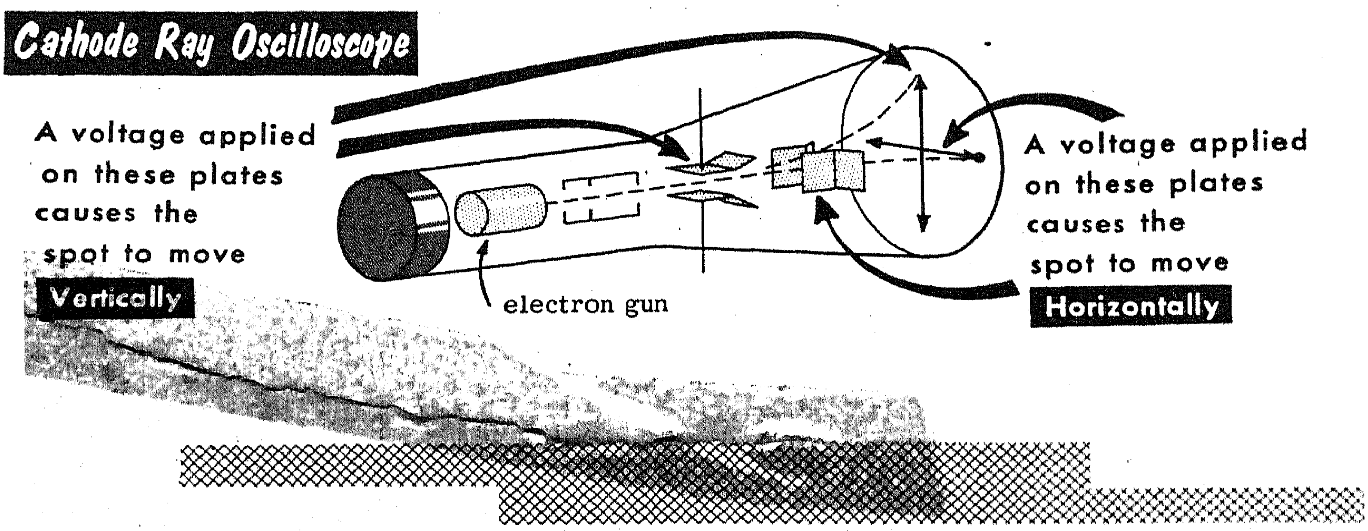
FULL-WAVE RECTIFIER METER OPERATION

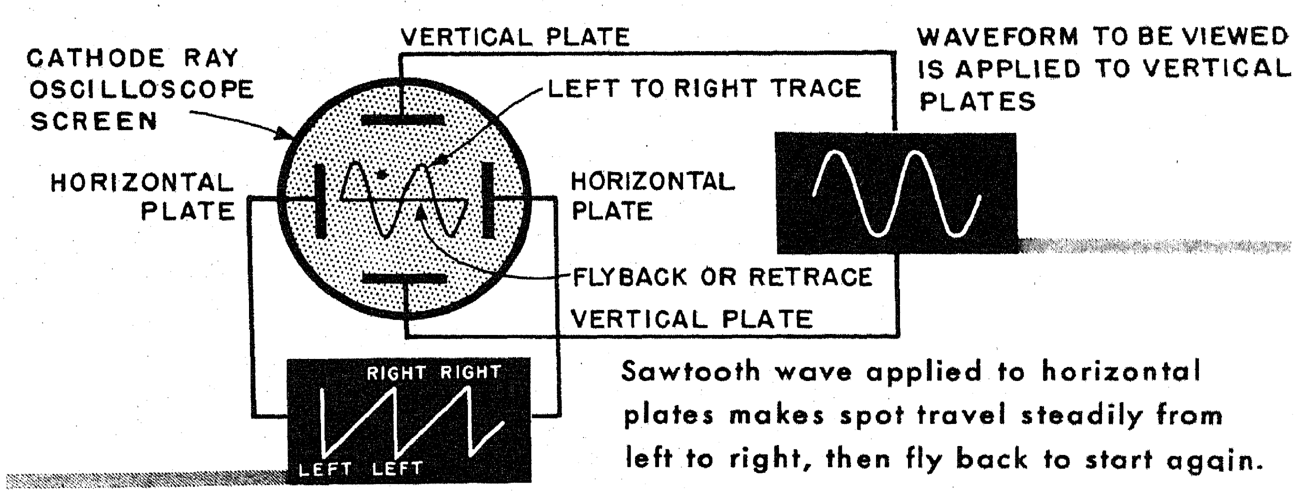


Measuring Waveforms

Waveforms can actually be examined by means of an instrument called the "oscilloscope." It uses a special tube in which a beam, or "pencil" of electrons, is focused to a point on a fluorescent screen that glows with the impact of the electrons. Two pairs of deflecting plates bend the beam in accordance with the voltages applied to them.

Applying different voltages to the plates at the sides of the beam will move the spot formed on the fluorescent screen sideways, and voltages applied to the upper and lower plates will deflect the beam up or down. If different fluctuating voltages are applied to both pairs of plates, the spot will trace a pattern on the screen representing the combined effect of the two voltage fluctuations.





If the fluctuation applied to the horizontal plates follows a "sawtooth" waveform, the spot will move steadily from left to right across the screen, and then rapidly return to its starting point. By using this waveform as a "time-base," in this way, the voltage applied to the vertical plates traces its own waveform. In this way, we can view electrical waveshapes and, by use of a scale with the scope, be able to measure actual values of voltages in circuits under examination.

In a parallel resonant circuit, the branch currents cancel, the line current is a minimum, impedance is a maximum, and the phase angle is 0°.

In the parallel L-C circuit, at frequencies above resonance, X_L is greater than X_C, I_C is greater than I_L, and the circuit acts capacitively; at frequencies below resonance, X_C is greater than X_L, I_L is greater than I_C, and the circuit acts inductively.

The Q of a parallel resonant circuit is the ratio of the current in the tank (I_L or I_C) to the current in the line.

A parallel circuit is resonant if $X_L = X_C$ and $I_L = I_C$.

The parallel resonant circuit may be used as a bandpass or band-rejection circuit.

In a parallel L-C circuit, the branch currents are 180° out of phase and can be subtracted directly: $I_t = I_L - I_C$.

The current in either branch of a parallel L-C circuit may be greater than the line current.

A tank circuit can be used to simulate the properties of either a capacitor or an inductor.

In a parallel L-C circuit, the closer X_L and X_C are to equality, the higher the circuit impedance Z_t and the lower the line current I_t.

In the parallel resonant circuit, the circulating current is the same everywhere in the L and in the C circuit.

Circulating current in a parallel resonant L-C circuit is an interchange of energy between the inductance and the capacitance.

The higher the Q of a parallel resonant L-C circuit, the narrower the bandpass.

Bandpass filters are used to permit the passage of a narrow band of frequencies while rejecting all other undesired frequencies; band-reject filters are used to block the passage of a narrow band of frequencies while passing all other frequencies.

A-C meters use the same D'Arsonval movement as is used in d-c meters, except that a rectifier circuit is added to convert a-c to d-c.

The most commonly used rectifier in a-c ammeters and voltmeters is the copper-oxide rectifier, which provides rectification up to about 20 kc.

REVIEW QUESTIONS

1. What is a parallel resonant circuit?
2. Give the conditions present in a parallel resonant circuit.
3. Describe the circulating current in a parallel resonant L-C circuit.
4. What is the formula for the impedance of a parallel resonant circuit?
5. What is a tank circuit and why is it so named?
6. How does a parallel L-C circuit act at frequencies above and below the resonant frequency? Why?
7. In a parallel resonant circuit, what relationship exists among the branch currents, the line current, and the impedance?
8. What is the nature of the impedance of the tank circuit at resonance?
9. In a parallel L-C circuit, what is the phase relationship between the branch currents?
10. Define bandpass and band-rejection filter circuits.
11. Basically, how do a-c meters differ from d-c meters?
12. What two types of rectifier circuits are principally used in meters?

NATURAL TRIGONOMETRIC FUNCTIONS

Angle	*Sine*	*Cosine*	*Tangent*	*Angle*	*Sine*	*Cosine*	*Tangent*
0°	0.000	1.000	0.000				
1°	.018	1.000	.018	46°	.719	.695	1.036
2°	.035	0.999	.035	47°	.731	.682	1.072
3°	.052	.999	.052	48°	.743	.669	1.111
4°	.070	.998	.070	49°	.755	.656	1.150
5°	.087	.996	.088	50°	.766	.643	1.192
6°	.105	.995	.105	51°	.777	.629	1.235
7°	.122	.993	.123	52°	.788	.616	1.280
8°	.139	.990	.141	53°	.799	.602	1.327
9°	.156	.988	.158	54°	.809	.588	1.376
10°	.174	.985	.176	55°	.819	.574	1.428
11°	.191	.982	.194	56°	.829	.559	1.483
12°	.208	.978	.213	57°	.839	.545	1.540
13°	.225	.974	.231	58°	.848	.530	1.600
14°	.242	.970	.249	59°	.857	.515	1.664
15°	.259	.966	.268	60°	.866	.500	1.732
16°	.276	.961	.287	61°	.875	.485	1.804
17°	.292	.956	.306	62°	.883	.470	1.881
18°	.309	.951	.325	63°	.891	.454	1.963
19°	.326	.946	.344	64°	.899	.438	2.050
20°	.342	.940	.364	65°	.906	.423	2.145
21°	.358	.934	.384	66°	.914	.407	2.246
22°	.375	.927	.404	67°	.921	.391	2.356
23°	.391	.921	.425	68°	.927	.375	2.475
24°	.407	.914	.445	69°	.934	.358	2.605
25°	.423	.906	.466	70°	.940	.342	2.747
26°	.438	.899	.488	71°	.946	.326	2.904
27°	.454	.891	.510	72°	.951	.309	3.078
28°	.470	.883	.532	73°	.956	.292	3.271
29°	.485	.875	.554	74°	.961	.276	3.487
30°	.500	.866	.577	75°	.966	.259	3.732
31°	.515	.857	.601	76°	.970	.242	4.011
32°	.530	.848	.625	77°	.974	.225	4.331
33°	.545	.839	.649	78°	.978	.208	4.705
34°	.559	.829	.675	79°	.982	.191	5.145
35°	.574	.819	.700	80°	.985	.174	5.671
36°	.588	.809	.727	81°	.988	.156	6.314
37°	.602	.799	.754	82°	.990	.139	7.115
38°	.616	.788	.781	83°	.993	.122	8.144
39°	.629	.777	.810	84°	.995	.105	9.514
40°	.643	.766	.839	85°	.996	.087	11.43
41°	.656	.755	.869	86°	.998	.070	14.30
42°	.669	.743	.900	87°	.999	.052	19.08
43°	.682	.731	.933	88°	.999	.035	28.64
44°	.695	.719	.966	89°	1.000	.018	57.29
45°	.707	.707	.000	90°	1.000	.000	∞

GLOSSARY

Alternating Current (a-c): Electric current which moves first in one direction for a fixed period of time and then in the opposite direction for the same period of time. Ac changes in value continuously and reverses direction at regular intervals.

Ampere-Turns: The unit of magnetomotive force. Equal to the number of amperes of current flowing in a winding, multiplied by the number of turns in the winding.

Autotransformer: A transformer in which part of the primary winding serves as the secondary or in which part of the secondary winding is also in the primary. It has good voltage regulation under varying load conditions.

Bandpass Filter Circuit: A filter circuit which passes a desired narrow band of frequencies while rejecting all other undesired frequencies.

Bandwidth: The number of cycles that receive approximately the same amplification in an amplifier.

Band-Rejection Filter Circuit: A filter circuit which rejects a desired narrow band of frequencies while passing all other desired frequencies.

Capacitance (C): That property of an electric circuit which tends to oppose a change in voltage.

Capacitive Reactance (X_C): The opposition offered by a capacitance to alternating current. Measured in ohms. $X_c = 1/(2\pi fC)$.

Capacitor: Any two conductors separated by a dielectric.

Copper-Oxide Rectifier: A rectifier made up of discs of copper, coated on one side with cuprous oxide. Allows current flow in one direction and opposes current flow in the other direction.

Counter EMF: A voltage produced by a changing current and which at every instant opposes the change of current that produces the voltage.

Dielectric: Any insulating or nonconducting material. Air, mica, glass, paper, oil, and rubber are common dielectrics.

Dielectric Constant: The ratio of the ability of a given material to establish electric lines of force between two conductors, as compared to dry air.

Distributed Capacitance: Stray or random capacitance that exists between connecting wires, between components located physically near to each other, and between different parts of a given component.

Eddy Currents: Small circulating currents (power losses) set up by the induced voltage in any conductor carrying alternating currents.

Electrolytic Capacitor: A type of fixed capacitor which shows polarity, and is used principally in relatively low-frequency filter circuits at voltages up to 600 volts.

Electromagnet: A coil of wire, usually wound, or a soft-iron core, which produces a strong magnetic field when current is sent through the coil.

Farad: The unit of measurement of capacitance. One million microfarads (μf) equals one farad.

Frequency: The number of complete cycles per second that an alternating current undergoes.

Galvanometer: A sensitive instrument used to measure small voltages and currents.

Henry: The unit of measurement of inductance. A thousand millihenries (mh) equals one henry. A million microhenries (μh) equals one henry.

High-Pass Filter: A type of filter which offers little opposition to the passage of high frequencies, and high opposition to the passage of low frequencies.

Hysteresis Losses: Energy lost in the core of a transformer by the constant reversing of the alternating current.

Impedance (Z): Opposition to the flow of alternating current that results from any combination of resistance, inductive reactance, and capacitive reactance, or any two of these factors.

Induced EMF: A voltage produced when a current-carrying conductor is moved through a magnetic field and cuts across the lines of force, or when the magnetic field is moved across the conductor.

Inductance (L): That property of an electric circuit or component which opposes any change in current.

Inductive Reactance (X_L): The opposition offered by an inductance to alternating current. Measured in ohms. ($X_L = 2\pi fL$).

Kirchhoff's Current Law: States that the sum of all the currents flowing to a point in a circuit must be equal to the sum of all the currents flowing away from that point.

Kirchhoff's Voltage Law: The sum of all the voltage drops around a closed circuit is equal to the applied voltage.

Left-Hand Rule for Motors: A means of showing the relative directions of magnetic field flux, current flow in a conductor, and motion of the conductor through the field.

Low-Pass Filter: A type of filter that offers little opposition to the passage of low frequencies, and high opposition to the passage of high frequencies.

Mutual Induction: Production of an alternating voltage that occurs when two coils are placed close to one another in such a manner that the magnetic flux set up by one coil links the turns of the other coil.

Parallel-Resonant Circuit: A circuit in which an inductor and a capacitor are connected in parallel and have such values that at the resonant frequency the inductive reactance and the capacitive reactance are equal. Line current is at a minimum.

Peak Voltage: The highest instantaneous voltage attained in a circuit in a given period of time. Equal to 1.414 times the rms value.

Peak-to-Peak Voltage: For any alternating waveform, the total potential difference between maximum voltage amplitudes of opposite polarities.

Phase: The time difference between any point on a cycle and the beginning of that cycle.

Phase Difference: The time difference between any two cycles.

Q: A measure of the "quality" of a circuit. Varies inversely with the resistance of the circuit. Equal to X_L/R.

Resonant Frequency: The single frequency at which $X_L = X_C$ in a circuit.

Right-Hand Rule for Generators: A means of showing the relative directions of magnetic field flux, motion of a conductor through the field, and of the current induced in the conductor.

RMS (Root-Mean-Square) Value: The effective value of an alternating voltage or current. Equal to 0.707 of maximum or peak value. Corresponds to the equivalent d-c value which produces the same heating effect.

Series-Resonant Circuit: A circuit in which an inductor and a capacitor are connected in series and have such values that at the resonant frequency the inductive reactance and the capacitive reactance are equal. Current is at a maximum.

Skin Effect: The name given to the tendency of high-frequency (r-f) currents to concentrate at the surface of a conductor. Caused by counter-emf's induced in the center of a conductor carrying high-frequency currents which forces them to travel at the surface.

Step-Down Transformer: One in which the voltage induced in the secondary is less than that applied to the primary.

Step-Up Transformer: One in which the voltage induced in the secondary is greater than that applied to the primary.

Tank Circuit: Any resonant circuit (usually applied to parallel circuits).

Transformer: A device which by electromagnetic induction converts an a-c input voltage higher or lower than the input voltage.

Turns Ratio: A comparison of the number of turns in the primary winding of a transformer to the number of turns in the secondary winding.

INDEX TO VOL. II

(Note: A cumulative index covering all six volumes in this series is included at the end of Volume VI.)

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